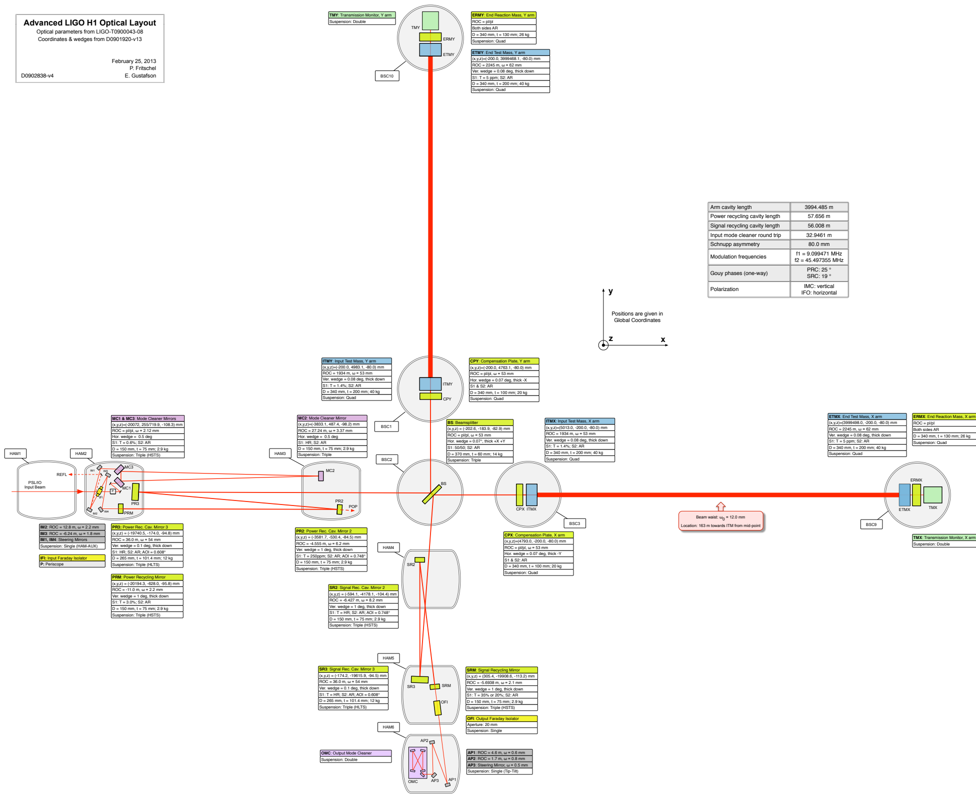
Mirror quality and interferometer performance

Hiro Yamamoto - Caltech LIGO

- Introduction
- Gaussian beam and Modal model
- Cavity basic
- PSD, BRDF and loss
- Mirror polishing and coating
- Beam splitter
- Thermal effects
- Stable cavity

Introduction
idealized vs reality



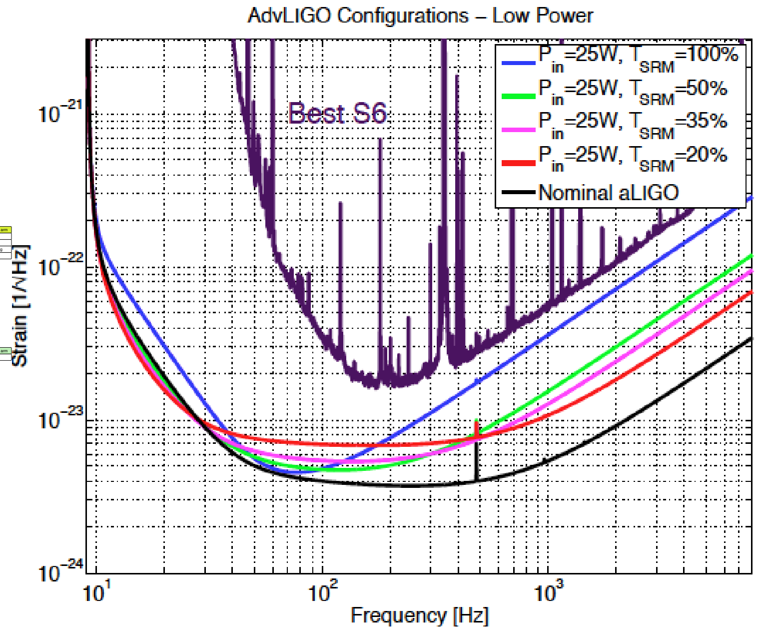
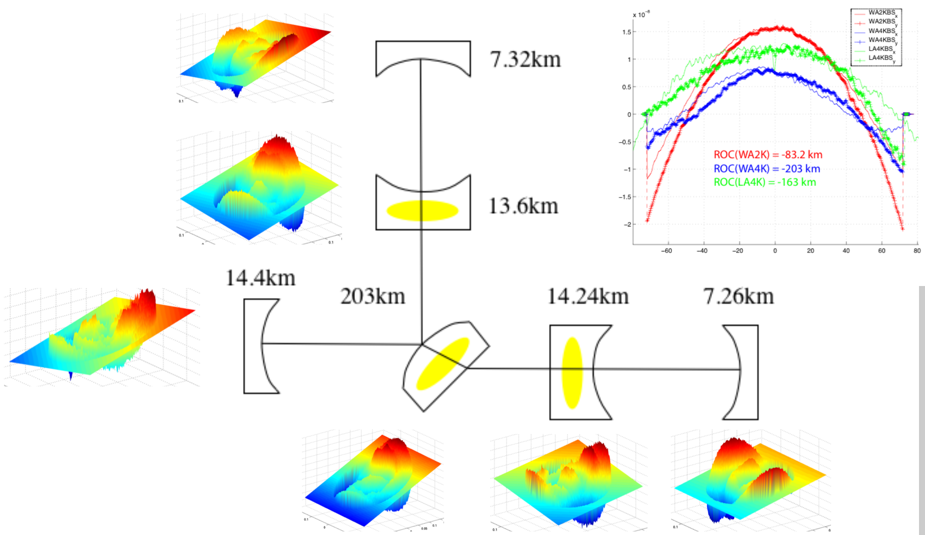
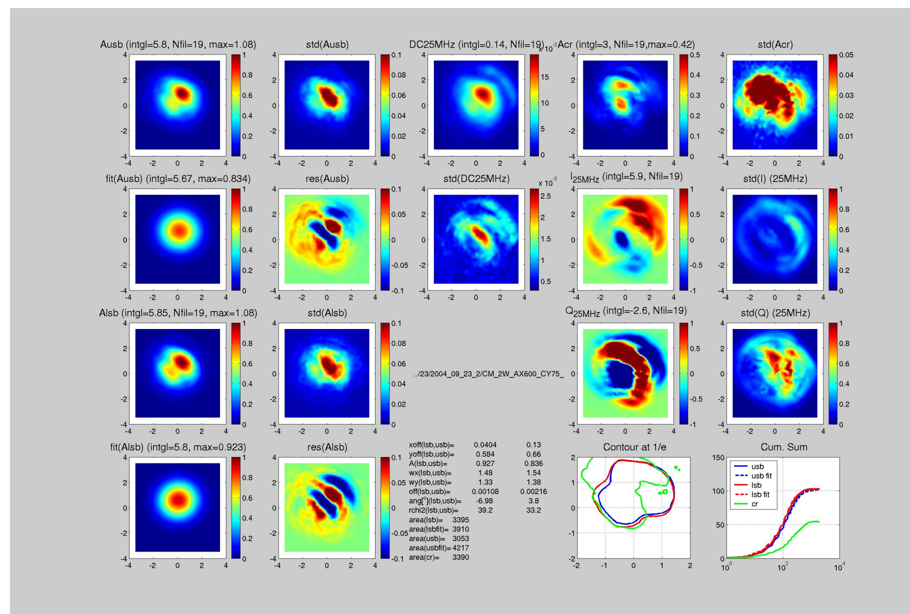


Figure 2: Possible configurations of aLIGO in the early commissioning phase, with 25W of input power. As reference, the nominal aLIGO curves and the best S6 sensitivity are also included.

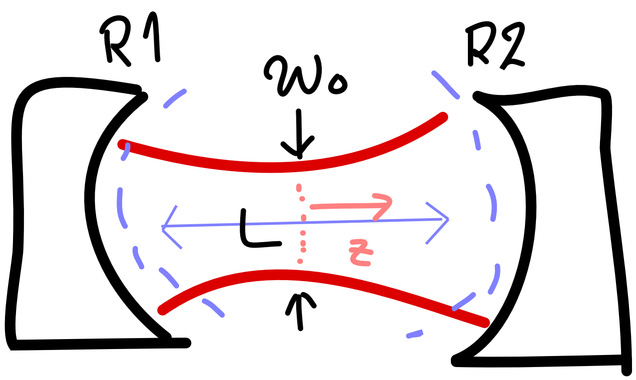
Initial LIGO optics and Fields idealized vs reality





Gaussian beam and Modal model

- Gaussian beam : stationary state in a two mirror cavity (FP)



$$G_{00}(x,y,z,t) = G_{00}(x,y,z)\exp[i(\omega \cdot t - k \cdot z)]$$

$$G_{00}(x,y,z) = \sqrt{\frac{2}{\pi}}\frac{1}{w(z)}\exp(-r^2(\frac{1}{w(z)^2} + i\frac{k}{2R(z)}) + i \cdot \eta(z))$$

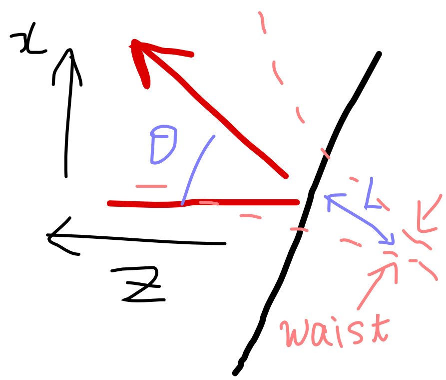
$$w(z)^2 = w_0^2(1+\frac{z^2}{z_0^2}),\ \ R(z) = z + \frac{z_0^2}{z},\ \ \eta(z) = a\tan(\frac{z}{z_0})$$

$$HG_{mn} = G_{00}(x,y,z,t)\sqrt{\frac{1}{2^{m+n}m!n!}}H_m(\frac{\sqrt{2}x}{w(z)})H_n(\frac{\sqrt{2}y}{w(z)})\exp[i(m+n)\eta(z)]$$

$$LG_{pm} = G_{00}(x,y,z,t)\sqrt{\frac{p!}{(p+|m|)!}}\exp(im\varphi)L_p^{|m|}(\frac{2r^2}{w(z)^2})\exp[i(2p+|m|)\eta(z)]$$

Gaussian beam and Modal model

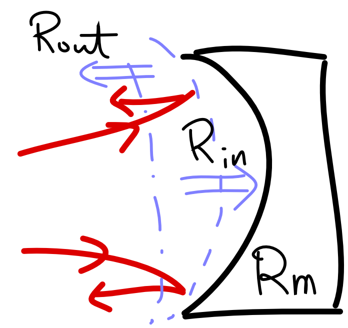
- Tilt



L is the distance to waist from the reflection point

$$E_{ref}(\theta) = G_{00}(\theta=0)\cdot \exp(i\omega t - i(x\cdot k_x + z\cdot k_z))$$
$$\sim G_{00}\cdot(1 - i\ \mathrm{x}\cdot\mathrm{k}\cdot\theta)$$
$$= \mathrm{G}_{00}\cdot(1 - i\cdot\frac{1}{\sqrt{2}}H_1(\frac{\sqrt{2}x}{w(z)})\cdot\frac{\theta}{\Theta(L)}exp[i(\eta(z)-\eta(L))])$$
$$= G_{00} - i\frac{\theta}{\Theta(L)}\cdot G_{10}$$
$$\Theta(z) = \frac{1}{\pi}\frac{\lambda}{w(z)},\quad \mathrm{H}_1(x) = 2x$$

- curvature mismatch



$$E_{ref}(\delta R) = G_{00}(R=R_{in})\cdot Exp[-ikr^2(-\frac{2}{2R_m})]$$
$$= G_{00}(R=\infty)\cdot Exp[ikr^2(\frac{1}{2R_{in}} - (\frac{1}{R_{in}} - \frac{1}{R_m}))]$$
$$\approx G_{00}(R=\infty)\cdot Exp[ir^2(\frac{1}{2R_{in}})]\cdot(1 - ikr^2\frac{\delta R}{R_{in}^2})$$
$$= G_{00}(R=-R_{in})(1 - ik\frac{w^2}{2}\frac{\delta R}{R_{in}^2}(1 - L_1^0(\frac{2r^2}{w^2})))$$
$$\approx G_{00}(R=-R_{in}) + i\pi\frac{w^2}{\lambda R_{in}}\frac{\delta R}{R_{in}}LG_1^0(R=-R_{in})$$
$$\delta R = R_m - R_{in},\quad L_1^0(r) = 1 - r$$

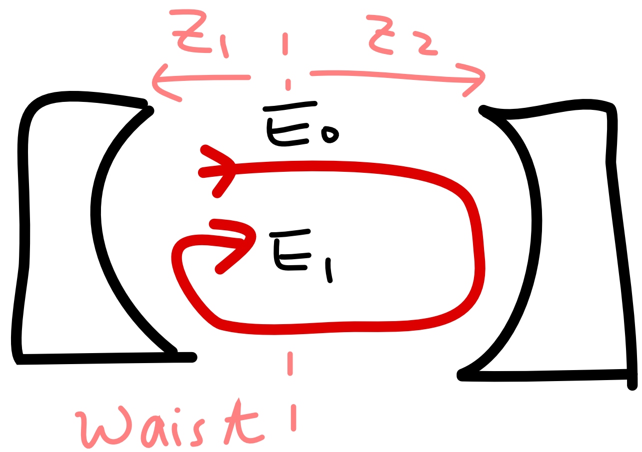
Gaussian beam and Modal model

- Cavity field and Gouy phase

$$E_1 = E_0 \cdot r_1 r_2 \cdot \exp[i2\phi]$$

$$\phi = \left\{ \begin{array}{c} m+n+1 \\ 2p+|m|+1 \end{array} \right\} \Delta\eta - kL$$

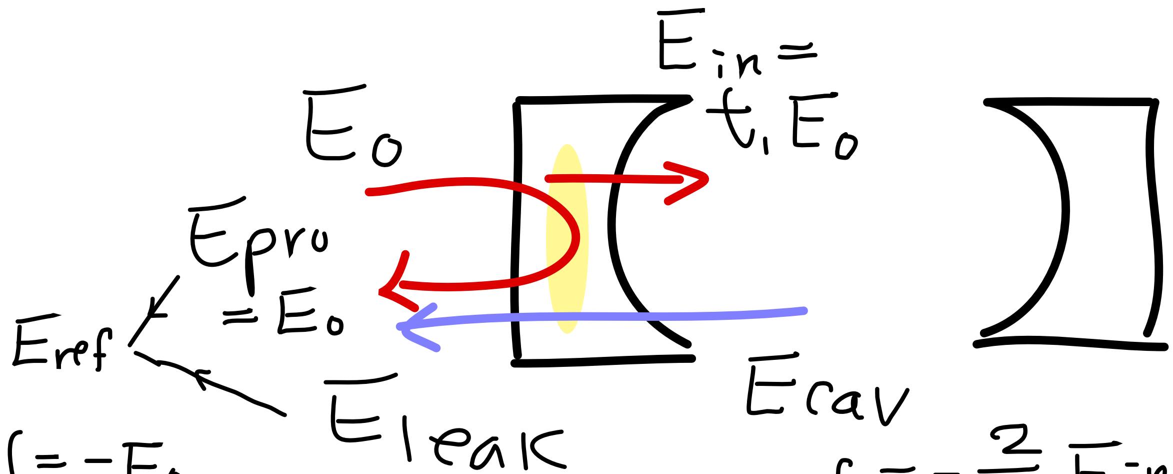
$$\Delta\eta = \eta(z_2) - \eta(-z_1) = a\cos(1 - \frac{L}{R}) \text{ for } R_1 = R_2 = R$$



- Resonance condition
  - » ϕ=nπ for main mode
  - » non resonant for other modes

Cavity basic

Resonant vs non-resonant



$$E_{ref} \begin{cases} = -E_0 \\ = +E_0 \end{cases}$$

$$E_{leak} = t_1 E_{cav} \begin{cases} = -2E_0 \\ \sim t_1^2 E_0 \end{cases}$$

$$E_{cav} \begin{cases} = -\frac{2}{t_1^2} E_{in} & \text{resonant} \\ \sim E_{in} & \text{non-res} \end{cases}$$

Mode of the reflected field when there is a lens in ITM with focal length of f_{lens}

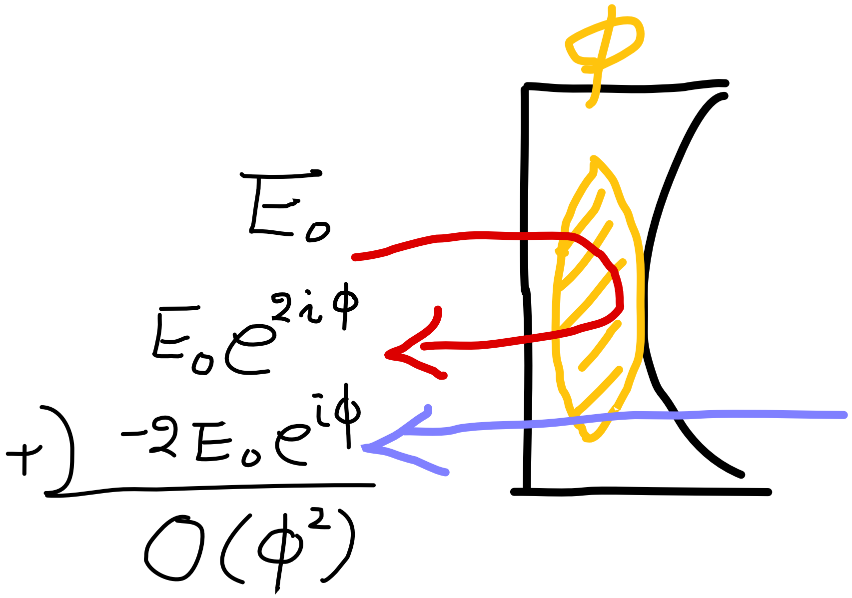
$$E_{non-resonant} = \frac{1}{1+i\alpha} E_{00} - \frac{i\alpha}{\sqrt{2}(1+i\alpha)^3}(E_{20} + E_{02}) + O(\alpha^2)$$

$$E_{resonant} = -\frac{1}{1+i\alpha} E_{00} + O(\alpha^2)$$

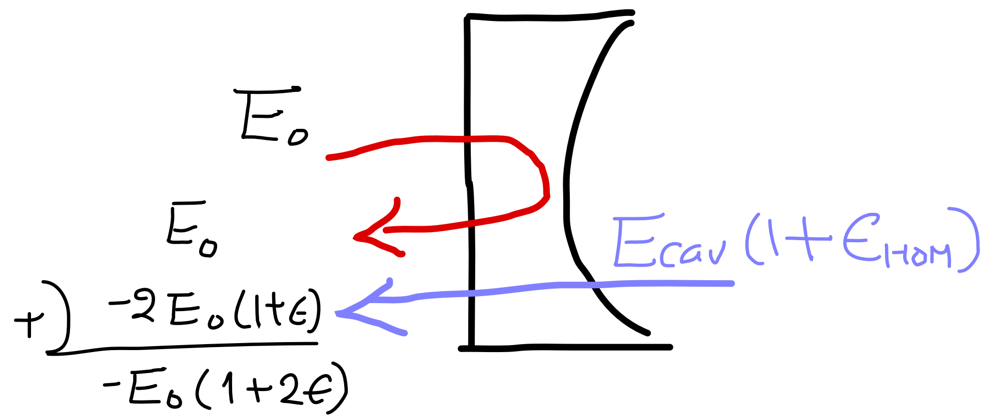
$$\alpha = -\frac{k \cdot w^2}{4 \cdot f_{lens}}$$

Phase cancelation and noise enhancement

Phase cancelation



Noise enhancement



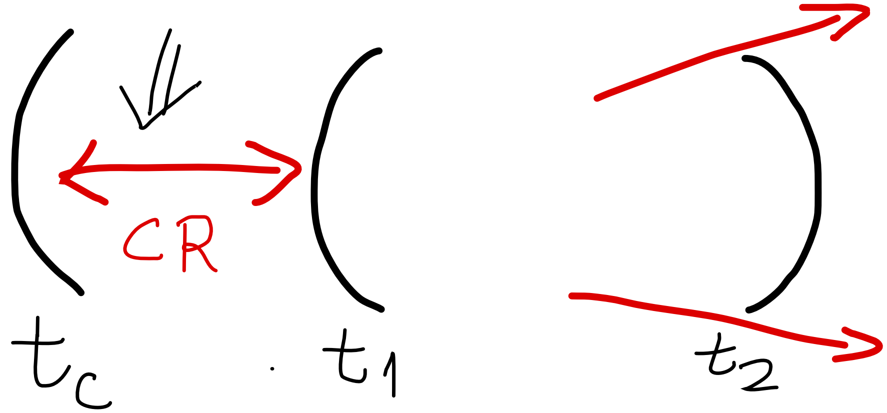
Only carrier (injected field which resonate in the cavity)
not sideband (not resonating in the cavity)
nor signal sideband (induced in the cavity)

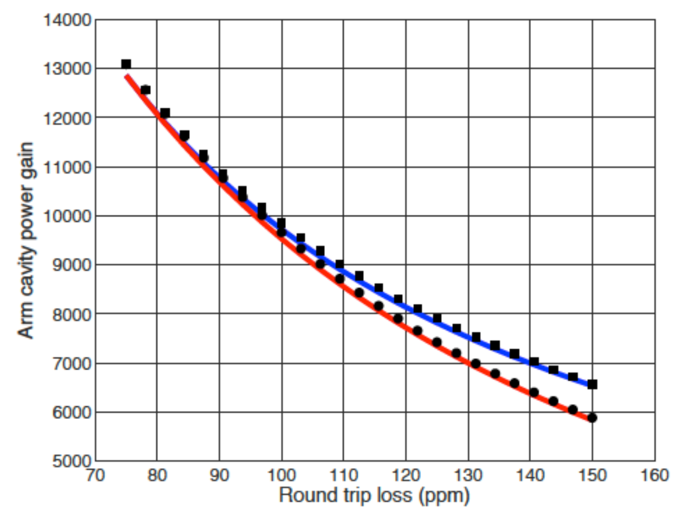
Coupled cavity

$$T_2 = t_2^2 + \text{loss in the arm}$$

$$-\frac{2}{tc} \Big/ \left\{\left(1 + \frac{Tc}{4}\right)\left(1 + \frac{4\,T2}{T1\,Tc}\right)\right\}$$

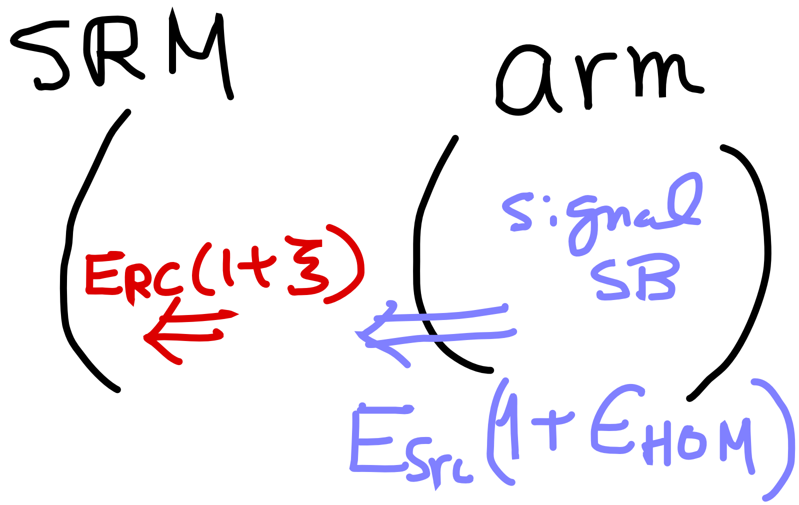
$$T_1 \cdot T_c = 5\times10^{-4} = 500\text{ppm}$$





Cavity basic

Mode healing



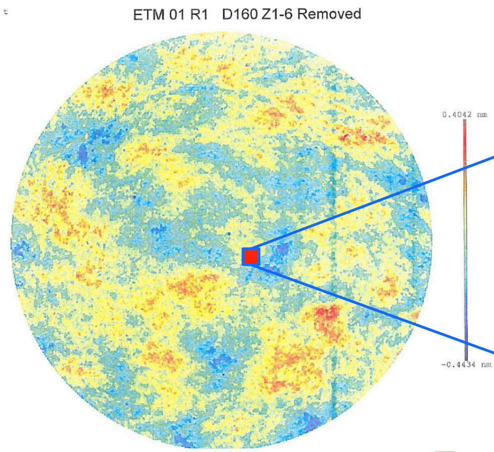
$$\frac{E_{RC}}{E_{scr}} = A$$

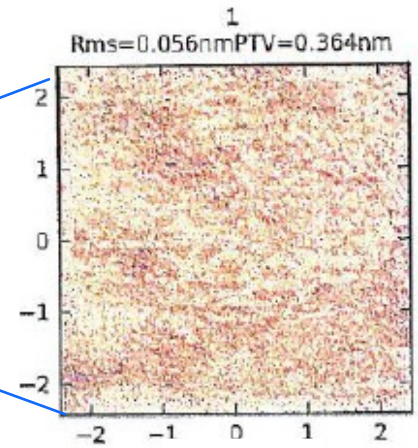
$$E_{RC}\cdot\xi \sim E_{SRC}\cdot\epsilon$$

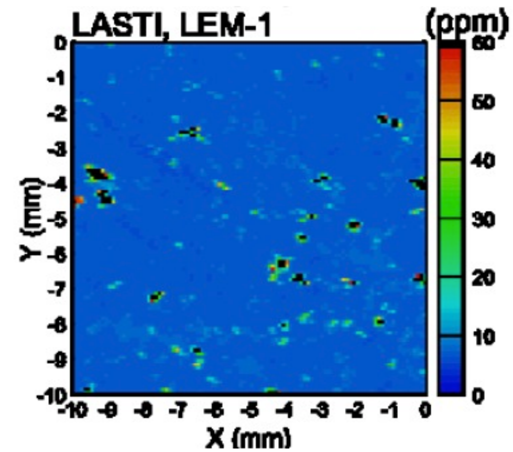
$$\frac{\xi}{\epsilon} = \frac{E_{SRC}}{E_{RC}} = \frac{1}{A}$$

Recycling gain ~ 5

Surface structure with different spatial distribution



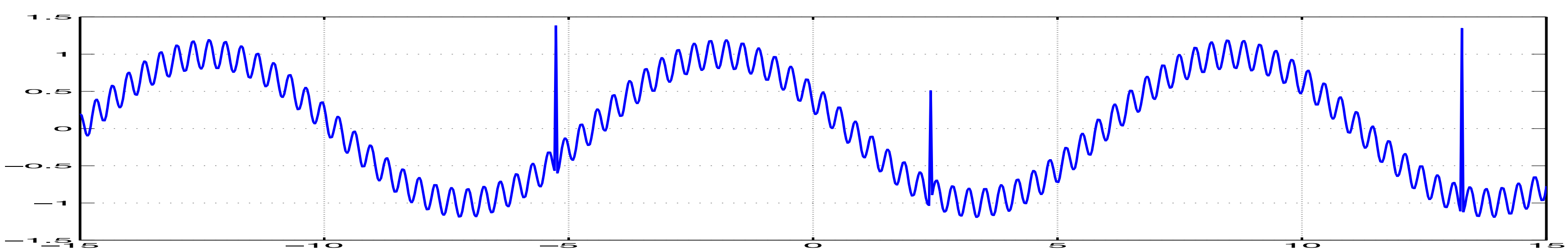




160mm — Fizeau IFO

5mm — Phase Measuring Microscope

10mm — Integrating Sphere

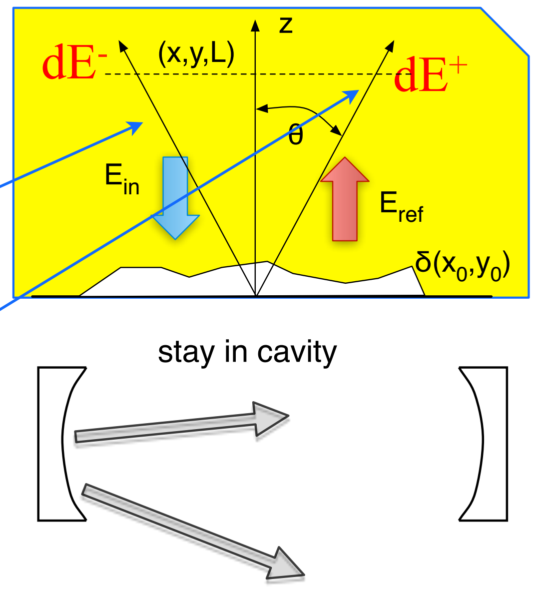


Scattering by aberration

$$E_{ref} = E_{ref}^{0} \cdot \exp(i2k\delta(x,y))$$
$$= E_{ref}^{0} \cdot (1 + i2k\delta - 2(k\delta)^2)$$
$$= E_{ref}^{0} \cdot (1 - 2(k\delta)^2) + E_{ref}^{0} \cdot i2k\delta$$

$$dP = \iint dx\,dy \left|E_{ref}^{0}\right|^2 4k^2 \delta(x,y)^2$$
$$= P_{ref}^{0} \left(\frac{4\pi\sigma}{\lambda}\right)^2 S$$
$$\sigma^2 \equiv \iint dx\,dy\,\delta(x,y)^2 / S$$
$$= \int df\, PSD_{1D}(f)$$

$$\theta = k_x / k = \lambda_{laser} / \lambda_{space}$$



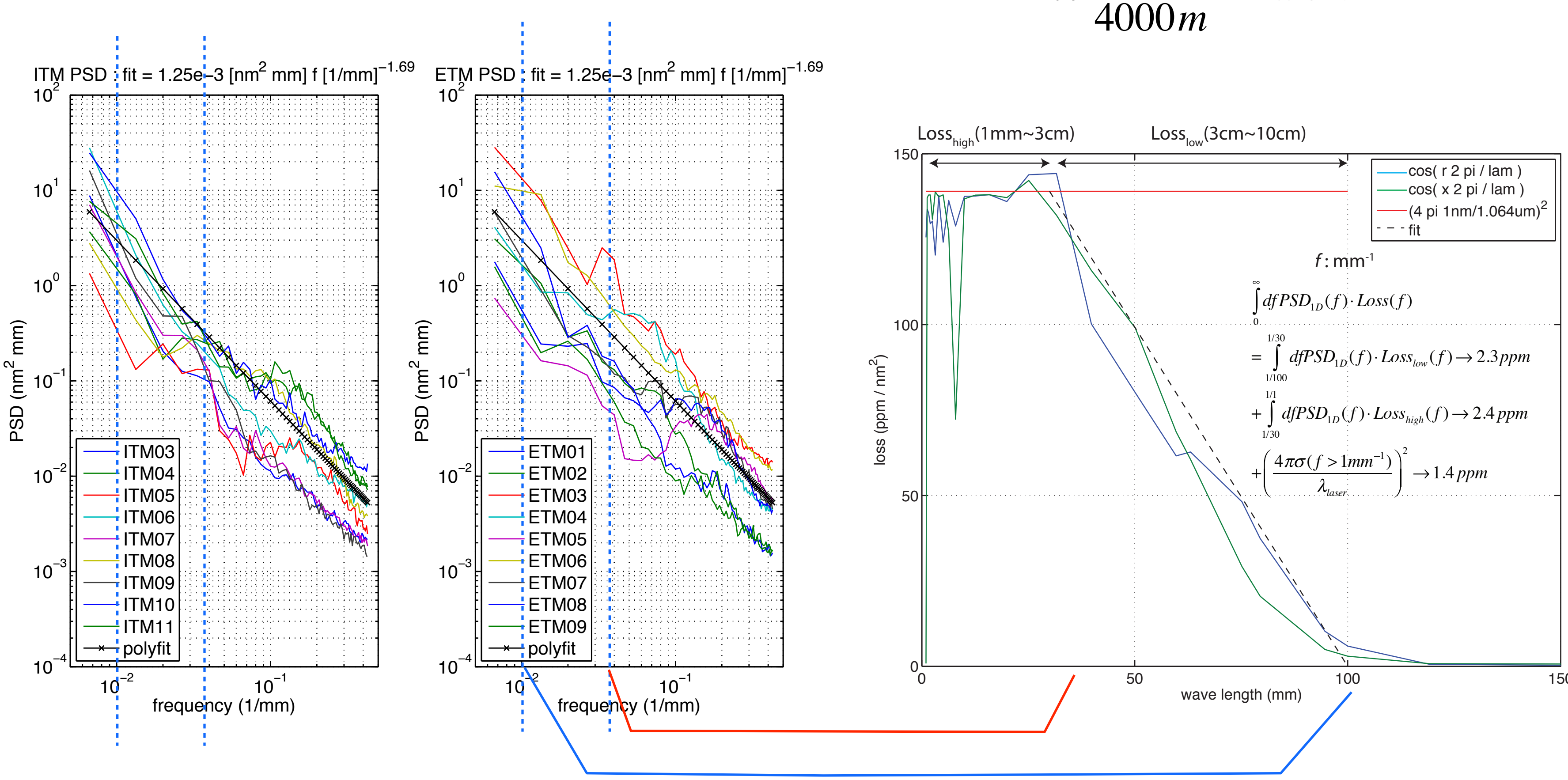
$$for\ \ \delta(x,y) = \delta_0 \sin(k_x x)$$
$$E_{ref}^{0} \cdot i2k\delta = E_{ref}^{0} \cdot i2k\delta_0 \sin(k_x x)$$
$$= E_{ref}^{0} \cdot (k\delta_0(\exp(ik_x x) - \exp(-ik_x x)))$$
$$= E^{0} \frac{2\pi\delta_0}{\lambda} [\exp(-i(kz - k_x x)) - \exp(-i(kz + k_x x))]$$

aLIGO optics scattering loss by polished surface

10cm 3cm

10cm 3cm

$$\lambda / \frac{17cm}{4000m} = 2.5cm$$



$$\int_0^\infty df PSD_{1D}(f) \cdot Loss(f)$$

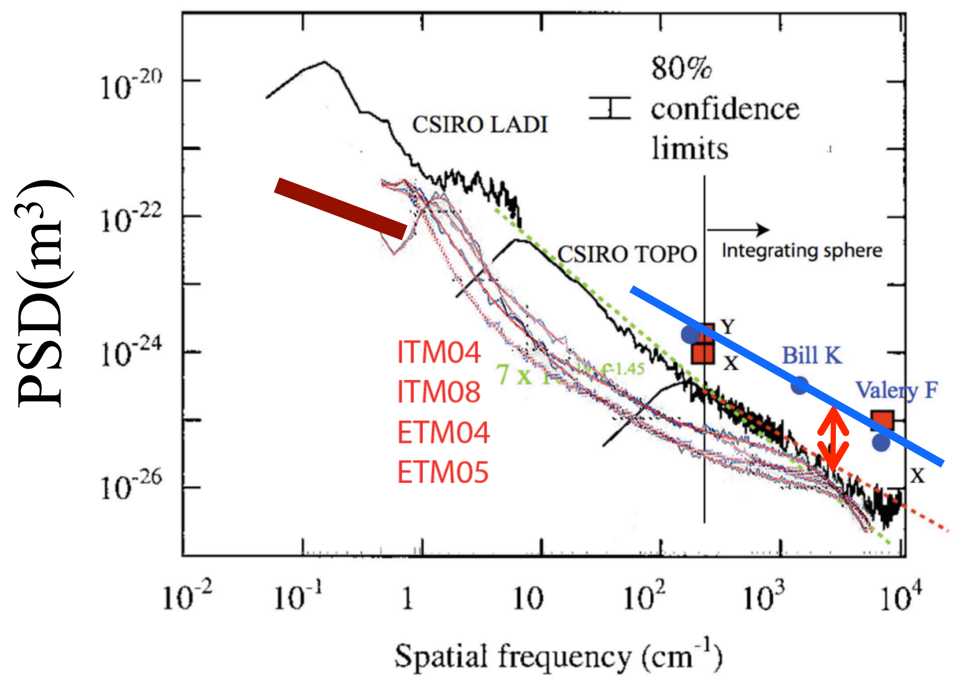
$$= \int_{1/100}^{1/30} df PSD_{1D}(f) \cdot Loss_{low}(f) \rightarrow 2.3ppm$$

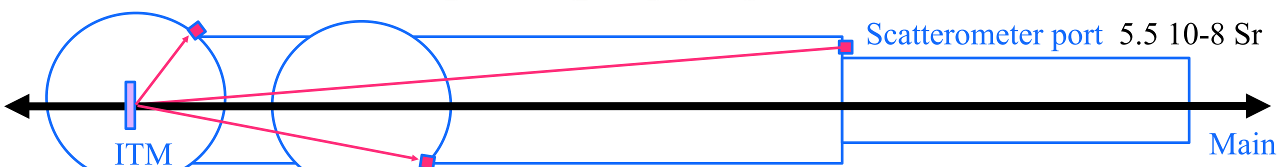
$$+ \int_{1/30}^{1/1} df PSD_{1D}(f) \cdot Loss_{high}(f) \rightarrow 2.4ppm$$

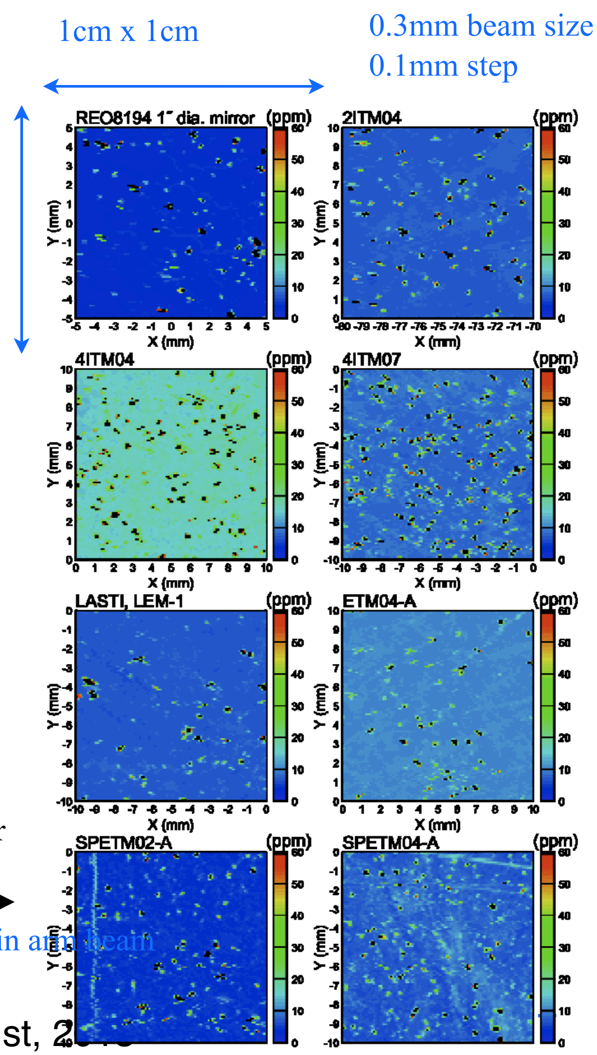
$$+ \left(\frac{4\pi\sigma(f > 1mm^{-1})}{\lambda_{laser}} \right)^2 \rightarrow 1.4ppm$$

Peeking at LIGO mirror profile

iLIGO vs aLIGO PSD







BRDF ≠PSD

- BRDF
  - » how light is reflected by an opaque surface
- PSD
  - » spectral density of the surface

$$BRDF(\theta) = (\frac{4\pi}{\lambda^2})^2 PSD_{2D}(f) = (\frac{4\pi}{\lambda^2})^2 C \frac{PSD_{1D}(f)}{f}$$

$$\lambda_s = \frac{1}{f} \quad \theta = \frac{\lambda_{laser}}{\lambda_{space}} = f \cdot \lambda_{laser}$$

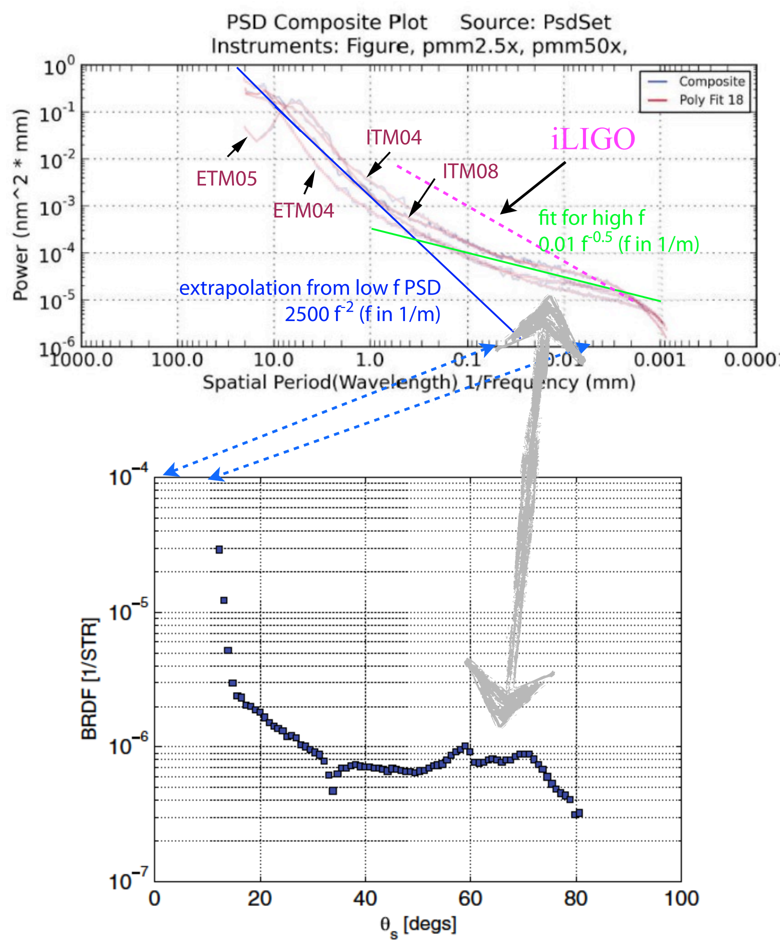
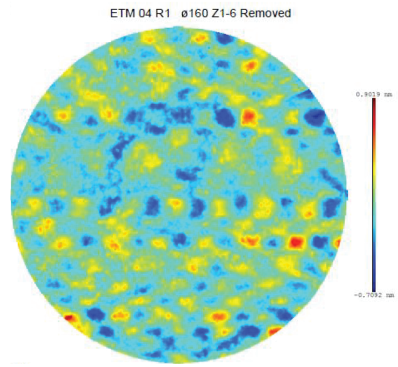
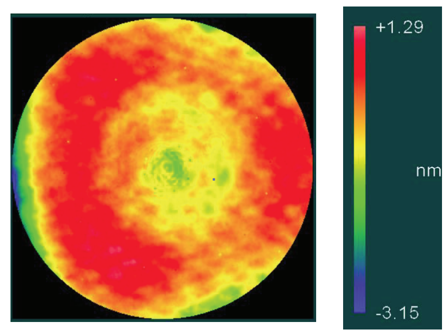


Fig. 9. (Color online) BRDF versus scattering angle for the HRM.

Polishing and coating
ETM04 : coating is tough

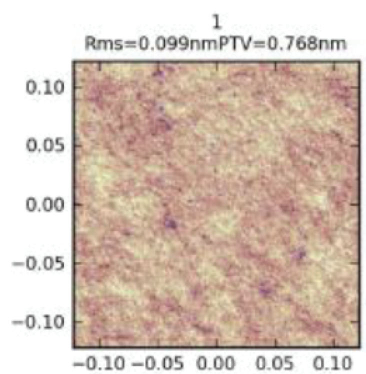


(1) Surface after polishing by ASML
Aperture size 160mm
RMS = 0.1732nm, PV=1.611nm

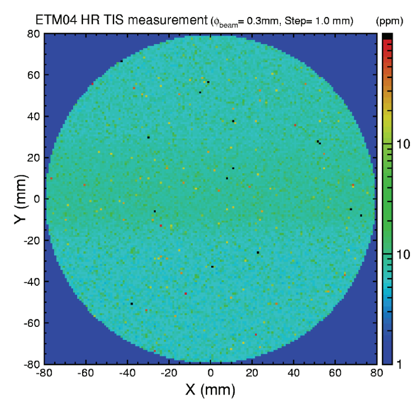


latest coating

(2) Surface after multilayer coating by ison spattering
Aperture160mm
RMS = 0.563nm, PV=4.436nm



(3) Surface after polishing measured by
PMM(phase measuring microscope)
with magnification of 50.
0.25mm x 0.25mm square near center.
RMS = 0.099nm, PV=0.768nm



(4) Reflectance measured by an integrating sphere
with the scattering angle larger than 1°. The size of the
laser is 0.3mm, with spacing 1mm.
RMS using all data points is 98ppm. RMS is 20ppm after
excluding 15 points with reflectance > 1000ppm.

latest coating
5~10ppm

Polishing by Coastline and ASML

Requirement and result of ITM04

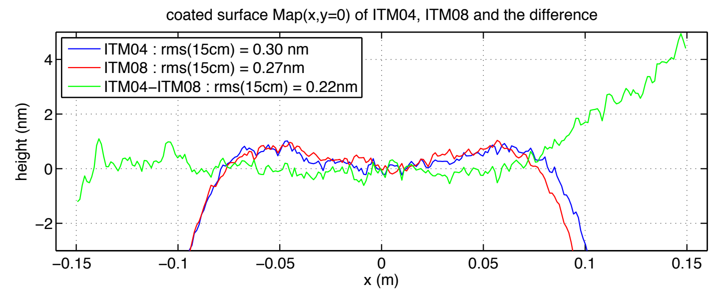
Surface	Specification Parameter	Location	Specification Value	Actual Value	Pass/Fail
1	Spherical, CC, RoC	Central 160 mm	1934 m - 5m/+15m	1938.61 m	PASS
	Radius Difference from all ITMs	Central 160 mm	1938.53 m ± 3 m	0.08 m	PASS
	Astigmatism Amplitude ($Z_{2,2}$)	Central 160 mm	σ_{RMS}< 3 nm	0.12 nm	PASS
	Figure Error (LSF) <$1mm^{-1}$	Central 300 mm	σ_{RMS}< 2.5 nm	0.37 nm	PASS
	$Z_{0,0}$, $Z_{1,1}$, $Z_{2,0}$, $Z_{2,2}$ Fit	Central 160mm	σ_{RMS}< 0.3 nm	0.15 nm	PASS
	Error (HSF) 1-750mm^{-1}	Center, Ø60 mm, Ø120 mm	σ_{RMS}≤ 0.16 nm	0.137 nm	PASS

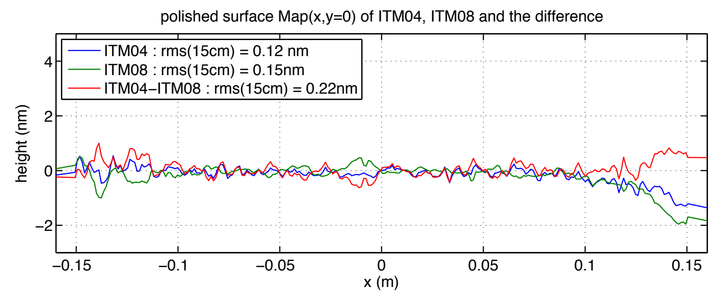
Requirement by simulation Actually delivered

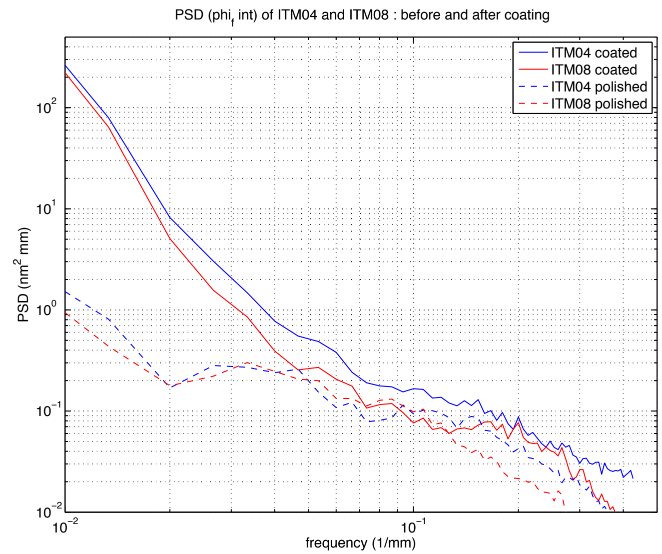
LSF(>2mm) : σ < 0.5nm for loss < 20ppm, σ = 0.15nm -> 2ppm

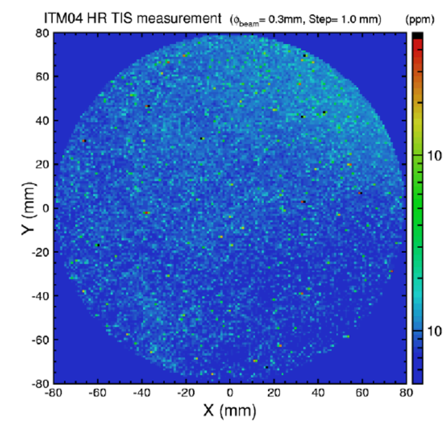
HSF(<1mm) : σ = 0.137nm -> ~3ppm (< 1mm), <6ppm (<2mm)

Coating by LMA
ITM04 and ITM08









Caltech : 10ppm
LMA : 4.5ppm

		Round trip loss (ppm)	Non 00 mode in cavity (ppm)	LG20 mode in cavity (ppm)
polished	ITM04	2.9	3.2	0
	ITM08	3.0	3.5	0
coated	ITM04	2.7	8.8	2.8
	ITM08	3.0	9.0	4.9

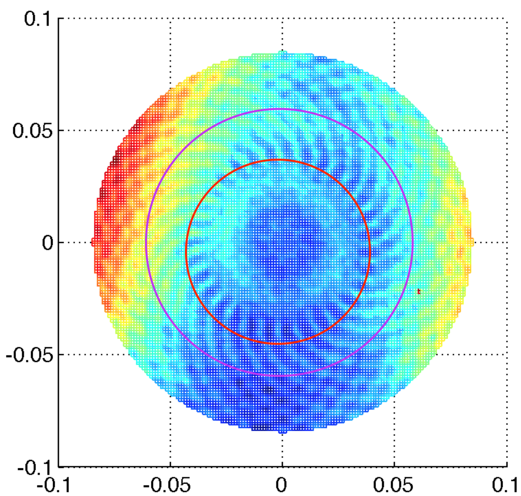
Table 1 Cavity quality factors

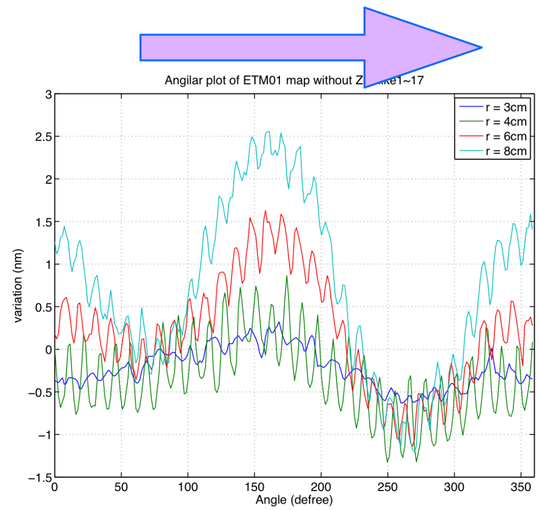
LIGO

Coating by LMA

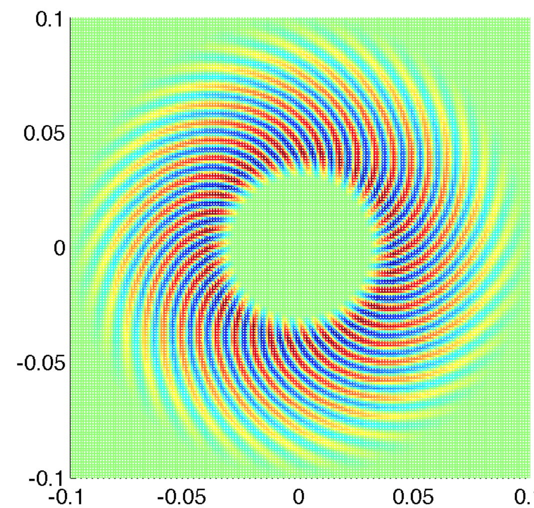
ETM01

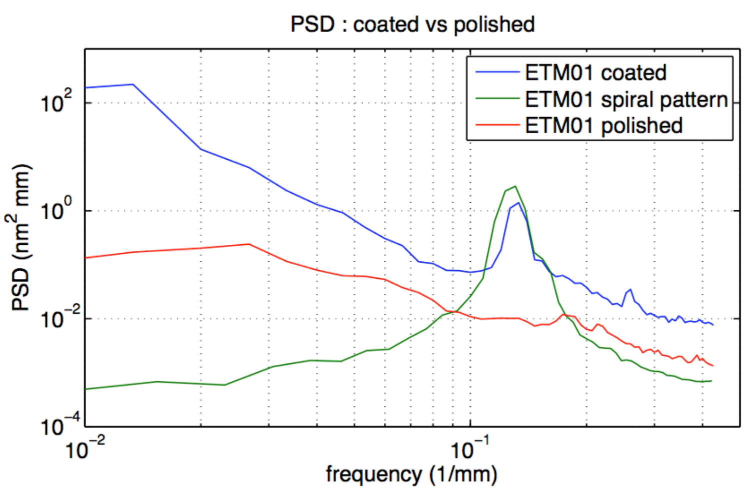
measured at Caltec - Z1~Z7





P-V 1 nm

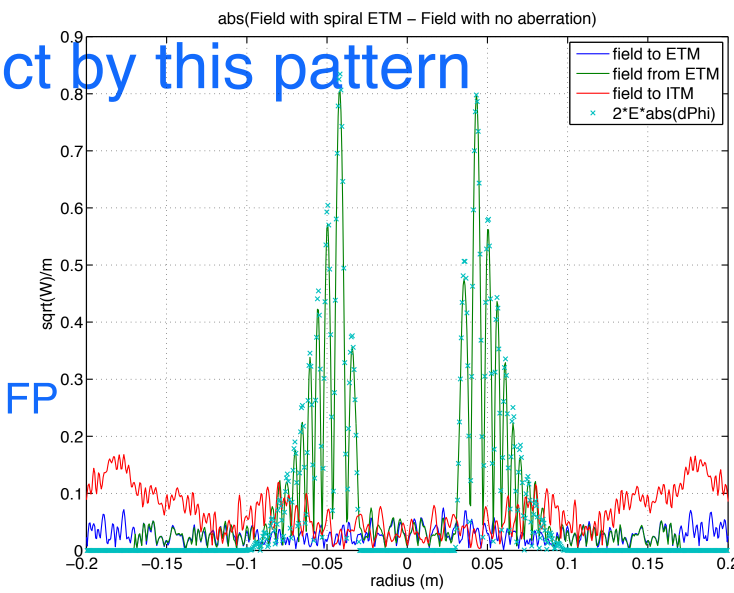


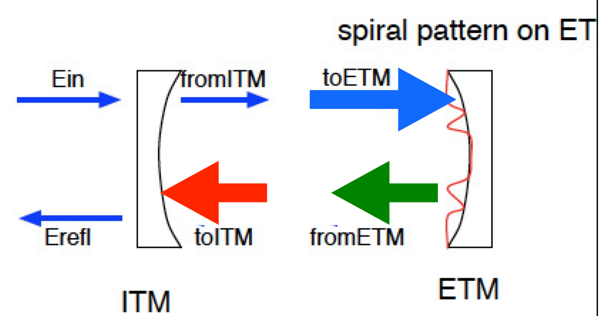


Using matlab to extract the spiral pattern, and use it as the phasemap in SIS

LMA ETM01 coating accepting test short wavelength spiral pattern

- SIS analysis to understand the effect by this pattern
- Round trip loss ~ 6ppm ← OK
- Any other effects
  - » Field aberration due to this pattern
    - – Field in FP with this map – Field in idealistic FP
    - – Very fine grid sizes to make sure FFT is OK
  - » Mode analysis if any mode could dominate
    - – No dominant mode for LGpm (2p+m<25) and HGmn (m+n<25)
  - » If ITM has similar pattern, can they interfere
    - – ITM = MAPPING (DATAFILE(“ETM01pattern.dat”), ”-x”,”y”) * 0.5
    - – Loss = loss by ETM + loss by ITM no additional by interference

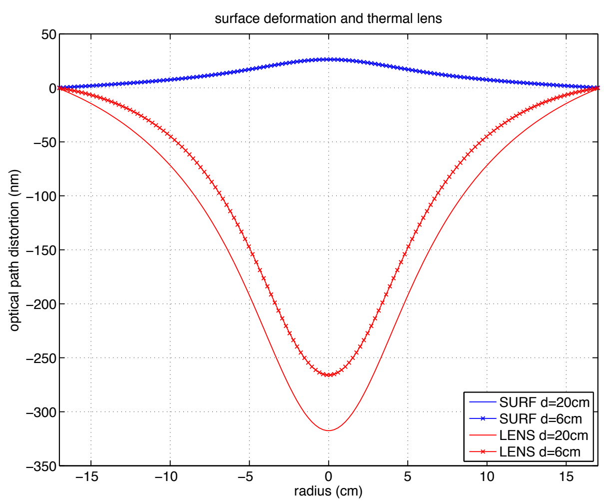


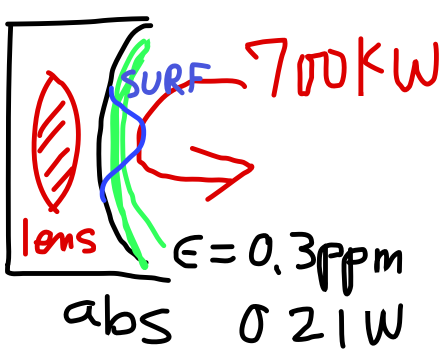


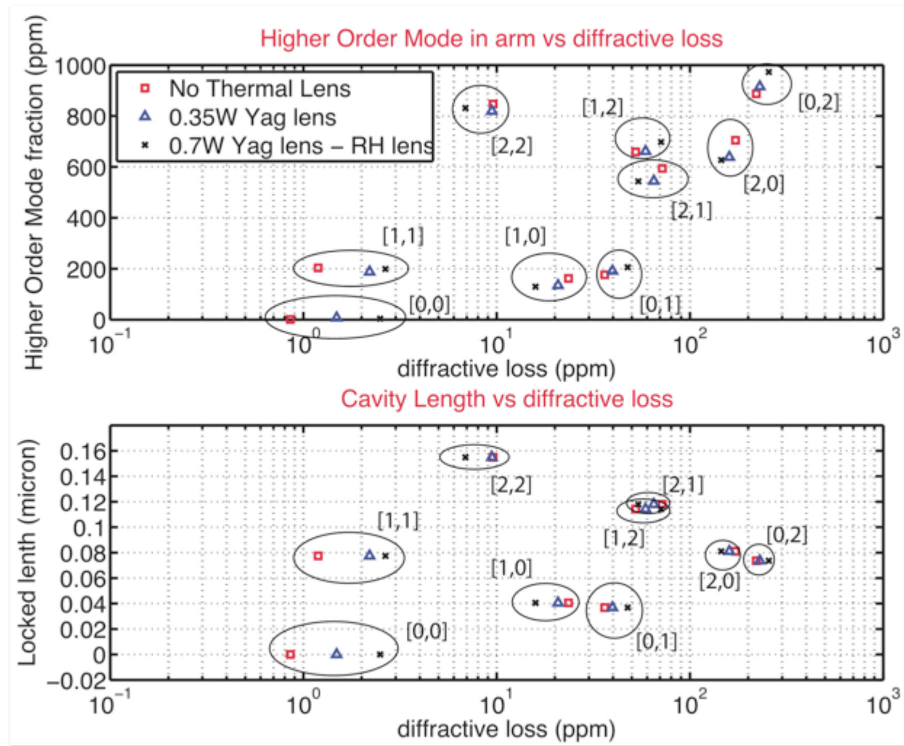
LMA ETM01 coating accepting test long wavelength central plateau

- Old coating system, one at a time
  - » The beam size on ETM is larger than that on ITM and the plateau size on ETM needs to be 20% wider, when coating to coating variation is taken into account
- New coating using the planetary system, a pair at a time
  - » Higher order mode, mostly LG20, in the FP cavity is ~100ppm
    - Better than old, 120ppm, and two ETMs will be “identical”, but is this **good enough?**
    - The plateau size is around the same as the old one
    - Astigmatism uncertainty due to the substrate is not a major issue
    - Asymmetry in the far outside is better (smaller) in the new coating
  - » Coupled cavity simulation
    - LG20 in SRC shows no increase of LG20 by the mode healing
      - Stable signal recycling cavity kills LG20 in SRC
    - LG20 in PRC is ~2000ppm increase by the ETM coating aberration

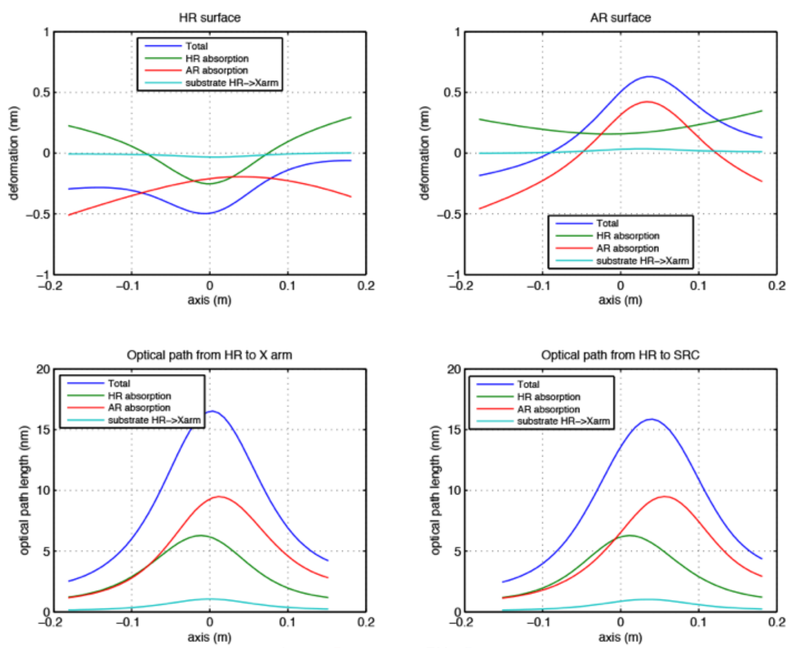
Thermal distortion test mass

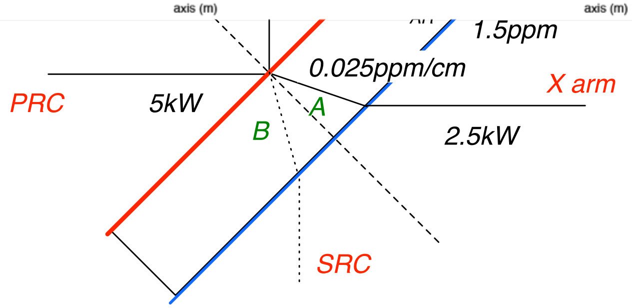


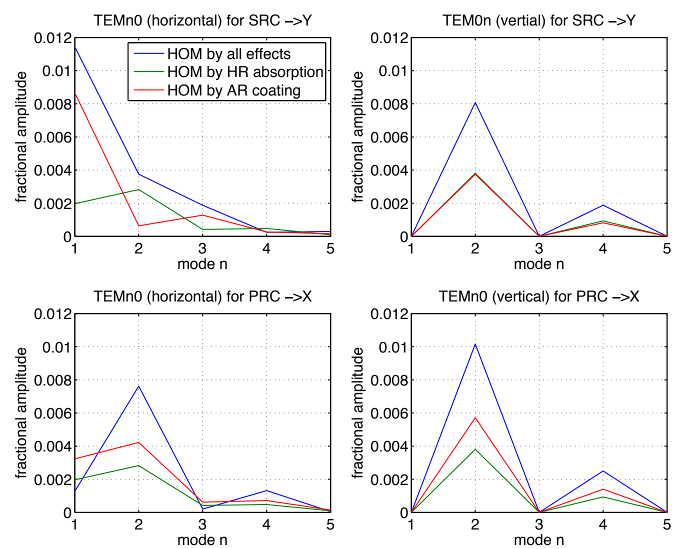




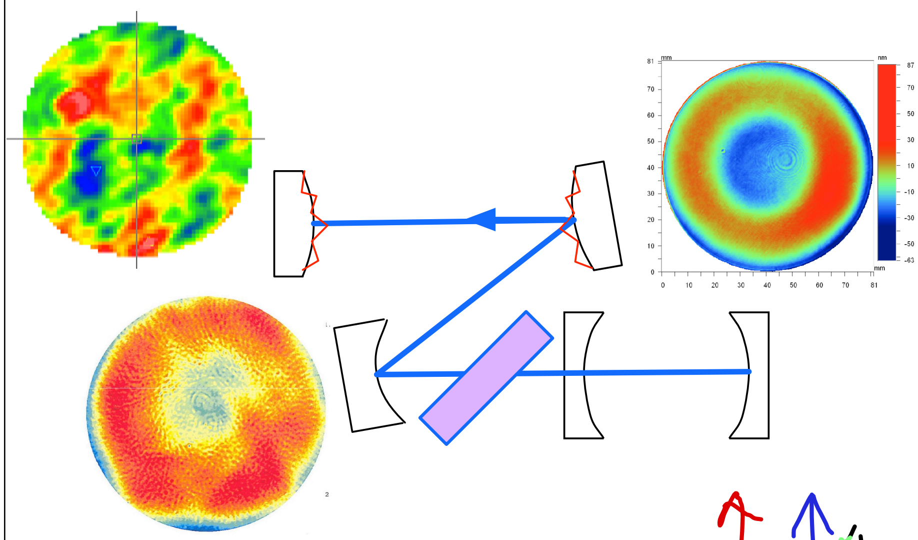
Thermal distortion BS

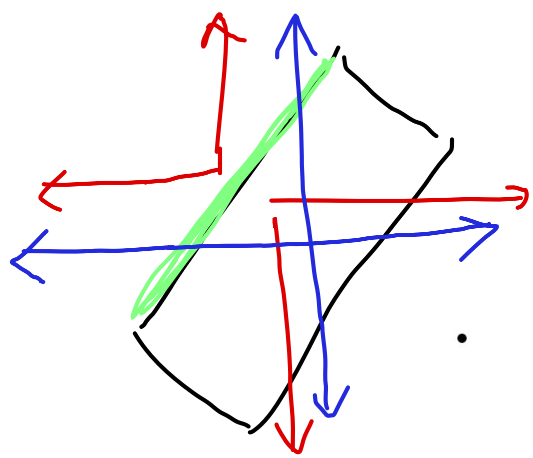




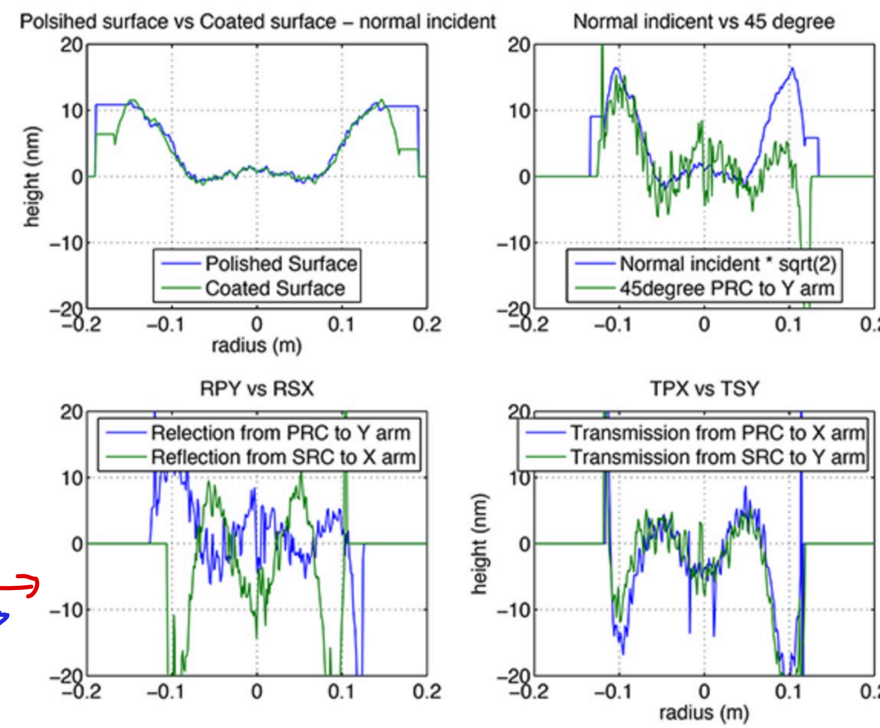


Not so nice looking mirrors





- **BS02 maps**



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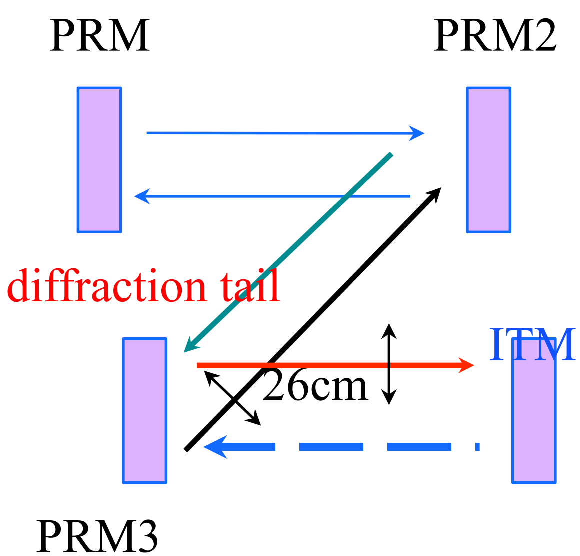
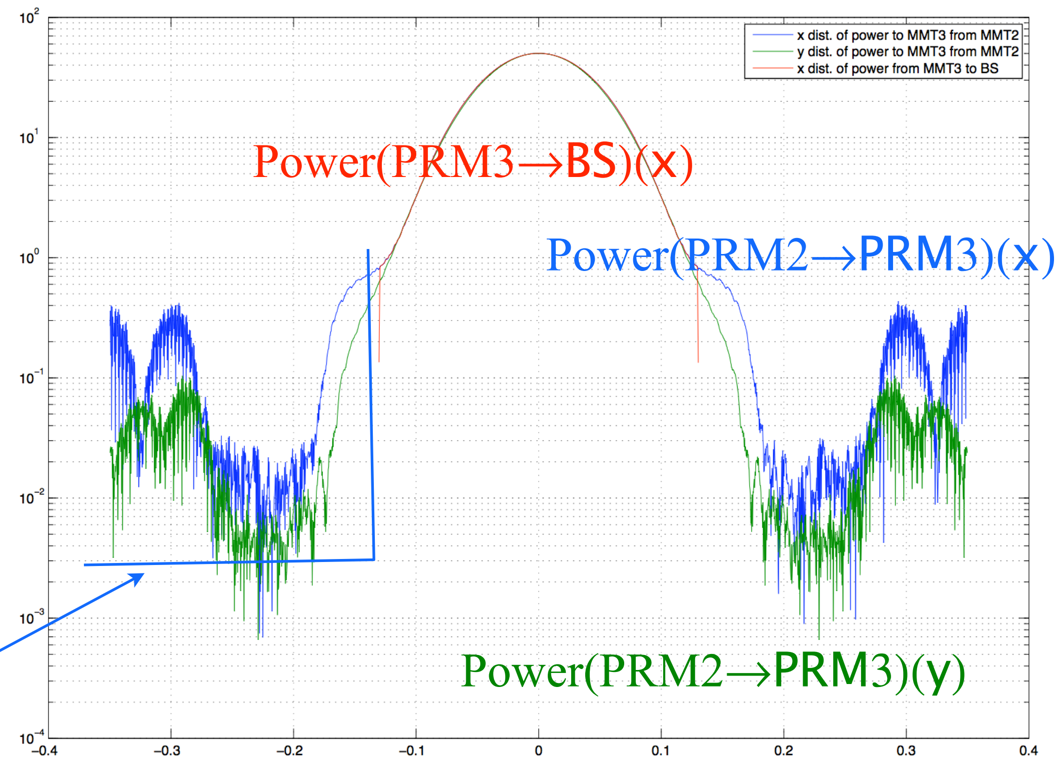
Why ROC(ITM) < ROC(ETM)
Power loss on RM3





loss = 330ppm
energy outside of
PRM3 surface

Higher order mode fraction on SRM