# Reducing Thermal Noise in Advanced GW Detector Mirror Coatings

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JGW – G12024

## Outlook

- b-KAGRA mirror noise budget Reducing coating noise
- Optimizing coating materials
  Modeling glassy mixtures
- nm-layered SiO2//TiO2 composites
- Optimizing coating geometry Minimum noise dichroic coating

#### **b-KAGRA Mirror Noise Budget**



#### Substrate Thermoelastic Noise PSD

$$S_{sub}^{(\text{TE})}(f) = \frac{4\sqrt{2}}{\pi^2} \alpha^2 (1+\sigma)^2 \frac{k_B T^2 w_m}{\kappa} J(f)$$
$$J(f) = \int_0^\infty du \int_{-\infty}^\infty dv \frac{u^3 \exp(-u^2/2)}{(u^2+v^2)[(u^2+v^2)^2+4\pi^2 f^2]}$$
$$\alpha = \text{thermoelastic constant} \\ \sigma = \text{Poisson modulus} \qquad \text{of substrate; } w_m = \text{spot width}$$

κ = thermal conductivity

Valid beyond Braginsky's "adiabatic" assumption  $(\kappa/\varrho Cf)^{1/2} < w_m$ 

[M. Cerdonio et al., Phys. Rev. D 63 (2001) 082003] (misprint in their eq. 21 fixed)

#### **Coating Brownian Noise PSD**

$$\begin{split} S_{coat}^{(B)}(f) &= \frac{2k_BT}{\sqrt{\pi^3}f} \frac{1-\sigma^2}{w_m Y} \phi_c \\ \phi_c &= \frac{d_1 + d_2}{\sqrt{\pi w_m}} \frac{1}{Y_\perp} \left\{ \left[ \frac{Y}{1-\sigma^2} - \frac{2\sigma_\perp^2 YY_\parallel}{Y_\perp (1-\sigma^2)(1-\sigma_\parallel)} \right] \phi_\perp \\ &+ \frac{Y_\parallel \sigma_\perp (1-2\sigma)}{(1-\sigma_\parallel)(1-\sigma)} (\phi_\parallel - \phi_\perp) + \frac{Y_\parallel Y_\perp (1+\sigma)(1-2\sigma)^2}{Y(1-\sigma_\parallel^2)(1-\sigma)} \phi_\parallel \right\} \\ Y_\perp &= \frac{d_1 + d_2}{Y_1^{-1}d_1 + Y_2^{-1}d_2} \ ; Y_\parallel = \frac{Y_1 d_1 + Y_2 d_2}{d_1 + d_2} \\ \phi_\perp &= Y_\perp \left( \frac{Y_1^{-1}\phi_1 d_1 + Y_2^{-1}\phi_2 d_2}{Y_1 d_1 + d_2} \right) \ ; \phi_\parallel = Y_\parallel^{-1} \left( \frac{Y_1\phi_1 d_1 + Y_2\phi_2 d_2}{d_1 + d_2} \right) \\ \sigma_\perp &= \frac{\sigma_1 Y_1 d_1 + \sigma_2 Y_2 d_2}{Y_1 d_1 + Y_2 d_2} \\ \frac{\sigma_1 Y_1 d_1}{(1+\sigma_1)(1-2\sigma_1)} + \frac{\sigma_2 Y_2 d_2}{(1+\sigma_2)(1-2\sigma_2)} = -\frac{Y_\parallel (\sigma_\perp^2 Y_\parallel + \sigma_\parallel Y_\perp) (d_1 + d_2)}{(\sigma_\parallel + 1)[2\sigma_\perp^2 Y_\parallel - (1-\sigma_\parallel) Y_\perp]} \implies \sigma_\parallel \\ \phi_c &= \frac{d_1 + d_2}{\sqrt{\pi} w_m} \left( \frac{Y}{Y_\perp} \phi_\perp + \frac{Y_\parallel}{Y} \phi_\parallel \right) \end{split}$$

## **Reducing Coating Brownian Noise**



## • Optimizing Coating Materials



**Low** mechanical loss-angle per unit thickness  $(\eta_H)$ Depends **both** on complex Young modulus  $\tilde{Y} = Y(1 - i\phi)$ , **and** refractive index  $n_H$ ;

**High** dielectric contrast  $(n_H/n_L)$ helps *reducing the number of layers* (coating thickness) needed for a prescribed coating transmittance;

**Low** dielectric losses  $(Im[n_H])$ *increases power-handling* capability.

## **Titania Doped Tantala**

To date, most successful attempt to reduce thermal noise by improving material properties is LMA's "formula 2"  $TiO_2$  - doped  $Ta_2O_5$  [G.M. Harry et al, Class. Quantum Grav. 24 (2007) 405].

How to optimize glassy-mixtures?

•Extensive Experimental Trial-and-Error (LMA, CSIRO);

- •Solid State Modeling/Simulation Glasgow (LIGO) Urbino (Virgo)
- •Effective Medium (Mixture) Approach. (Sannio)

We need reliable values for the material parameters !

## **Loss Angle Reduction in Doped Glasses**

Acoustic oscillations (thermal phonons) in a *symmetric* double-well potential drive well-to-well jumps  $\implies$  dissipation.

Doping (and/or other structural stresses) may destroy the potential symmetry, resulting into *well-trapping*  $\implies$  *reduced* dissipation...

[J.S. Wu and C.C. Yu, "How Stress can Reduce Dissipation in Glasses, " Phys. Rev. B84 (2011) 174109]





## **Trustable Numbers Needed**

Frequently quoted values must be handled w. care, especially for Titania ...

|                 | SiO <sub>2</sub>                     | TiTa <sub>2</sub> O <sub>5</sub> | Ta <sub>2</sub> O <sub>5</sub>      | TiO <sub>2</sub>                     |  |
|-----------------|--------------------------------------|----------------------------------|-------------------------------------|--------------------------------------|--|
| Loss angle      | 0.4×10 <sup>-4</sup><br>[37]         |                                  |                                     |                                      |  |
|                 | 0.5×10 <sup>-4</sup>                 | 2.3×10 <sup>-4</sup>             | 3.8×10 <sup>-4</sup> <sup>[1]</sup> | 6.3×10 <sup>-3</sup><br>deduced from |  |
|                 | 10 <sup>-3 on</sup><br>sapphire [47] | 2×10 <sup>-4</sup> [36]          |                                     | [48]                                 |  |
| Young's modulus | 72 [1, 10, 37]                       | 140 [37]                         | 140 [37]                            | 290 <sup>[12]</sup>                  |  |
| (GPa)           | 40-60                                |                                  |                                     |                                      |  |
| ratio           | 0.17 [1, 10]                         | 0.23 [6]                         | 0.23 [37]                           | 0.28 [12]                            |  |
|                 |                                      |                                  |                                     |                                      |  |

J. Franc et al., ET-021-09 (2009)

Values from a *single* experiment: 25-doublets QWL Silica/Titania coating [P. Amico et al., J. Phys. Conf. Ser. 32 (2006) 413]. Thickness of Titania layers was 116nm. Well above limit-thickness for preventing crystallization upon annealing [S. Chao et al., J. Opt. Soc. Am. A16 (1999) 1477]. Reported loss angle most likely due to crystallization.

In the amorphous phase Y<sub>TiO2</sub> is 160 - 170 Gpa. [T. Modes et al., Surf. Coat. Technol. 200 (2005) 306] Quoted Y = 290 Gpa is OK for the crystalline (Rutile) phase [O. Zywitzki, et al., Surf. Coat. Technol. 180 (2004) 538].

Large spreads among values obtained from different measurement techniques...

Conjectured values - No direct measurements of Y or  $\sigma$  on doped Tantala reported yet.

[I. Pinto et al., LIGO-G1100586]

### **Material Loss Angles from Coating Noise PSD**

Use TNI measurements and results :

| 1 | QWL       | Silica | Tantala       | REO | 8.25 ± 0.3 |
|---|-----------|--------|---------------|-----|------------|
| 2 | Optimized | Silica | Tantala       | LMA | 6.85 ± 0.2 |
| 3 | QWL       | Silica | Doped Tantala | LMA | 6.0 ± 0.25 |
| 4 | Dichroic  | Silica | Doped Tantala | LMA | 5.5 ± 0.5  |

and G. Harry's mainstream formula for coating noise (vanishing Poisson limit)

$$\phi_{c} = b_{L}d_{L}\phi_{L} + b_{H}d_{H}\phi_{H}$$
$$b_{L,H} = (Y_{L,H} / Y_{s} + Y_{s} / Y_{L,H}) / (w\sqrt{\pi})$$

to retrieve the loss angles from fiducially known thicknesses etc.

| Coating # | $d_L$   | $d_{H}$ |
|-----------|---------|---------|
| 1         | 2.72 µm | 1.83 µm |
| 2         | 4.05 µm | 1.36 µm |
| 3         | 2.54 µm | 1.67 µm |
| 3         | 2.36 µm | 1.45 µm |

A. Villar et al., LIGO-G1000937 (2010)



#### **Material Loss Angles from TNI Measurements**



## Material Loss Angles from Coating Noise PSD

Silica  $(TiO_2)$  and plain Tantala  $(Ta_2O_5)$ 



[A. Villar et al., LIGO-G 1101096]

## Loss Angles from Coating Noise PSD, contd.

Doped Tantala (TiO<sub>2</sub>::Ta<sub>2</sub>O<sub>5</sub>)



[A. Villar et al., LIGO-G 1101096]

## **Comparison between TNI and Q-based Material Loss Angle Estimates**

Good agreement for Silica;

Somewhat larger loss angle for Tantala (plain & doped).

Doped/Undoped loss-angle ratio is the same

Reason of discrepancy yet unclear

[A. Villar et al., LIGO-G 1101096]



## **Diffusion at Interfaces...**



#### Tomography & Density Profiles

[F. Granata et al. (LMA) , LIGO/G120514]

... diffusion length largely annealing-schedule dependent...

### ... may Explain in part the Discrepancy



## • Simple Mixture Modeling



# Simple Mixture Modeling Effective Medium Theories (EMT)





Composite materials (mixtures) can be modeled by an appropriately-weighted average of macroscopic properties of both components.

Replace actual, "composite" system with a homogeneous, "effective" medium.

Effective for a wide variety of properties dielectric constant index of refraction elastic modulus loss angle etc.

Results depend somewhat on inclusion concentration, morphology, orientation.

[A. Villar et al., LIGO-G 1101096]

## **EMT for Composite Optical Properties**

#### Bruggemann formula:



Chien-Jen Tang, "Analysis of  $Ta_2O_5$ -TiO<sub>2</sub> and  $Ta_2O_5$ -SiO<sub>2</sub> Composite Films Prepared by Ion-Beam Sputtering Deposition," PhD Dissertation, Taiwan National Central University (2006).

## **EMT for Composite Viscoelastic Properties**

Barta's microscopic derivation of Bruggemann-like mixture formulas for viscoelastic parameters of a glassy-oxide composite yields

$$\begin{cases} (1-\eta_2)\frac{X-X_1}{2X+(X_1/y_1)(\sigma_1+1)} + \eta_2\frac{X-X_2}{2X+(X_2/y_2)(\sigma_2+1)} = 0\\ (1-\eta_2)\frac{(X/y)-(X_1/y_1)}{2X+(X_1/y_1)(\sigma_1+1)} + \eta_2\frac{(X/y)-(X_2/y_2)}{2X+(X_2/y_2)(\sigma_2+1)} = 0 \end{cases}$$

$$X = \frac{\sigma Y}{\sigma + 1}, \ y = \sigma - 2$$

System can be solved in closed form . [S. Barta, «Effective Young modulus and Poisson's ratio for the particulate composite," J. Appl. Phys. 75 (1994) 3258].

[A. Villar et al., LIGO-G 1101096]

## **TNI Result vs EMT Prediction**

We can compare the doped Tantala loss angle distribution obtained from TNI measurements to the prediction of EMT, using Scott-MacCrone loss angle for Titania, and the TNI result for plain Tantala

TNI : distribution deduced from doped coating measurement, using the marginal distribution of Silica loss angle from the undoped coating measurements.

Bruggeman-Barta : distribution deduced using Scott-MacCrone value for Titania loss angle, with plain Tantala loss-angle distribution from undoped coating measurements.



[A. Villar et al., LIGO-G 1101096]

#### • How to do better than TiO2::Ta2O5 ?



## A Direct Measurement of $\phi_{TiO_2}$

THE REVIEW OF SCIENTIFIC INSTRUMENTS

VOLUME 39, NUMBER 6

JUNE 1968

#### Apparatus for Mechanical Loss Measurements in Low Loss Materials at Audio Frequencies and Low Temperatures\*

W. W. SCOTT AND R. K. MACCRONE\*\*

Department of Metallurgy and Materials Science, University of Pennsylvania, Philadelphia, Pennsylvania 19104 (Received 8 January 1968; and in final form, 14 February 1968)

A new apparatus for measuring mechanical loss in low loss materials at low temperatures is decribed. The method has several advantages over existing techniques. Using this apparatus, losses as low as  $2 \times 10^{-4}$  with a resolution of  $10^{-7}$  have been reproducibly measured at  $4.2^{\circ}$ K in TiO<sub>2</sub> (rutile).



- Introduces an apparatus conceptually similar to the familiar «cantilever».
- Little details about tested materials. Speaks of *lightly reduced TiO2* in the main body, but mentions Rutile in the title;
- Reported results over a wide range of temperatures from 5K to 400K...



## **Titania is Nasty**



J. Szczyrbowski, Surf. Coat. Technol. 112 (1999) 261–266.

# A SiO<sub>2</sub>//TiO<sub>2</sub> Coating Prototype

P. Amico et al., J. Phys. Conf. Ser. 32 (2006) 413;

F. Travasso, ILIAS 05 Conference presentation (2005).



1 sq. inch, 50 $\mu$ m thick membranes coated with 25 x TiO<sub>2</sub> (pure) / SiO<sub>2</sub> QWL-doublets.

Coatings obtained by e - beam evaporation. Manufactured by italian Company SILO (www.silo.it). *No* annealing details. *No* TEM images.

Thickness of pure TiO<sub>2</sub> QWL layers  $\cong$  116nm. Crystallization expected.

Used to estimated Titania loss angle in [Franc et al., ET-021-09 (2009)]

## **Trustable Numbers Needed**

Frequently quoted values must be handled w. care, especially for Titania ...

|                             | SiO <sub>2</sub>   | TiTa <sub>2</sub> O <sub>5</sub> | Ta <sub>2</sub> O <sub>5</sub>      | TiO <sub>2</sub>                     | - |
|-----------------------------|--|----------------------------------|-------------------------------------|--------------------------------------|---|
| Loss angle                  | 0.4×10 <sup>-4</sup><br>[37]                               |                                  |                                     |                                      | - |
|                             | 0.5×10 <sup>-4</sup>                                       | 2.3×10 <sup>-4</sup>             | 3.8×10 <sup>-4</sup> <sup>[1]</sup> | 6.3×10 <sup>-3</sup><br>deduced from |   |
|                             | 10 <sup>*3 on</sup><br>sapphire [47]                       | 2×10 <sup>-4</sup> [36]          | $\checkmark$                        | [48]                                 |   |
| Young's<br>modulus<br>(GPa) | 72 <sup>[1, 10, 37]</sup><br>40- <b>60</b> <sup>[14]</sup> | 140 [37]                         | 140 [37]                            | 290[12]                              |   |
| Poisson's<br>ratio          | 0.17 <sup>[1,10]</sup>                                     | 0.23 [6]                         | 0.23 [37]                           | 0.28 <sup>[12]</sup>                 |   |

J. Franc et al., ET-021-09 (2009)

Values from a *single* experiment: 25-doublets QWL Silica/Titania coating [P. Amico et al., J. Phys. Conf. Ser. 32 (2006) 413]. Thickness of Titania layers was 116nm. Well above limit-thickness for preventing crystallization upon annealing [S. Chao et al., J. Opt. Soc. Am. A16 (1999) 1477]. Reported loss angle most likely due to crystallization.

In the amorphous phase Y<sub>TiO2</sub> is 160 - 170 Gpa. [T. Modes et al., Surf. Coat. Technol. 200 (2005) 306] Quoted Y = 290 Gpa is OK for the crystalline (Rutile) phase [O. Zywitzki, et al., Surf. Coat. Technol. 180 (2004) 538].

Large spreads among values obtained from different measurement techniques...

Conjectured values - No direct measurements of Y or  $\sigma$  on doped Tantala reported yet.

[I. Pinto et al., LIGO-G1100586]

# Silica Doped Titania (SiO<sub>2</sub>::TiO<sub>2</sub>)

Extinction coefficient, and its absorption

and scattering components, vs Tannealing.



W.H. Wang and S. Chao, Optics Lett., 23 (1998) 1417;

S. Chao, W.H. Wang, M.-Y. Hsu and L.-C. Wang, J. Opt. Soc. Am. A16 (1999) 1477; S. Chao, W.H. Wang and C.C. Lee, Appl. Opt., 40 (2001) 2177;

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# A SiO2//TiO2::SiO2 Coating Prototype

R.P. Netterfield and M. Gross, "Investigation of Ion Beam Sputtered Silica Titania Mixtures for Use in GW Interferometer Optics," Optical Interference Coatings (OIC) Conference, Tucson AZ, USA, 2007, paper Thd2.



# SiO<sub>2</sub>//TiO<sub>2</sub> nm-Multilayers - Rationale



Use a *sub-wavelength* layer *stack* (SWS) made of two (or more...) different refractive materials to *synthesize* a *high-index*, *low mechanical loss* material.

Expect *larger* n<sub>eff</sub>, *smaller* b<sub>eff</sub>, compared to "isotropic" mixture With same stoichiometry;

nm – thick TiO<sub>2</sub> layers should *not* crystallize significantly upon annealing. nm-thick silica layers act as separa-Tors, hopefully preventing Titania from crystallizing

## **Annealing of TiO2 nm-layers**



## **Modeling nm - Multilayers**



## **Co-sputtered vs nm-layered Silica//Titania**



Layered SiO2::TiO2 mixture "bertter" than isotropic mixture at all stoichiometries

## **Co-sputtered Titania//Tantala** vs nm-layered Silica//Titania



Layered SiO2::TiO2 outperforms by large isotropic Ta2O5-TiO2 mixtures

Several alternative sandwich thickness choices possible, yielding the same effective medium properties;

Accuracy in single layer thickness NOT important, provided -Each layer in sandwich is subwavelength, -The thickness fraction of each constituent is accurate;

Possibly *mild* technological challenges: layers a few nm thick are currently manufactured for e.g. X-ray mirrors.



#### **nm-Layered Prototypes**

Nano-layer sample #2 TEM pictures



Sample #2



Sample #2 with thickness



close-up view of the layer interfaces



Electron diffraction pattern of the layers (amorphous)

[courtesy S. Chao]

## **nm-Layered Prototypes**



#### Nano-layer sample #2

[courtesy S. Chao]

Silicon wafer

## Chao's Lab, Tsing Hua Univ., Taiwan





Kaufman-type ion beam sputter system in a class 100 clean compartment within a class 10,000 clean room, used to coat low loss mirror for ring-laser gyroscope





Three sets of exchangable target holder, each holding two sputter targets



Sputter target and single axis substrate rotation



Kaufman iongun with plasma bridge neutralizer

[courtesy S. Chao]

**Clamped cantilever Q mesurement setup** 

## **The Clamped Coated Cantilever**



Chao's Q measurements will yield both composite and pure TiO2 loss angle estimates

## **Cryogenic Peak**



## • Optimizing Coating Geometry Minimal Noise Dichroic Design



## "Naive" Reference Dichroic Design



...this simple argument ignores material dispersion, e.g.

| wavelength<br>material | 532     | 670     | 946     | 1064    | 1319    | (courtesy  |
|------------------------|---------|---------|---------|---------|---------|------------|
| Silica                 | 1.47809 | 1.47337 | 1.47044 | 1.46995 | 1.46937 | IVI.Gross, |
| Doped Tantala          | 2.13890 | 2.10980 | 2.09570 | 2.09418 | 2.09238 | CSIRO)     |

## "Naive" Reference Dichroic Design, contd.

#### Design goal compliant and minimal noise designs (dispersion included)

| N₂<br>N₁ | 10      | 11      | 12      | 13      | 14      | 15      | 16                    | 17      | 18      | 19       | 20       |
|----------|---------|---------|---------|---------|---------|---------|-----------------------|---------|---------|----------|----------|
| 1        | 3316,2  | 1635,3  | 806,01  | 397,2   | 195,72  | 96,432  | 47,513                | 23,409  | 11,534  | 5,6826   | 2,7998   |
| '        | 0,3621  | 0,3909  | 0,41563 | 0,43632 | 0,45306 | 0,46597 | 0,47517               | 0,48076 | 0,48279 | 0,4813   | 0,47626  |
| 2        | 2107,8  | 1039    | 512,07  | 252,32  | 124,33  | 61,256  | 30,181                | 14,87   | 7,3264  | 3,6097   | 1,7785   |
| 2        | 0,61658 | 0,64006 | 0,65946 | 0,67515 | 0,68746 | 0,69667 | 0,70299               | 0,70657 | 0,70752 | 0,70584  | 0,70149  |
| 2        | 1306,5  | 643,93  | 317,31  | 156,35  | 77,035  | 37,955  | 18,701                | 9,2136  | 4,5395  | 2,2366   | 1,1019   |
| 3        | 0,79441 | 0,80856 | 0,82004 | 0,82916 | 0,8362  | 0,84139 | 0,84486               | 0.84675 | 0,84709 | 0,84591  | 0,84316  |
| 4        | 800,48  | 394,47  | 194,37  | 95,771  | 47,187  | 23,249  | 11,45                 | 5,6436  | 2,7806  | 1,37     | 0,67497  |
| 4        | 0,89572 | 0,90326 | 0,90932 | 0,9141  | 0,91776 | 0,92043 | 0,922 <mark></mark> 2 | 0,92314 | 0,92326 | 0,92258  | 0,92107  |
| 5        | 487,7   | 240,32  | 118,41  | 58,341  | 28,745  | 14,162  | 6,9777                | 3,4379  | 1,6938  | 0,83453  | 0,41117  |
| 5        | 0,94848 | 0,95229 | 0,95534 | 0,95773 | 0,95956 | 0,96089 | 0,96176               | 0,96222 | 0,96226 | 0,96191  | 0,96113  |
| 6        | 296,29  | 145,99  | 71,932  | 35,441  | 17,462  | 8,6033  | 4,2388                | 2,0884  | 1,029   | 0,50696  | 0,24977  |
| Ŭ        | 0,97487 | 0,97674 | 0,97824 | 0,97942 | 0,98031 | 0,98096 | 0,98139               | 0,98161 | 0,98163 | 0,98145  | 0,98106  |
| 7        | 179,74  | 88,56   | 43,634  | 21,498  | 10,592  | 5,2187  | 2,5712                | 1,2668  | 0,62415 | 0,30751  | 0,15151  |
| '        | 0,98781 | 0,98873 | 0,98946 | 0,99003 | 0,99046 | 0,99078 | 0,99099               | 0,99109 | 0,9911  | 0,99101  | 0,99082  |
| 8        | 108,94  | 53,677  | 26,447  | 13,03   | 6,4199  | 3,163   | 1,5584                | 0,76781 | 0,37829 | 0,18638  | 0,09183  |
| 0        | 0,99411 | 0,99455 | 0,9949  | 0,99518 | 0,99539 | 0,99554 | 0,99564               | 0,99569 | 0,9957  | 0,99566  | 0,99556  |
| 0        | 66,001  | 32,519  | 16,022  | 7,8939  | 3,8893  | 1,9162  | 0,94411               | 0,46515 | 0,22918 | 0,11291  | 0,055632 |
| 9        | 0,99715 | 0,99737 | 0,99754 | 0,99767 | 0,99777 | 0,99785 | 0,9979                | 0,99792 | 0,99792 | 0,9979   | 0,99786  |
| 10       | 39,974  | 19,695  | 9,7038  | 4,781   | 2,3556  | 1,1606  | 0,5718                | 0,28172 | 0,1388  | 0,068387 | 0,033694 |
| 10       | 0,99863 | 0,99873 | 0,99881 | 0,99888 | 0,99893 | 0,99896 | 0,99899               | 0,999   | 0,999   | 0,99899  | 0,99897  |

Table 1 – ETM reference "hybrid" SD design. In each cell: ETM power transmission coefficient @1064 nm [ppm] (1<sup>st</sup> line) and power reflection coefficient @532 nm (2<sup>nd</sup> line) for different values of N<sub>1</sub> and N<sub>2</sub>

#### [M. Principe and I. Pinto, LIGO-T080337]

## "Naive" Reference Dichroic Design, contd.

 $|cap| (|\lambda_0^{(H)}/4|\lambda_0^{(L)}/4|)^{14} (|\lambda_0^{(H)}/8|3\lambda_0^{(L)}/8|)^6 |\lambda_0^{(H)}/8|substrate$ 



## **Genetically Engineered Dichroic Design**



(50 iterations, 25 cycles, other settings default; layers rescaled modulo  $\lambda/2$ ; layers below  $5\lambda$  /1000 deleted. 38 layers left.

(... controlled ignorance attitude...)

Neat trend toward a *stacked-doublet core* ...



[I. Pinto et al., LIGO-G0900205]

## Simplest SD-Core Dichroic Design



 $|cap|(|d_{H}|d_{L}|)^{N}|d_{H}|substrate$ 

[I. Pinto et al., LIGO-G0900205]

## **Isoreflective Contours (N=18)**

$$cap \left| \left( \left| d_{H} \right| d_{L} \right) \right|^{18} \left| d_{H} \right| substrate$$



## **Minimal Noise SD Dichroic Design**

Require, e.g.,  $\tau_p = 5.5$  ppm @ 1064nm and  $\Gamma_p = 0.95$  @ 532nm



## **Minimal Noise SD Design**

 $|cap|(|d_{H}|d_{L}|)^{N}|d_{H}|$  substrate



 $\xi_L = 0.02008, \xi_H = 0.029362$  $d_L = 195.493nm, d_H = 112.1nm$ N=18

| λ [nm] |           |                      |
|--------|-----------|----------------------|
| 1064   | 5.5       | τ <sub>p</sub> [ppm] |
| 532    | 0.949261  |                      |
| 670    | 0.0889853 |                      |
| 946    | 0.994795  | Γ                    |
| 980    | 0.999955  | ſ                    |
| 1319   | 0.181319  |                      |
| 1550   | 0.0347272 |                      |

Tweak (adjust) the thicknesses of the *outermost* (first, last) layers, e.g. Let:

$$d_{first}^{(H)} = \frac{\lambda_0}{n_H} \left(\frac{1}{4} - \xi_{first}\right), d_{last}^{(L)} = \frac{\lambda_0}{n_L} \left(\frac{1}{4} + \xi_{last}\right)$$

Keep  $\tau_p @1064nm, \ \Gamma_p @532nm$  unchanged



Tweak so that peak electric field at coating face achieves its *infimum*,

$$|E| = |E^{inc}||1 + \Gamma| =$$
$$= |E^{inc}|(1 - |\Gamma|) \cong |E^{inc}|\frac{\tau_p}{2}$$

For our N=18 SD-ETM design, get

$$\xi_{last} = -0.22862, \ \xi_{first} = 0.030761$$
  
 $d_{last} = 111.389nm, \ d_{first} = 15.4765nm$ 

$$\left| E_{face} \right| \cong 2.7 \cdot 10^{-6} \left| E^{inc} \right|$$

... additional constraints (e.g., flatness) can be accommodated by tweaking additional (outermost) layers ...

## **SD-Core, Tweaked Optimal Design**



LMA started from this to engineer the current AdvLIGO ETM and ITM coatings. They enforced further band-centering tweaks . Details are covered by secret .

## Conclusions

Several tools have been developed in the LVC (and the Scientific Community at large) which may suit the specific mirror noise issue of b-KAGRA, including

-EMT modeling of glassy mixtures

-The idea of nm-layered composites

-A simple systematic procedure for coating thickness optimization