Vertical Suspension-Point Interferometer

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イントロダクション（余談）

Outline

- Original motivation and design of VSPI
- Possible application in KAGRA and TOBA
- Analytical calculation and results
- Conclusion

This presentation may contain incomplete or incorrect descriptions. The purpose is not to show an achievement, but to stimulate discussions for new ideas.

Original Motivation of VSPI

Y. Aso, Ph.D thesis (2006)

- VSPI (Vertical Suspension Point Interferometer)

- Active control scheme to suppress vertical fluctuation, mainly for seismic noise.
- Control upper mass to follow the better-isolated lower mass.
- Fundamental performance is limited by sensor noise .
  → Interferometric sensor.

- Sensitivity is relaxed by
  the H-V coupling factor.

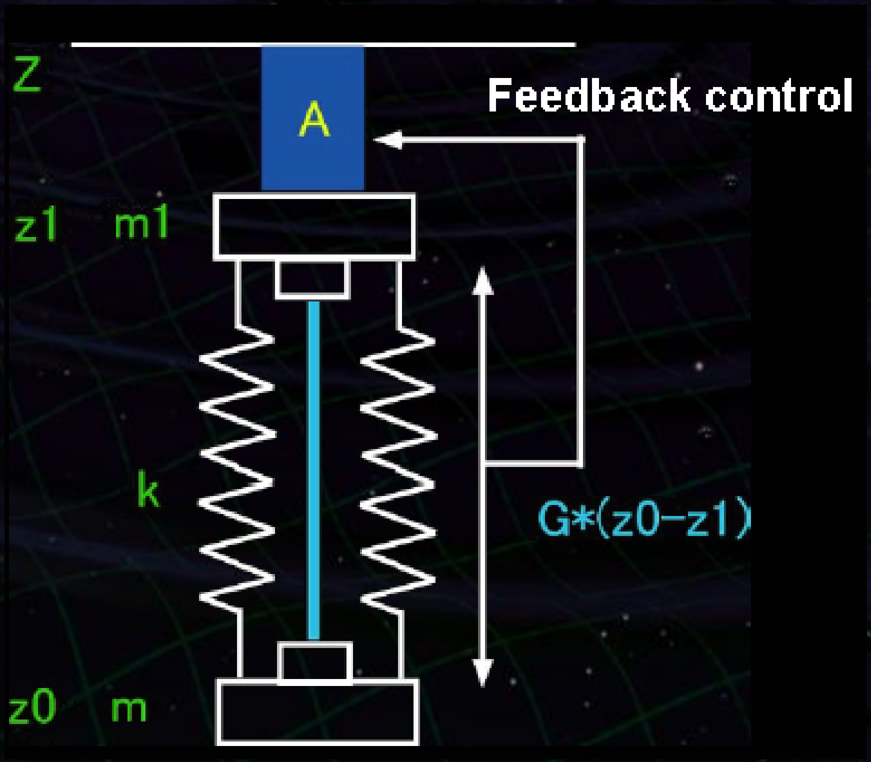


Fig. : Nishida Talk at Elba (2011)

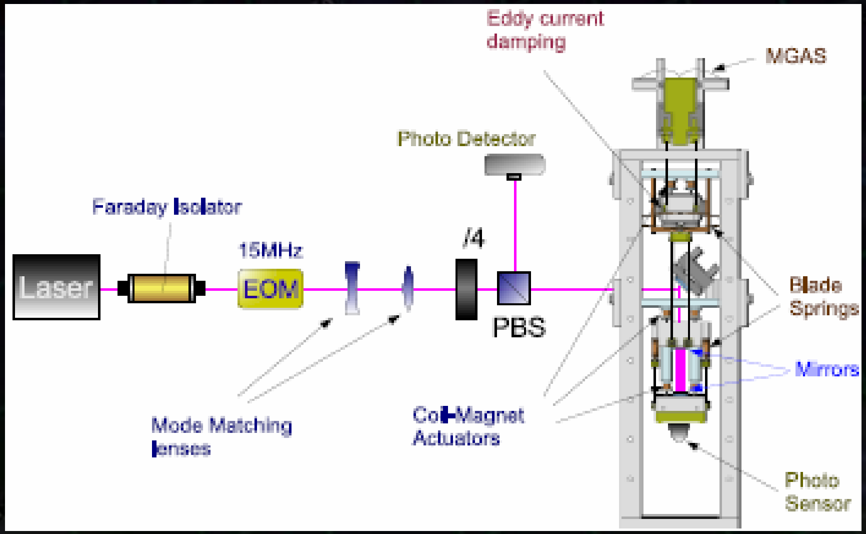
Resonant Frequency

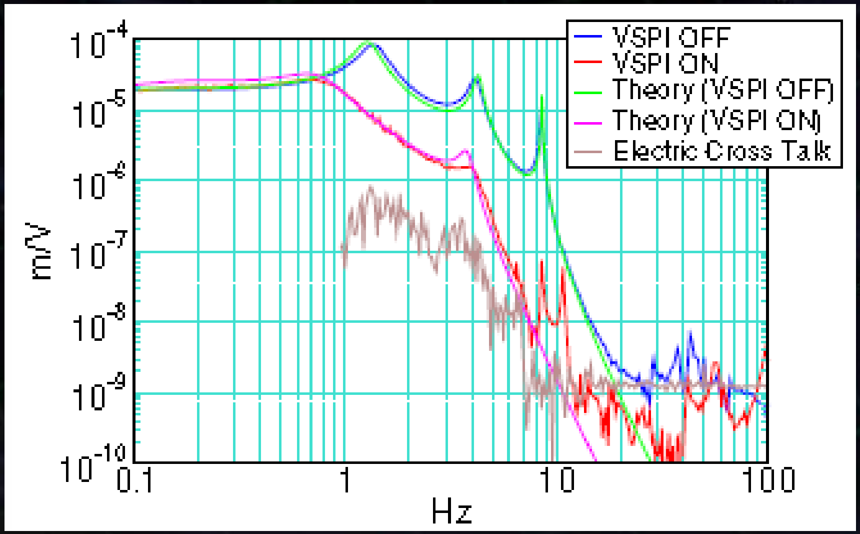
- Reduction of resonant frequency by VSPI
  - Smaller restoring force at suspension → lower resonant freq.
  - Reduction factor : $\sqrt{1+G}$ for res. freq. and Q-value.

Ref: Y. Aso, Appendix D in Ph.D thesis (2006).
K. Kawabe, in 'Detection of GW' p.257 (1998) in Japanese.
N. Mio, in 'GW handbook' p.201 (1992) in Japanese.

VSPI response measurement by Aso

Y. Aso, Ph.D thesis (2006)



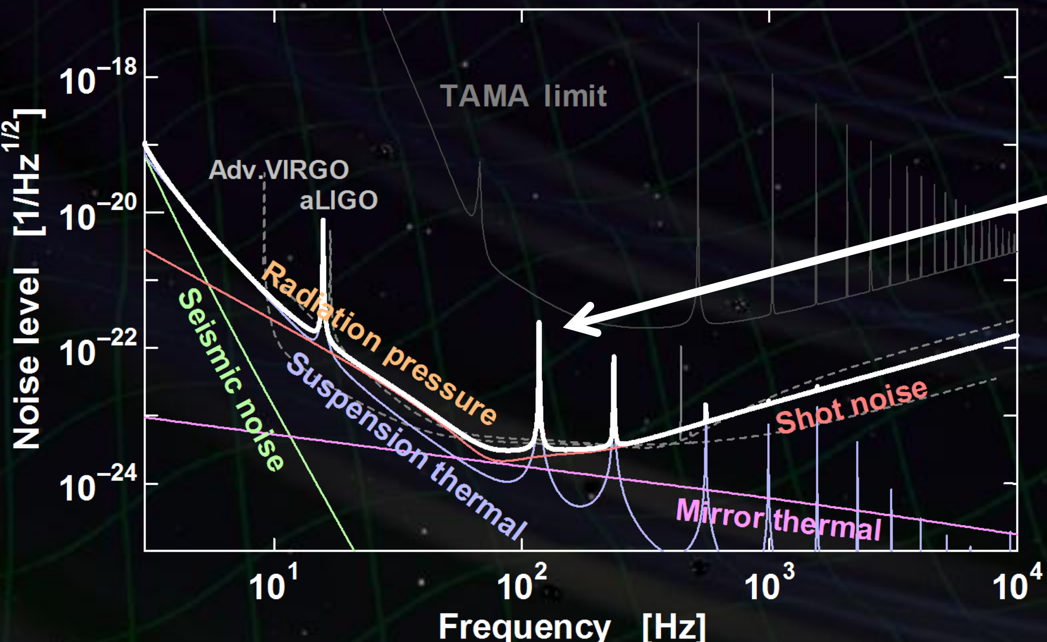


VSPI possibility in KAGRA??

E.Nishida Talk at GWADW Elba (2011)

- Thermal noise peak at ~120Hz, by a vertical resonance at the sapphire suspension

  ⇨ VSPI may solve this problem



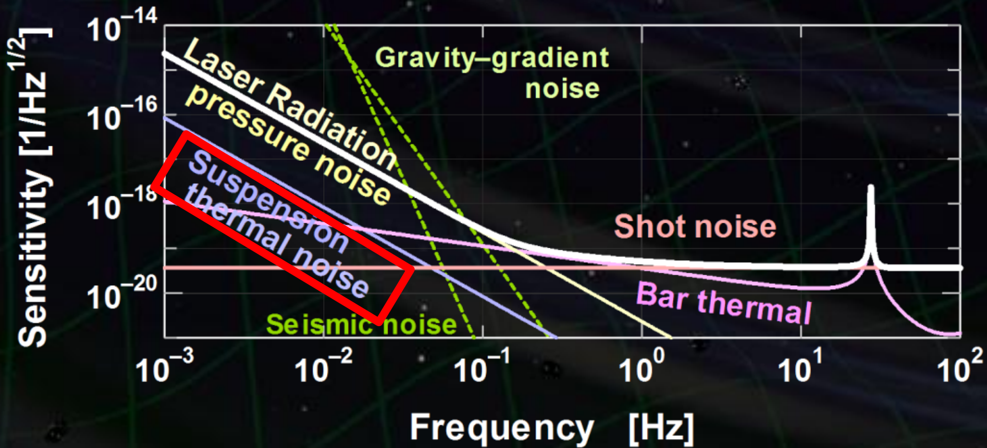
- Almost at the center of the KAGRA obs. band.
- Difficult to shift the freq.
  $f_{reso} \propto 1$ / (cross section)
  → constraints from the heat-extraction and vibration-isolation design.

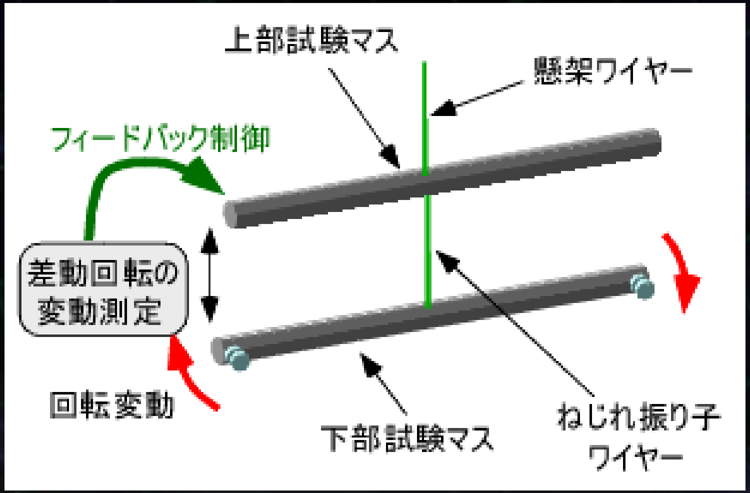
SPI in TOBA

M.Ando (2011)

- ‘Tuning bar’ technique in TOBA,
  same concept as VSPI, but for rotational DoF.
  - Suppression of the suspension thermal noise?
  - Lower resonant frequency
  - Seismic noise suppression

⇨ Very low-freq. GW observation





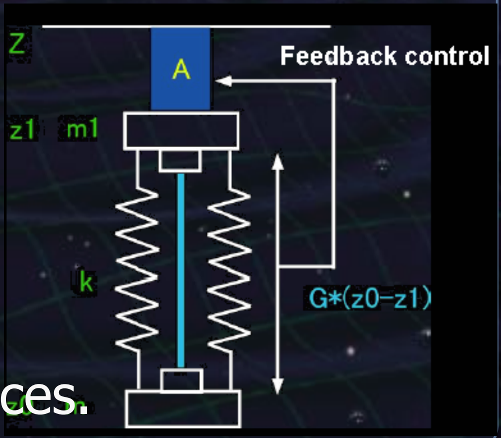
Physical Explanation

- Why thermal noise is suppressed by SPI???

- Explanation 1 : FDT (Fluctuation-Dissipation Theorem)

According to FDT, thermal noise depends on the imaginary part of the mechanical conductance:

$$\tilde{x}^2_{\mathrm{thermal}} = -\frac{4k_{\mathrm{B}}T}{\omega}\mathrm{Im}\left[\frac{\tilde{x}}{\tilde{f}}\right]$$

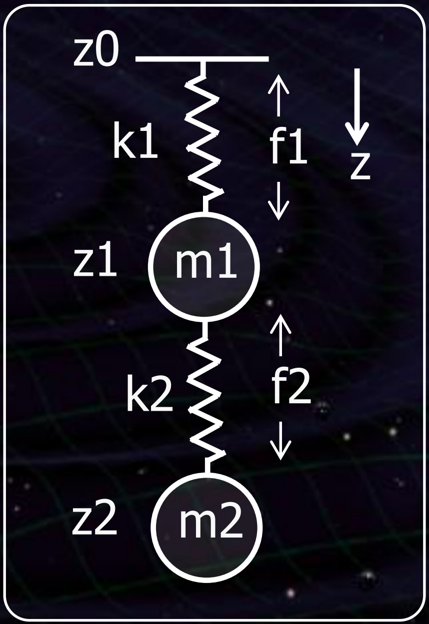


By SPI, mechanical response becomes zero, no deformation in the suspension wire by external forces.

- Explanation 2 : Reduction of effective mass

SPI effectively reduces the inertial mass of the upper mass. If the mass of the upper mass is small enough, the lower mass does not move by the force noise at the suspension wire.

Calculation Model



- Equation of Motion

$$\begin{cases} m_1\ddot{z}_1 = -k_1(z_1 - z_0) + k_2(z_2 - z_1) + f_1 - f_2 + f_c \\ m_2\ddot{z}_2 = -k_2(z_2 - z_1) + f_2 \\ f_c = G(z_2 - z_1) \end{cases}$$

(Control force)

- Fourier transformation to obtain

$$\begin{pmatrix} G - m_1\omega^2 + k_1 + k_2 & -G - k_2 \\ -k_2 & -m_2\omega^2 + k_2 \end{pmatrix} \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{pmatrix} = \begin{pmatrix} k_1\tilde{z}_0 + \tilde{f}_1 - \tilde{f}_2 \\ \tilde{f}_2 \end{pmatrix}$$

⇨ Can be solved for z_2

Calculation Results

KAGRA

- Response

VSPI control gain

$$\tilde{z}_2 = \frac{m_1 m_2 \omega_1^2 \omega_2^2}{D}\tilde{z}_0 + \frac{m_2 \omega_2^2}{D}\tilde{f}_1 + \frac{G - m_1(\omega^2 - \omega_1^2)}{D}\tilde{f}_2$$

Seismic noise — Upper TN — Lower TN

- Determinant

$$D = -m_1 m_2 \omega_1^2(\omega^2 - \omega_2^2) - m_2\omega^2\left\{G - m_1\omega^2 + (m_1 + m_2)\omega_2^2\right\}$$

- Resonant frequencies (for single pendulum)

$$k_1 = m_1\omega_1^2 \qquad k_2 = m_2\omega_2^2$$

Results (1/3)

- High-frequency and infinity gain limit:

$$\omega \gg \omega_1, \omega_2, \ (k_1 = m_1\omega_1^2, \ k_2 = m_2\omega_2^2)$$

$$G \to \infty$$

⇩

- Seismic noise (z0) and thermal noise at the upper stage (f1) are suppressed by the VSPI.

- Thermal noise at the lower stage (f2) is not suppressed.

$$\tilde{z}_2 \sim -\frac{\tilde{f}_2}{m_2\omega^2}$$: Same as the free-mass limit

Results (2/3)

- At the resonance of lower suspension

$$\omega \simeq \omega_2$$

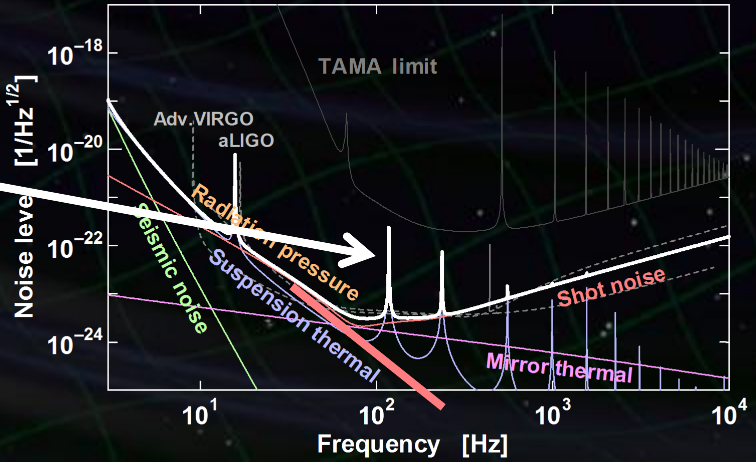
$$G \rightarrow \infty$$

⇩

- VSPI is effective to damp the resonance.

$$\tilde{z}_2 \sim -\frac{\tilde{f}_2}{m_2 \omega_2^2}$$: Same as the free-mass limit

Eliminate this peak



Results (3/3)

- Tune the VSPI control gain

$$\omega \gg \omega_1,\ \omega_2,\ \ (k_1 = m_1\omega_1^2,\ k_2 = m_2\omega_2^2)$$

$$G \sim m_1\omega^2$$

⇩

- Thermal noise at the lower stage (f2) is cancelled.

$$\tilde{z}_2 = 0$$

- Thermal noise at the upper stage (f1) is not cancelled.

$$\tilde{z}_2 = -\frac{\tilde{f}_1}{(m_1 + m_2)\omega^2}$$

*Thermal noise fluctuation is monitored and fed back to the upper stage to cancel its motion. Same suppression may possible by off-line data processing.

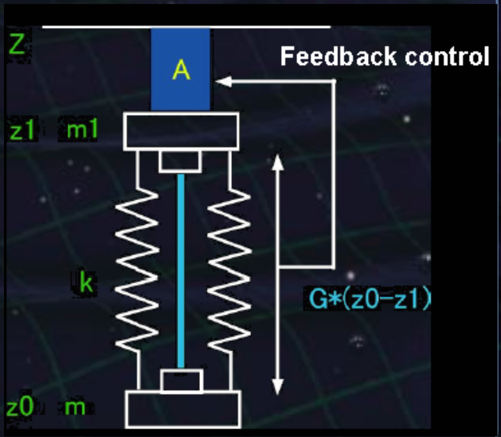
Physical Interpretation

- What is the SPI's effect???

- Explanation 3 : Change the mechanical response

SPI changes the spring constant of the suspension wire : $k_2 \rightarrow 0$. The thermal fluctuation force by the suspension remain.

⇨ Free mass with suspension thermal noise.



Discussion

- Consistency between physical explanations ????

(1) FDT, (2) Effective mass, (3) Mechanical response.

→ K.Yamamoto's comment

- FDT is valid only in a thermal-equilibrium system, and cannot be applied to an actively-controlled system like this.
- Langevin equation would be valid even in this case.

- Use VSPI in KAGRA ???

Please discuss considering:

necessity, technical readiness, interface constraints, …

Conclusion

KAGRA

- VSPI is effective to suppress vibrations from upper stages, such as seismic noise, thermal noise and so on.
- Simple VSPI control ($G \rightarrow \infty$) would not be effective to suppress the thermal noise at the lower-stage suspension thermal noise. However it damps the resonant peak.
- With gain tuning, it is possible to cancel the lower-stage thermal noise. However, upper-stage thermal noise would be a problem in this case.
- For TOBA, SPI (‘tuning-bar technique’) will be useless for thermal noise, but wil be useful to expand the observation band to lower frequencies.

Conclusion

End