
Vertical Suspension- Point Interferometer

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This presentation may contain incomplete or incorrect descriptions. The purpose is not to show an achievement, but to stimulate discussions for new ideas.

Outline

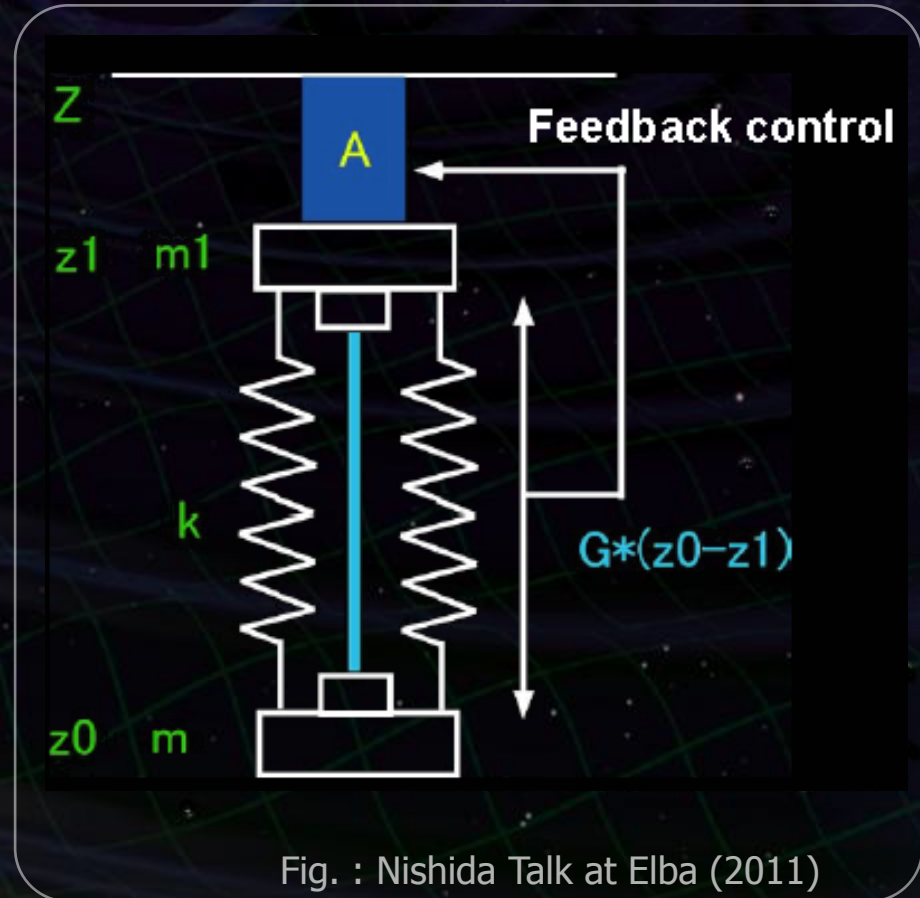
- Original motivation and design of VSPI
- Possible application in KAGRA and TOBA
- Analytical calculation and results
- Conclusion

Original Motivation of VSPI

Y. Aso, Ph.D thesis (2006)

• VSPI (Vertical Suspension Point Interferometer)

- Active control scheme to suppress vertical fluctuation, mainly for seismic noise.
- Control upper mass to follow the better-isolated lower mass.
- Fundamental performance is limited by sensor noise .
 - Interferometric sensor.
- Sensitivity is relaxed by the H-V coupling factor.



Resonant frequency

- Reduction of resonant frequency by **VSPI**
 - Smaller restoring force at suspension \rightarrow lower resonant freq.
 - Reduction factor : $\sqrt{1 + G}$ for res. freq. and Q-value.

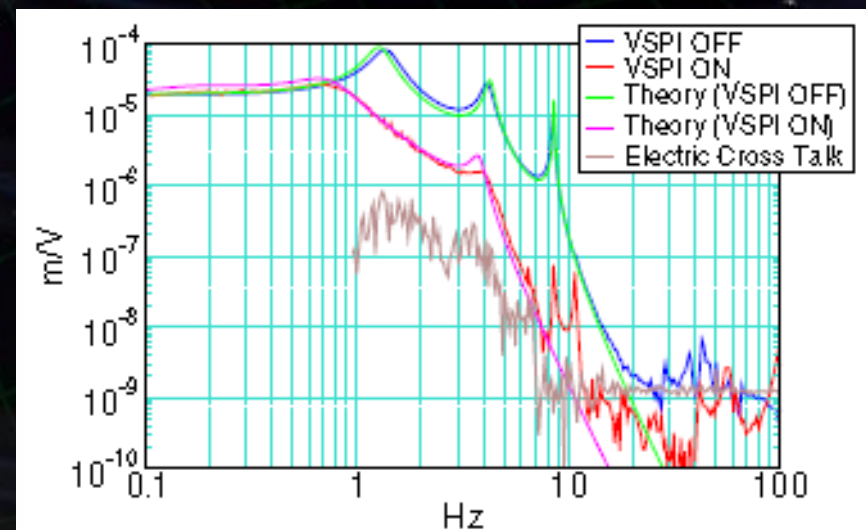
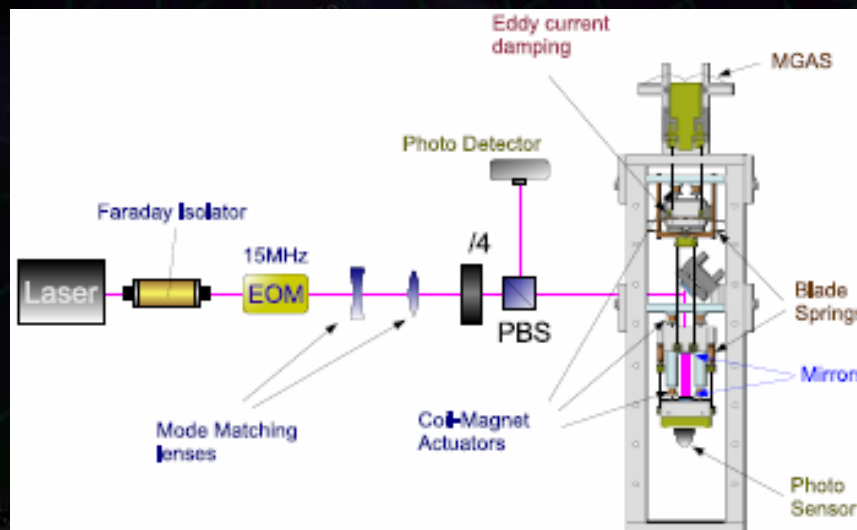
Ref: Y. Aso, Appendix D in Ph.D thesis (2006).

K. Kawabe, in 'Detection of GW' p.257 (1998) in Japanese.

N. Mio, in 'GW handbook' p.201 (1992) in Japanese.

VSPI response measurement by Y.Aso

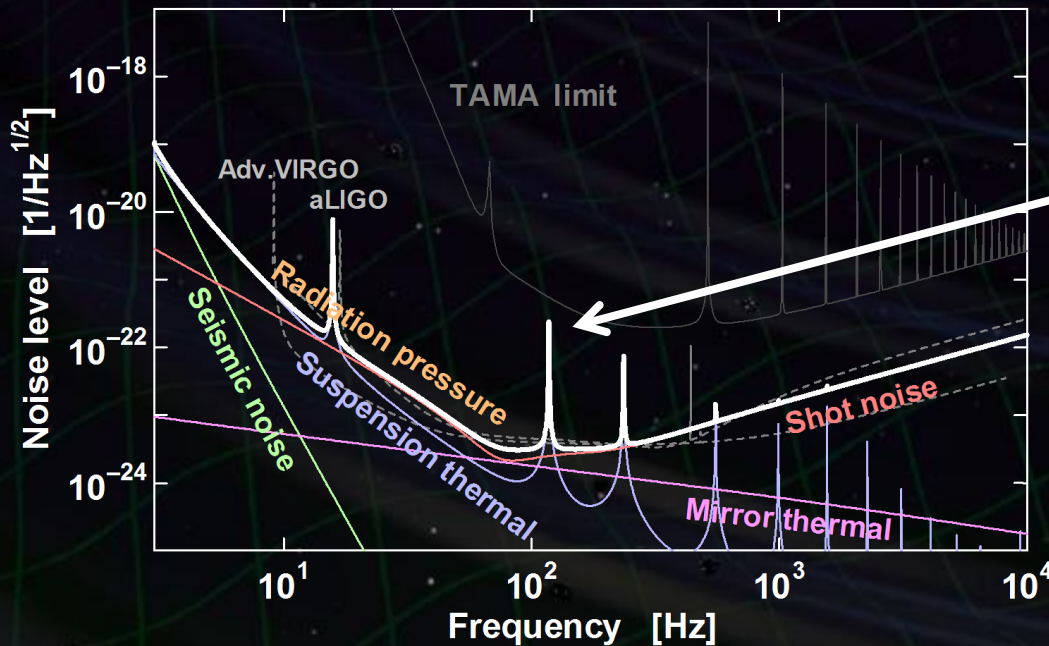
Y. Aso, Ph.D thesis (2006)



VSPI possibility in KAGRA??

E.Nishida Talk at Elba (2011)

- Thermal noise peak at $\sim 120\text{Hz}$,
by a vertical resonance at the sapphire suspension
⇒ VSPI may solve this problem



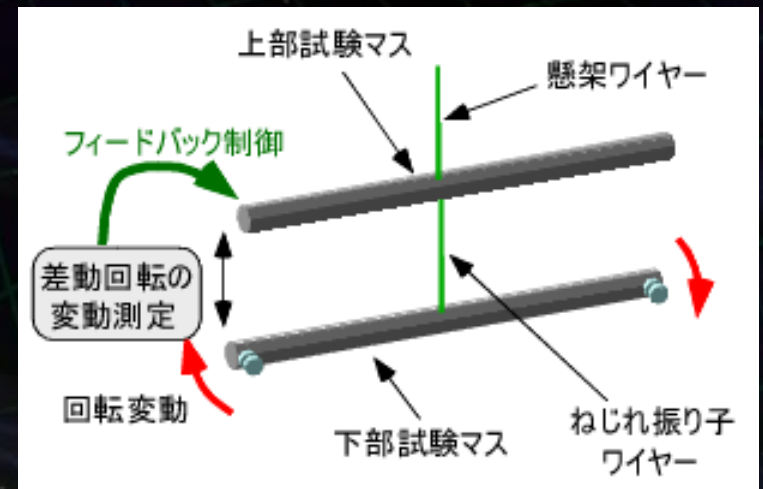
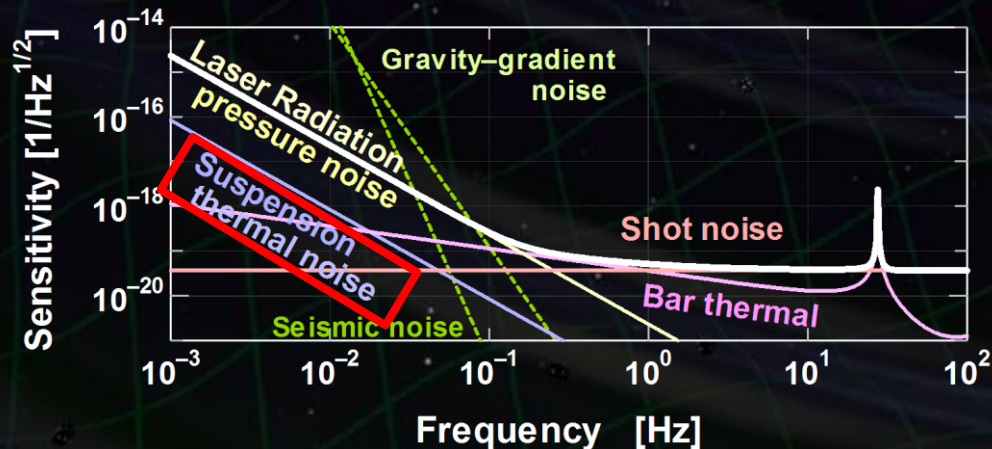
- Almost at the center of the KAGRA obs. band.
- Difficult to shift the freq.
 $f_{\text{reso}} \propto 1 / (\text{cross section})$
→ constraints from the heat-extraction and vibration-isolation design.

SPI in TOBA

M.Ando (2011)

- 'Tuning bar' technique in TOBA, same concept as VSPI, but for rotational DoF.
 - Suppression of the suspension thermal noise?
 - Lower resonant frequency
 - Seismic noise suppression

⇒ Very low-freq. GW observation



Physical Interpretation

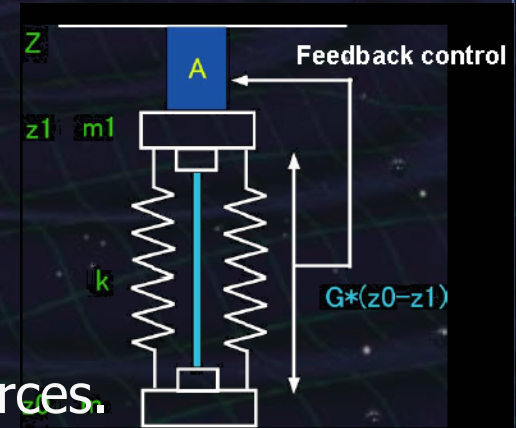
• Why thermal noise is suppressed by SPI???

- Explanation 1 : FDT (Fluctuation-Dissipation Theorem)

According to FDT, thermal noise depends on the imaginary part of the mechanical conductance:

$$\tilde{x}_{\text{thermal}}^2 = -\frac{4k_{\text{B}}T}{\omega} \text{Im} \left[\frac{\tilde{x}}{\tilde{f}} \right]$$

By SPI, mechanical response becomes zero, no deformation in the suspension wire by external forces.



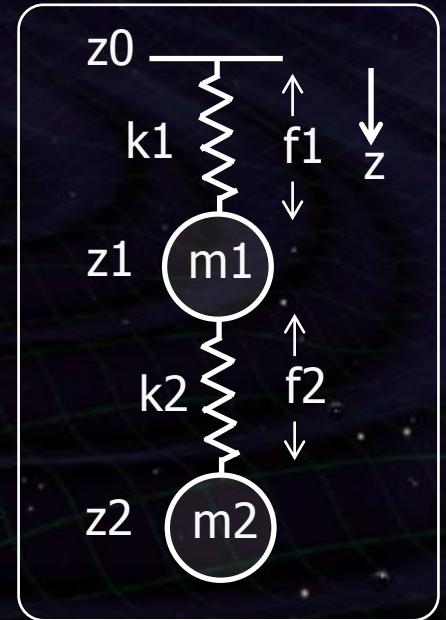
- Explanation 2 : Reduction of effective mass

SPI effectively reduces the inertial mass of the upper mass. If the mass of the upper mass is small enough, the lower mass does not move by the force noise at the suspension wire.

Calculation Model

- Equation of Motion

$$\begin{cases} m_1 \ddot{z}_1 = -k_1(z_1 - z_0) + k_2(z_2 - z_1) + f_1 - f_2 + f_c \\ m_2 \ddot{z}_2 = -k_2(z_2 - z_1) + f_2 \\ f_c = G(z_2 - z_1) \quad (\text{Control force}) \end{cases}$$



- Fourier transformation to obtain

$$\begin{bmatrix} G - m_1 \omega^2 + k_1 + k_2 & -G - k_2 \\ -k_2 & -m_2 \omega^2 + k_2 \end{bmatrix} \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} k_1 \tilde{z}_0 + \tilde{f}_1 - \tilde{f}_2 \\ \tilde{f}_2 \end{bmatrix}$$

⇒ Can be solved for z_2

Calculation Results

- Response

$$\tilde{z}_2 = \underbrace{\frac{m_1 m_2 \omega_1^2 \omega_2^2}{\Delta} \tilde{z}_0}_{\text{Seismic noise}} + \underbrace{\frac{m_2 \omega_2^2}{\Delta} \tilde{f}_1}_{\text{Upper TN}} + \underbrace{\frac{G - m_1(\omega^2 - \omega_1^2)}{\Delta} \tilde{f}_2}_{\text{Lower TN}}$$

VSPI control gain

- Determinant

$$\Delta = -m_1 m_2 \omega_1^2 (\omega^2 - \omega_2^2) - m_2 \omega^2 \{G - m_1 \omega^2 + (m_1 + m_2) \omega_2^2\}$$

- Resonant frequencies (for single pendulum)

$$k_1 = m_1 \omega_1^2 \quad k_2 = m_2 \omega_2^2$$

Results (1/3)

- High-frequency and infinity gain limit:

$$\omega \gg \omega_1, \omega_2, (k_1 = m_1\omega_1^2, k_2 = m_2\omega_2^2)$$

$$G \rightarrow \infty$$



- Seismic noise (z_0) and thermal noise at the upper stage (f_1) are suppressed by the VSPI.
- Thermal noise at the lower stage (f_2) is not suppressed.

$$\tilde{z}_2 \sim -\frac{\tilde{f}_2}{m_2\omega^2} \quad : \text{ Same as the free-mass limit}$$

Results (2/3)

- At the resonance of lower suspension

$$\omega \simeq \omega_2$$

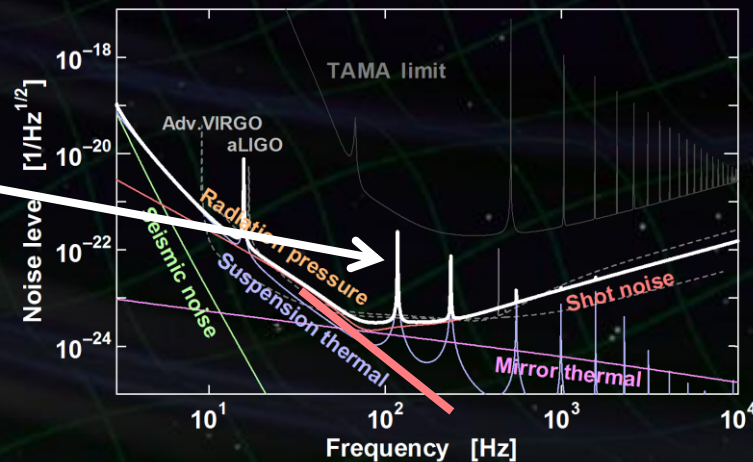
$$G \rightarrow \infty$$



- VSPI is effective to damp the resonance.

$$\tilde{z}_2 \sim -\frac{\tilde{f}_2}{m_2 \omega^2} \quad : \text{ Same as the free-mass limit}$$

Eliminate this peak

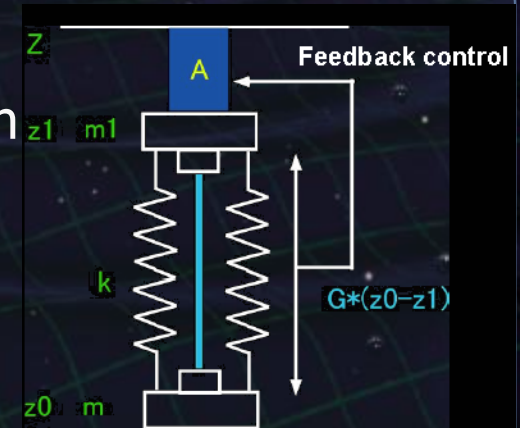


Physical Interpretation

- What is the SPI's effect???

- Explanation 3 : Change the mechanical response
SPI changes the spring constant of the suspension wire. The imaginary part and thermal fluctuation force remain.

⇒ Free mass with suspension thermal noise.



Consistency with Explanation 1 and 2 ????

Results (3/3)

- Tune the VSPI control gain

$$\omega \gg \omega_1, \omega_2, (k_1 = m_1\omega_1^2, k_2 = m_2\omega_2^2)$$

$$G \sim m_1\omega^2$$



- Thermal noise at the lower stage (f2) is cancelled.

$$\tilde{z}_2 = 0$$

- Thermal noise at the upper stage (f1) is not cancelled.

$$\tilde{z}_2 = -\frac{\tilde{f}_1}{(m_1 + m_2)\omega^2}$$

- *Thermal noise fluctuation is monitored and fed back to the upper stage to cancel its motion. Same suppression may possible by off-line data processing.

Conclusion

- VSPI is **effective** to suppress vibrations at upper stages, such as seismic noise, thermal noise and so on.
- Simple VSPI control ($G \rightarrow \infty$) would **not be effective** to suppress the thermal noise at the lower-stage suspension thermal noise. However it **damps** the resonant peak.
- With gain tuning, it is possible to **cancel** the lower-stage thermal noise. However, upper-stage thermal noise would be a **problem** in this case.
- For TOBA, SPI ('tuning-bar technique') will be **useless** for thermal noise, but will be **useful** to expand the observation band to lower frequencies.