Seamless time series generator for KAGRA noise and GW numerical simulation

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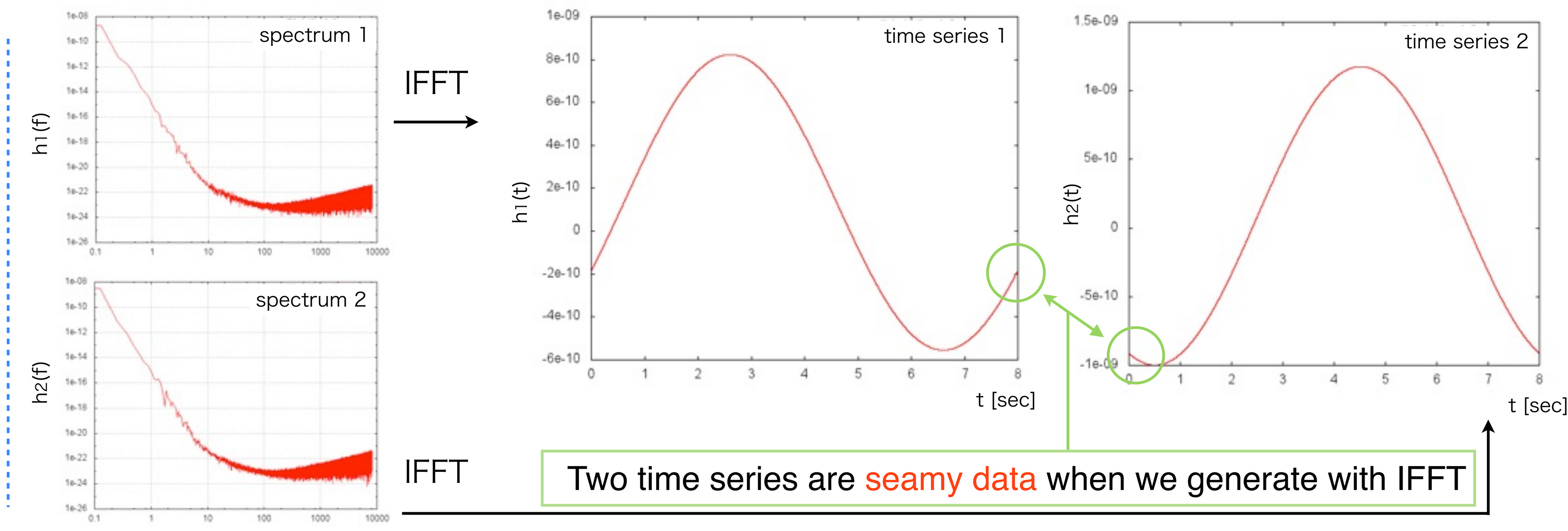
1. Introduction

We often generate time series signal which consists of noise and gravitational wave for the interferometer using FFT/IFFT. However, since FFT/IFFT method treat the data with finite time chunks, it is not good to use for the study of continuous processing of signals across long duration over the chunks. Moreover these generated noises will not joint smoothly to neighbor chunks. To mimic the realistic raw data taking and calibration process, we would like to generate seamless time series data.

We try to generate seamless gaussian noise to interface transfer function with white gaussian noise in time region. Where, transfer function is interferometer sensitivity written in complex frequencies. In this calculation we use Laplace transform and matrix operation. About another noises (thermal noise, line noise etc) we can generate time series directly considering Q-value, repetition of excitation. We will display these methods in detail and show result of simulation.



2A. Gaussian noise

We get colored noise $x(t)$ on time series by convolution of white noise $w(t)$ and impulse response $g(t)$.

$$x(t)=\int_{-\infty}^{t} g(t-\tau)w(\tau)d\tau$$

In fourier region colored spectrum $S_x(w)$ written in scalar product of white spectrum $S_w(w)$ and transfer function $G(w)$

$$S_x(\omega)=|G(\omega)|^2 S_w(\omega)$$

We required two conditions of transfer function, and operating transfer function to white noise [1][2].
1) Transfer function written in idempotent of integer of complex frequencies s.
2) When $s \to \pm\infty$, $G(s) \to 0$.

$$G(s)=\frac{P(s)}{Q(s)}=\frac{a_0+a_1s+\cdots+a_ms^m}{b_0+b_1s+\cdots+b_ns^n}$$

$$s=i\omega, n>m \qquad a_n, b_n=\text{const.}$$

Time Series $x(t)$ can be written with differential operator.

$$x(t)=\frac{P(D)}{Q(D)}w(t) \qquad \left(D=\frac{d}{dt}\right)$$

$$\Leftrightarrow \begin{array}{ll} Q(D)\phi(t)=w(t) & \cdots ① \\ x(t)=P(D)\phi(t) & \cdots ② \end{array}$$

The first we make up vector $z(t)$, vector $f(t)$ and matrix A to compute $\phi(t)$ [3].

$$① \Leftrightarrow b_0\phi+b_1\phi'+\cdots+b_n\phi^{(n)}=w(t)$$

$$\Leftrightarrow \frac{dz(t)}{dt}=Az(t)+f(t)$$

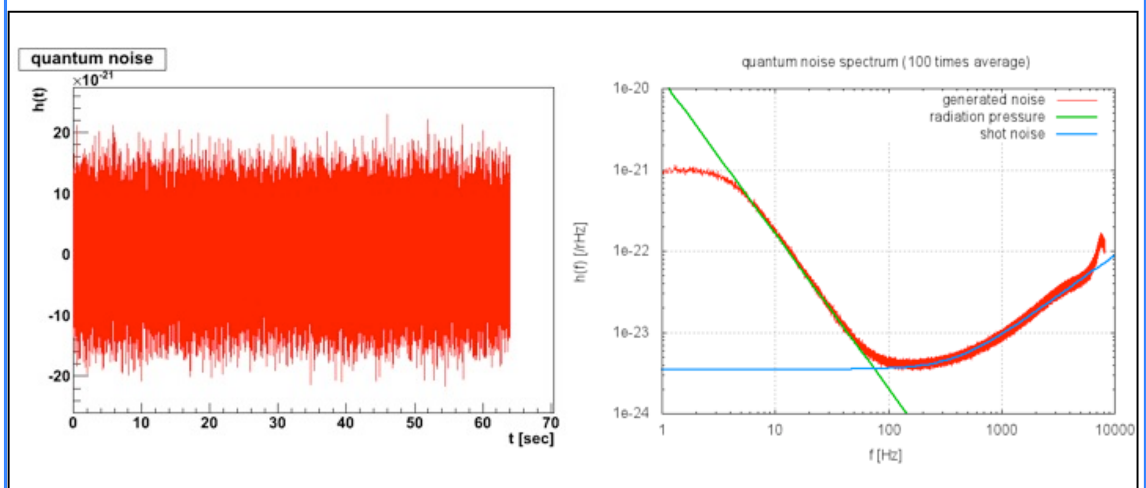
$$z=[\phi(t)\ \ \phi'(t)\ \ \cdots\ \ \phi^{(n-1)}(t)]^T \qquad A=\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -b_0 & -b_1 & -b_2 & \cdots & -b_{n-1} \end{pmatrix}$$

$$f(t)=[0\ 0\ \cdots\ w(t)]^T$$

To compute this differential equation, we can calculate $\phi(t)$.

Next using $\phi(t)$ solved from differential equation, We compute noise signal $x(t)$.

$$② \Leftrightarrow x(t)=a_0\phi+a_1\phi'+\cdots+a_m\phi^{(m)}$$



Left figure is generated time series and right is 100 times average of spectrum. We see generated spectrum(red) is along shot noise(blue) and radiation pressure(green) curve.
We checked gaussianity of generated noise with Kolomogrov-Smirnov test and Anderson-Darling test [4].

2B. Quasi-stationary noise

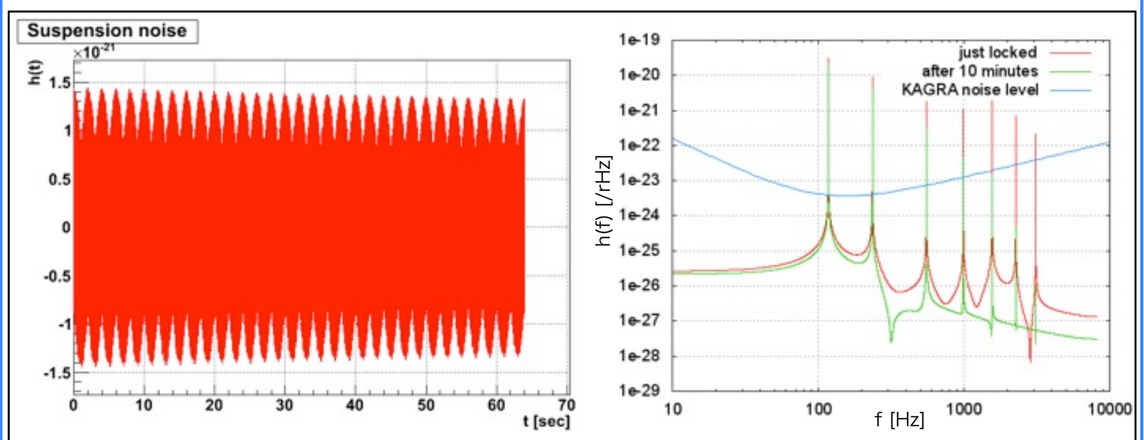
1）Suspension noise (violin mode)

When Interferometer locked, suspension wires excited. And these wires excited all of the time by thermal.

One excitation depend on Q-value of wires, peak frequency w_0 and initial phase $\phi_{\rm ini}$. A is amplitude.

$$x(t)=Ae^{-\frac{\omega_0 t}{2Q}}\sin(\omega_0 t+\phi_{\rm ini})$$

Excitation at lock of interferometer is generated with this equation where amplitude and initial phase are given a random number.
At this time lock timing match in onset generating noises.



Left figure is suspension noise and right is spectrum. Spectrum of just locked(red) decayed to one of after 10 minutes(green). Noise include from 1st to 7th violin mode.

If We generate Excitation by thermal in same method, calculation cost becomes very expensive. So We add in Brownian motion.
Thermal noise x(t) change complex signals X(t), Y(t), Z(t).

$$X(t)=\int_t^{t+\delta t} x(t')\cos(\omega_0 t')\mathrm{d}t'$$

$$Y(t)=\int_t^{t+\delta t} x(t')\sin(\omega_0 t')\mathrm{d}t'$$

$$Z(t)=X(t)+iY(t)$$

Z(t) moves δZ(t) during sampling interval, where δZ(t) is rayleigh distribution.

$$Z(t_{i+1})=Z(t_i)e^{-\frac{\omega_0\delta t}{2Q}}+\delta Z(t_{i+1})$$

$$P_{\rm rayleigh}(\delta Z)=\frac{\delta Z}{\sigma^2}e^{-\frac{\delta Z^2}{2\sigma^2}}$$

where σ^2 is variance of gaussian distribution (Re[δZ], Im[δZ]).

$$<\delta Z^2>=<\delta Z>^2+\sigma_{\delta Z}^2 \qquad <x> \text{ is expected value of } x$$

$$=\mu_{\rm rayleigh}^2+\sigma_{\rm rayleigh}^2$$

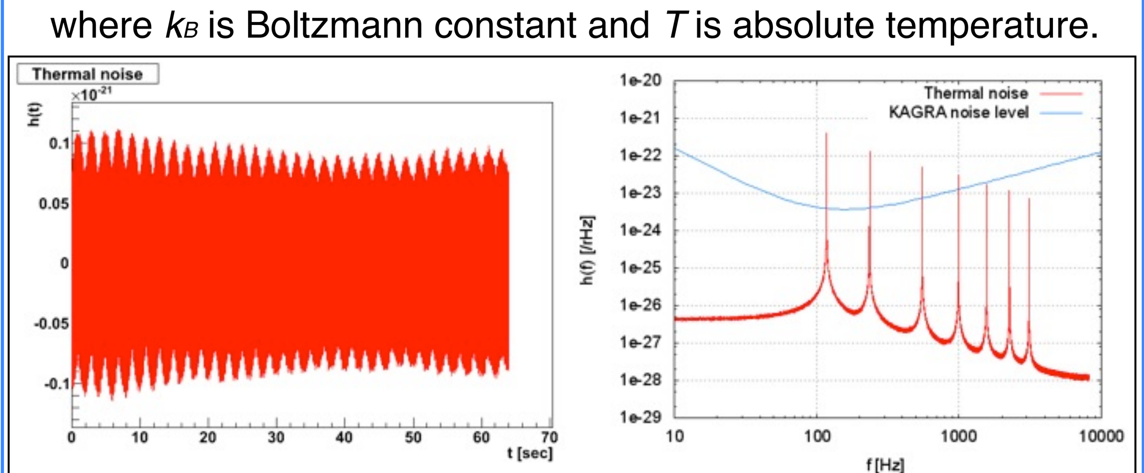
$$=\frac{\pi}{2}\sigma^2+\frac{4-\pi}{2}\sigma^2$$

$$\sigma^2=\frac{<\delta Z^2>}{2}$$

Expectation of Brownian motion is written in sampling interval δt.

$$<\delta Z^2>\sim\frac{k_BT}{Q}\left(\delta t+\frac{\delta t}{2}(1-e^{-2})\right)$$

where k_B is Boltzmann constant and T is absolute temperature.



These figures are time series of thermal noise(left) and 100 times average of spectrum of one(right).

2C. Non-stationary noise

We modeled one spike noise following equation.

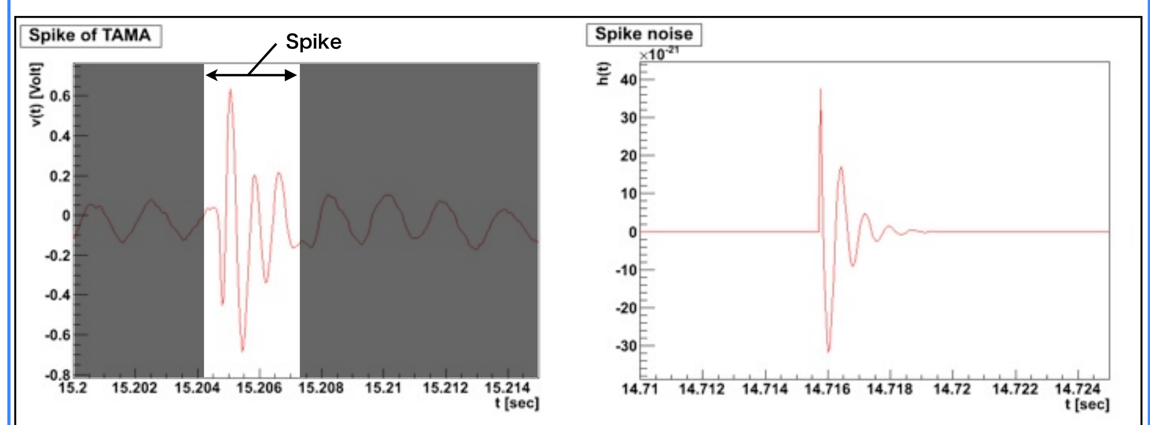
$$x(t)=Ae^{-\frac{t-t_0}{\tau}}\cos(2\pi f_0(t-t_0))$$

where A is amplitude, t_0 is start of spike signal, τ is damping time constant and f_0 is typical frequency.
τ_0 and f_0 are given base on TAMA data. τ0 is similar to 0.6 msec and f0 is similar to 1300Hz

Interval of two spike signals allowed exponential distribution.

$$p(x)\mathrm{d}x=\frac{1}{T}e^{-\frac{x}{T}}\mathrm{d}x$$

T is expected value of signal interval in our simulation.



Left figure is TAMA spike noise of which f_0 similar to 1300Hz.
Right is generated spike consulting parameter of noise of TAMA.

2D. Line noise

Line noise is sine curve fluctuating frequency. At KAMIOKA center frequency is 60 Hz. This noises are given assuming that frequency is constant during sampling interval δt.

$$x(t)=A\sin(\phi(t))$$

where A is amplitude and $\phi(t)$ is phase of signal.
Advance of phase per δt is calculated with following formula.

$$\phi(t_{i+1})=\phi(t_i)+\omega(t_{i+1})\delta t$$

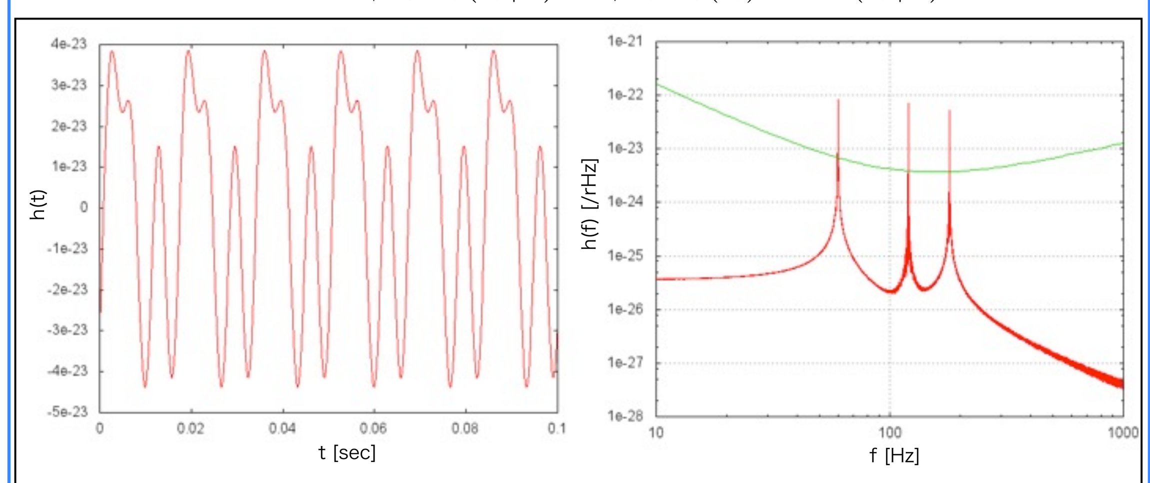
$$\omega(t)=2\pi f(t)$$

$$f(t) \text{ is random number around 60Hz}$$

We suppose frequencies of Harmonics signals depend on frequencies of fundamental wave.

e.g) 2 time wave $\quad x_{\rm 2time}(t)=B\sin(\phi_{\rm 2time}(t))$

$$\phi_{\rm 2time}(t_{i+1})=\phi_{\rm 2time}(t_i)+2\omega(t_{i+1})\delta t$$

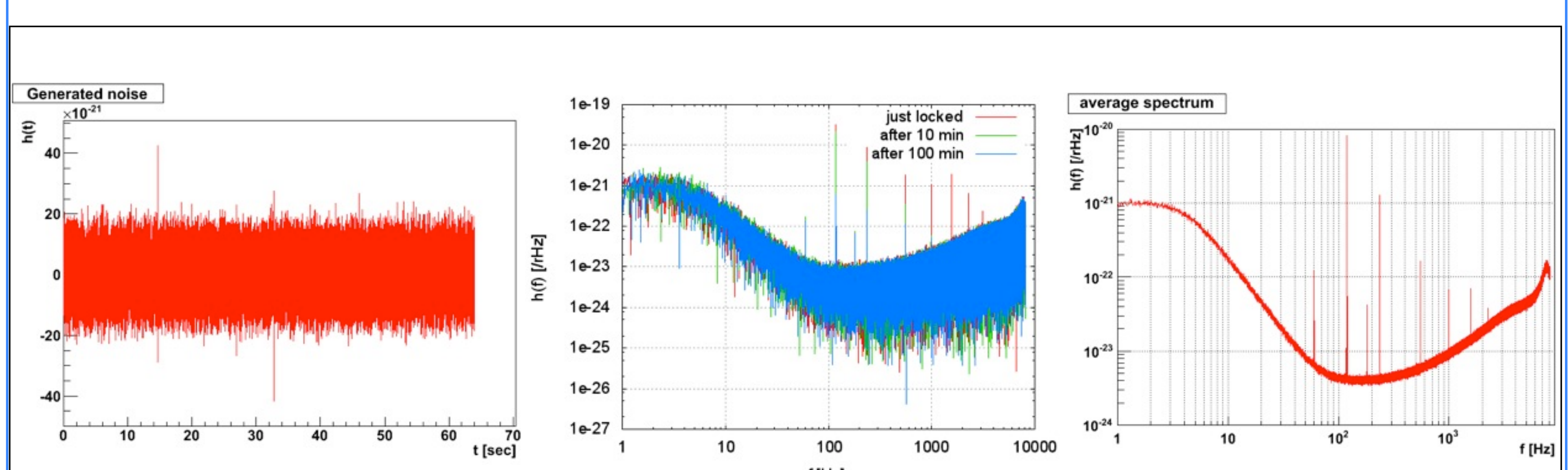


Left figure is time series of line noise and right is spectrum with Quantum noise curve(green).
Today this noise include fundamental, 2 time and 3time wave.

3. Generated noise

Interferometer's noise is linear sum of shown noises. All type of noises generated without IFFT, so we can use noise datas without regard to seam. Also to memorize initial parameter of generating routine and data length of generated, we can generate continuation of previous time.

Following figures are generated total noises include quantum, suspension, thermal, spike, line noise. The left is time series and center is spectrum of just lock interferometer(red), 10 minutes after from lock and 100 minutes after from lock.
Right figure is 100 times average of spectrum of generated noise.
These noises allow KAGRA spectrum and mimic to decay excitation of suspension wire.



4. Summary and Future work

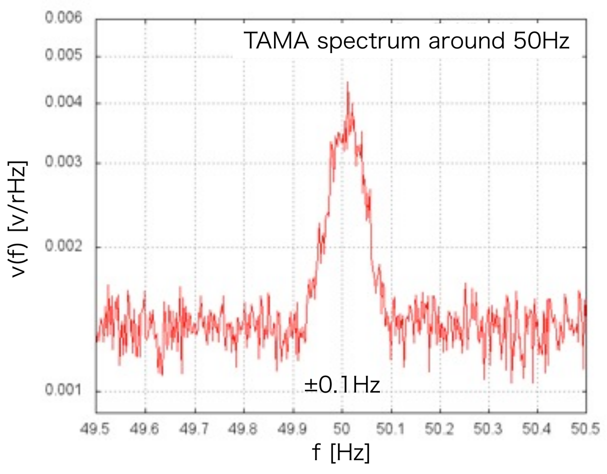
Summary
・We showed seamless simulation noise allowed KAGRA spectrum.
・These noise includes gaussian-stationary, quasi-stationary and non-stationary noises.
・We modeled quasi-stationary and non-stationary noises with TAMA signals.

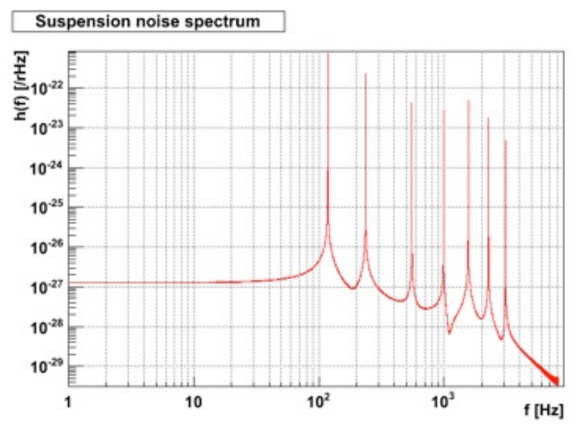
Future work
・To generate Interferometer's row data from KAGRA spectrum noise.
・To read out generated noises in frame format to use analysis simulation.

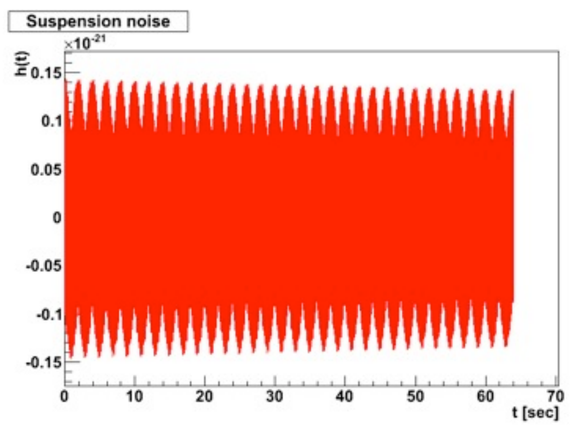
5. References

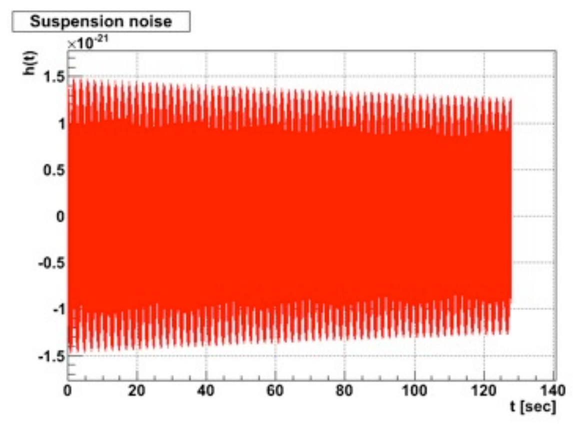
[1] Heinzel, G. (2007) S2-AEI-TN-3034.
[2] Franklin, J. N. (1965) SIAM Review, **7**, 68-80.
[3] Franklin, J. N. (1963) The Annals of Mathematical Statistics, **34**, 1259-1264
[4] T. W. Anderson (1954) Journal of the American Statistical Association, **49**, 765-769

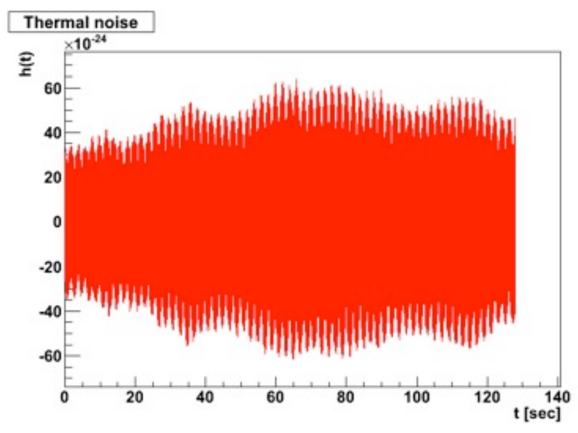
TAMA line


TAMA spectrum around 50Hz
v(f) [v/rHz]
f [Hz]
±0.1Hz


Suspension noise spectrum
h(f) [rHz]
f [Hz]


Suspension noise
h(t)
t [sec]


Suspension noise
h(t)
t [sec]


Thermal noise
h(t)
t [sec]