Continuum, Grain and Burst Behaviors of Stochastic Gravitational Waves from Cosmic Strings by Numerical Simulation

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☑ Abstract

With network of ground-based detectors (KAGRA, aLIGO, aVirgo, AIGO), we calculate overlap reduction function and estimate upper limit for isotropic stochastic gravitational wave background (SGWB), $\Omega_{gw}h_0^2 > 10^{-9}$ .Compared upper limit estimated with theoretical predictions of SGWB, we may detect gravitational wave(GW) from cosmic string. Although there are cusp on loop and kink on finite string which emit GW from cosmic string, in this study we focus on GW from cusp. Beamed GW burst are emitted from cusp. GW of bigger amplitude much than detector noise is seen as burst GW , and GW of smaller amplitude appear as continuum of stochastic background. We are also interested in the behavior of small GW components - "grains of small bursts" -, which amplitudes are similar to the detector instrumental noise level and its rate is order of 1Hz. Less enough analysis has performed for such GW before.The waveform of GW from cosmic string in time domain is known. Then we generate simulation data by method that one GW superpositions many times. In this poster, we calculate waveform of GW from cusp and some values which characterize waveform.

☑ Property of Cosmic String

Cosmic string is one dimensional topological detect which can be formed in the Early Universe. Strings form the network including infinite strings and closed loops. Beamed GW burst are emitted from cusp.

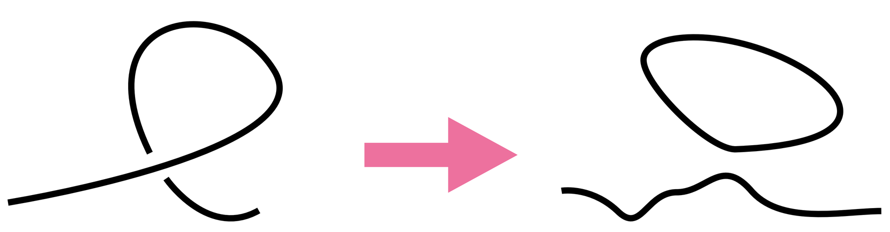


FIG.1 Loops are produced when infinite strings collide and reconnect with themselves.

Cosmic string is characterized three parameters,

- μ : energy per length in string
- α : loop size for Hubble scale
- p : reconnection probability. (p=1 fixed)

When loop generated, typical loop size is αl_H . As loop emit GW, loop lose its length and finally vanish. Although α is unknown, $G\mu/c^2$ is constrained by LIGO search [3] for GW burst from cosmic string and by CMB observation [4].

$$G\mu/c^2 < 2.3\times10^{-7}\ (95\%\ \mathrm{c.l.})$$

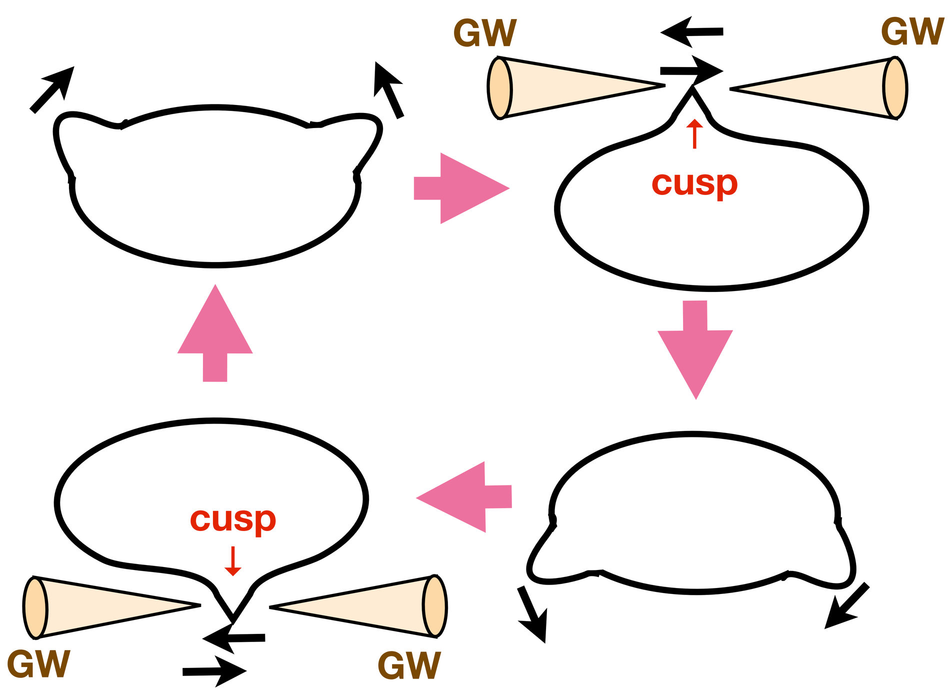
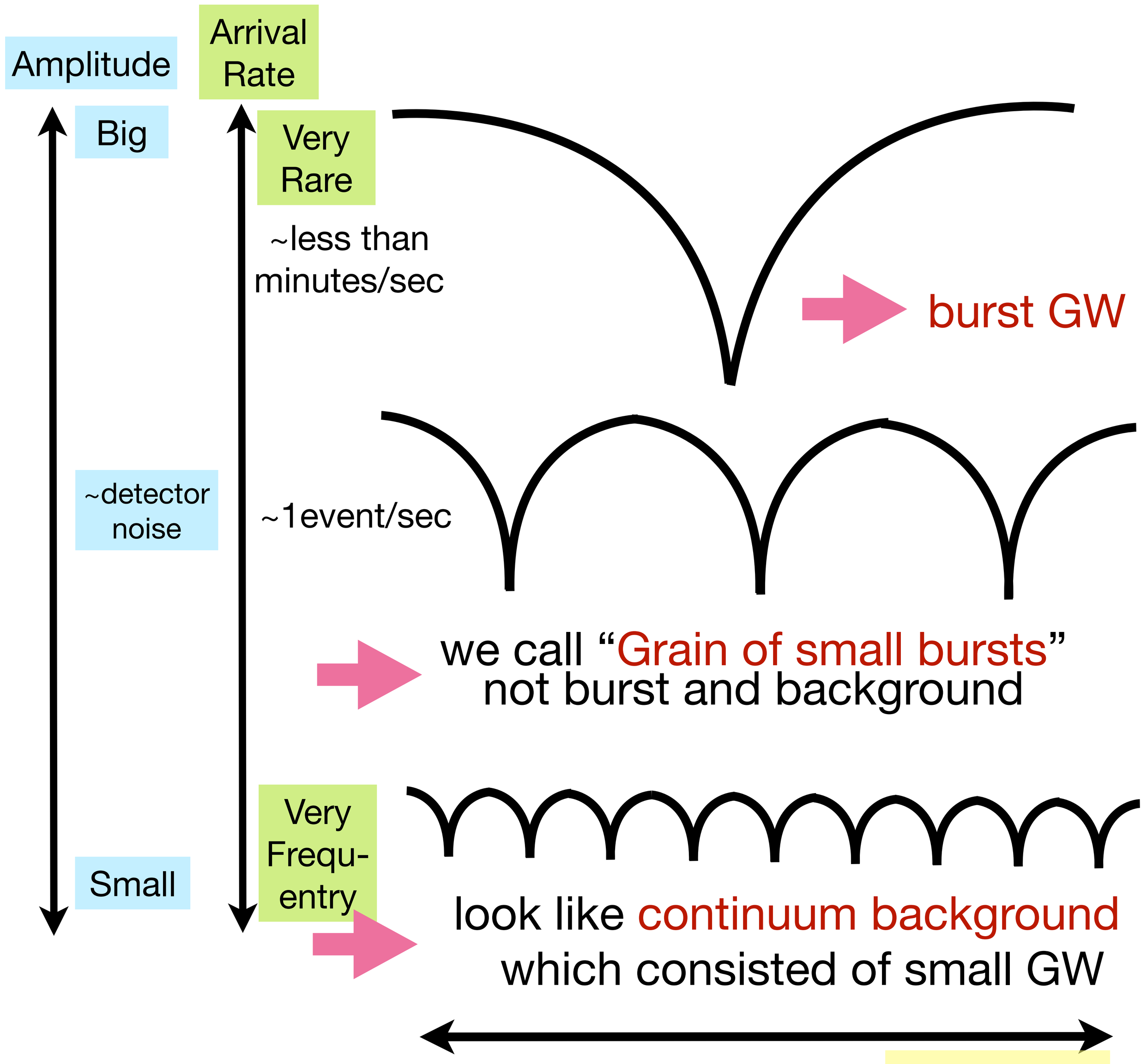


FIG.2 When two curves which move to opposite direction intersect on loop, cusp is generated and emits GW. While circumference of loop L >0, these events are repeated.

☑What's "Grains of small bursts" ?

Usually gravitational wave is categorized by size of amplitude. As there are loops having various circumferences in Universe, the amplitude of GW from cusp covers a very wide range.



☑Gravitational Wave from cusp

The waveform of GW from loop which is generated at z_{make} and is radiated at z_{rad} is known as [2][5]

$$h(t) = \frac{G\mu l^{2/3}(\alpha, z_{make}, z_{rad})c^{1/3}}{r(z_{rad})}|t(1+z)|^{1/3}$$

Loop release GW periodically and shrink. As Hubble scale l_H , time evolution of loop circumference is expressed as

$$l(z_{make}, z_{rad}) = \alpha l_H(z_{make}) - \frac{\Gamma G\mu}{c^2}(t_{make} - t_{rad})$$

and its period is $\tau = (1+z_{rad})\dfrac{l(z_{make}, z_{rad})}{2c}$

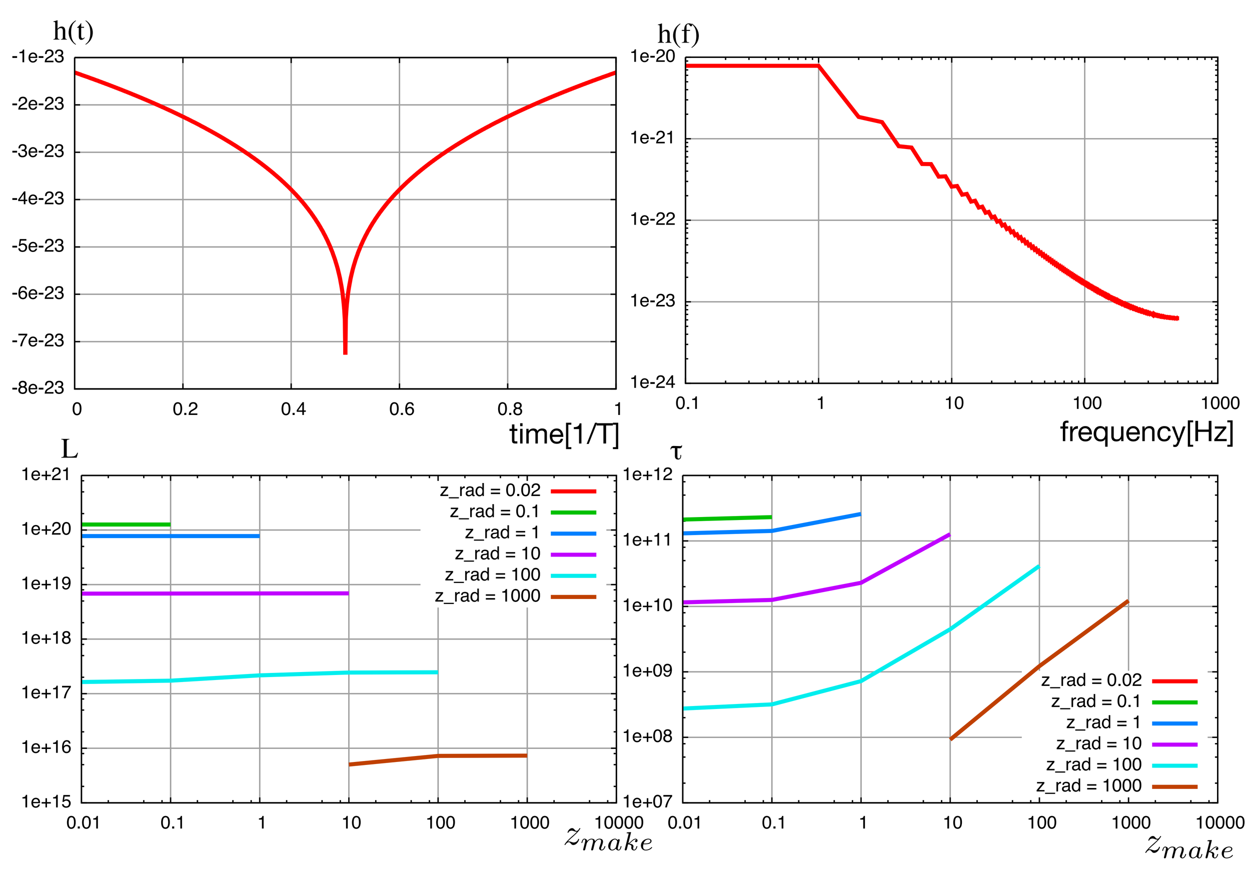
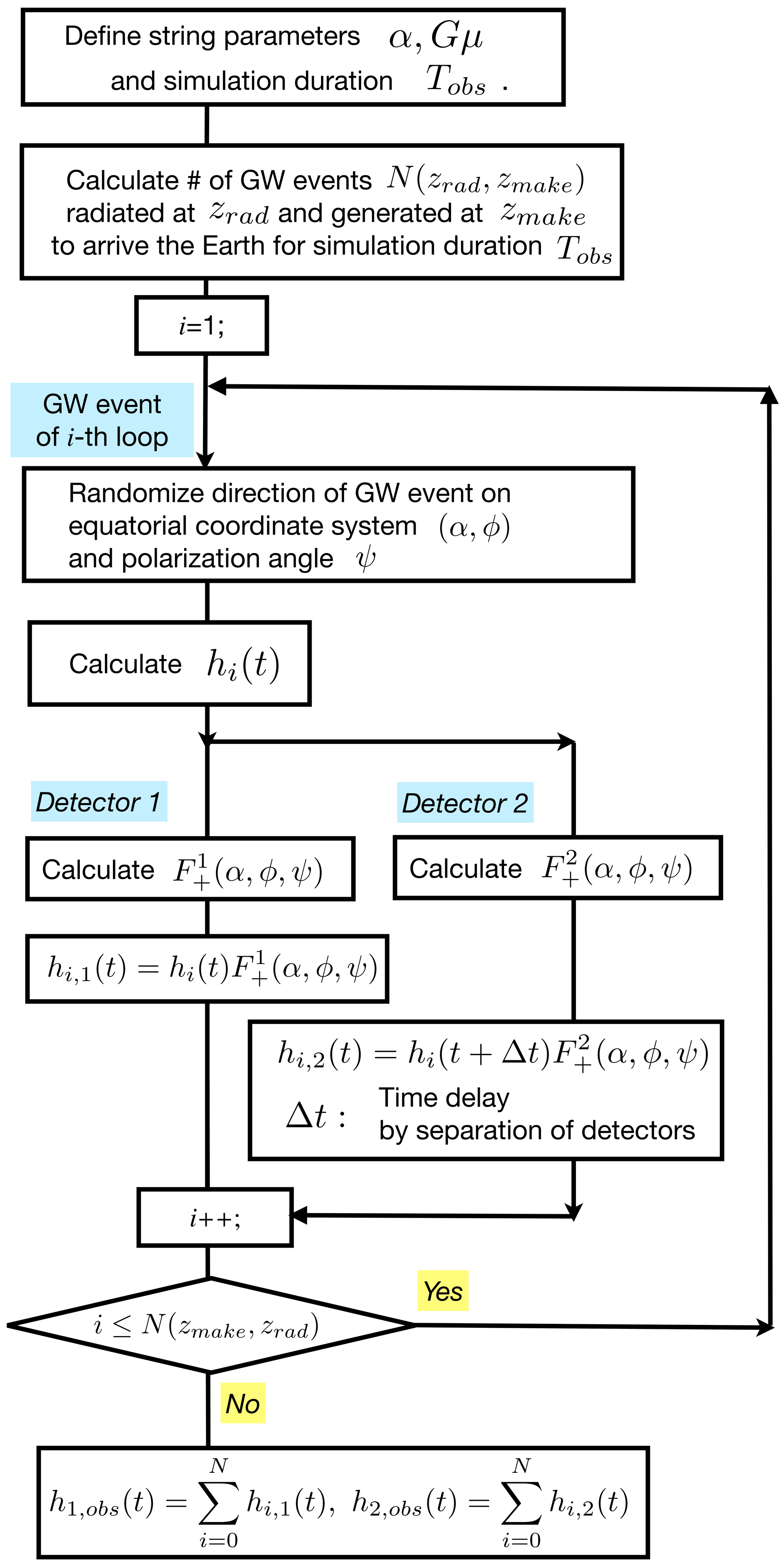


FIG.4 (top left) time[1/T] vs GW waveform h(t)
(top right) frequency vs spectrum h(f), we confirm $h(f) \propto f^{-4/3}$
(bottom left) z_{make} vs circumference of loop $l(z_{make}, z_{rad})$
(bottom right) z_{make} vs period τ

☑Flow Chart



☑Summary & Future Work

We calculate overlap reduction function γ(f) and the upper limit with ground-based detectors (KAGRA, aLIGO, aVirgo, AIGO). We calculate GW waveform h(t) from cusp and time evolution of circumference and period at each redshift z.

We will calculate arrival event rate to the Earth. And we will generate simulation data along flow chart and arrival rate and search new analysis method for "grains of small burst".

☑Reference

[1] Bruce Allen and Joseph D. Romano, *Phys. Rev. D*, 59, 102001
[2] Thibault Damour and Alexander Vilenkin, *Phy. Rev. D*,64,064008
[3] B. P. Abbott *et al.*, *Phys, Rev. D*, 80, 062002
[4] Uros Seljak, Anze Slosar and Patrick McDonald, astro-ph/0604335
[5] Joey Shapiro key and Neil J. Cornish gr-qc/0812.1590

☑ Overlap Reduction Function

This function quantifies two effects:[1]

1. separation time delay between two detectors.
2. non-parallel alignment of the detector arms.

$$\gamma(f) = \frac{5}{8\pi}\sum_A \int_{S^2} d\hat{\Omega}\ e^{i2\pi f\hat{\Omega}\cdot\Delta\vec{x}/c}F_1^A F_2^A$$

$F_i^A(\hat{\Omega})$ is the response of *i*-th (*i*=1,2) to the A = +,× polarization.

$\hat{\Omega}$ is a unit vector specifying a direction.

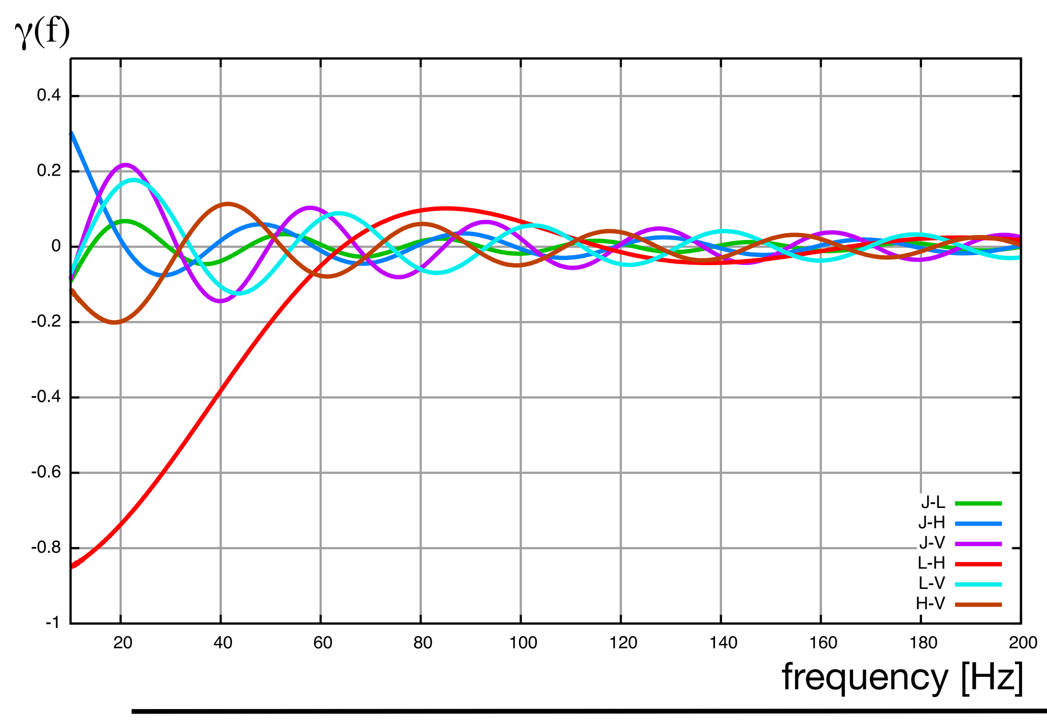


FIG.5 Frequency vs overlap reduction function γ(f).
On-going detectors are used to calculate.
Labels mean that J is KAGRA, L is LIGO Livingston, H is Hanford, V is Virgo. Only detectors under going on are showed.

☑ Upper Limit

Upper limit for isotropic stochastic background is estimated by following inequality, [1]

$$\Omega_{gw}(f) \ge \frac{1}{\sqrt{T_{tot}}}\frac{10\pi^2}{3H_0^2}\left[\frac{\gamma^2(f)}{f^6 P_1(f)P_2(f)}\right]^{-1/2} \times \sqrt{2}[\mathrm{erfc}^{-1}(2\alpha) - \mathrm{erfc}^{-1}(2\gamma)]$$

H_0 is a Hubble constant. $\gamma(f)$ is overlap reduction function.
$P_i(f)$ is the noise power spectra. α is a false alarm rate.
γ is a desired detection rate. (not overlap reduction function)
Here, we fix α=5% and γ=95%.

And we can extend this upper limit from two detector to the network,

$$(h_0^2\Omega_{gw})^{-2} = \sum_{j=0}^{n}\sum_{j<i}^{n}({}^{(i,j)}h_0^2\Omega_{gw})^{-2}$$

Taking combination of many detectors will improve upper limit. In this case, we calculate with KAGRA, LIGO Livingston, LIGO Hanford, Virgo, AIGO.

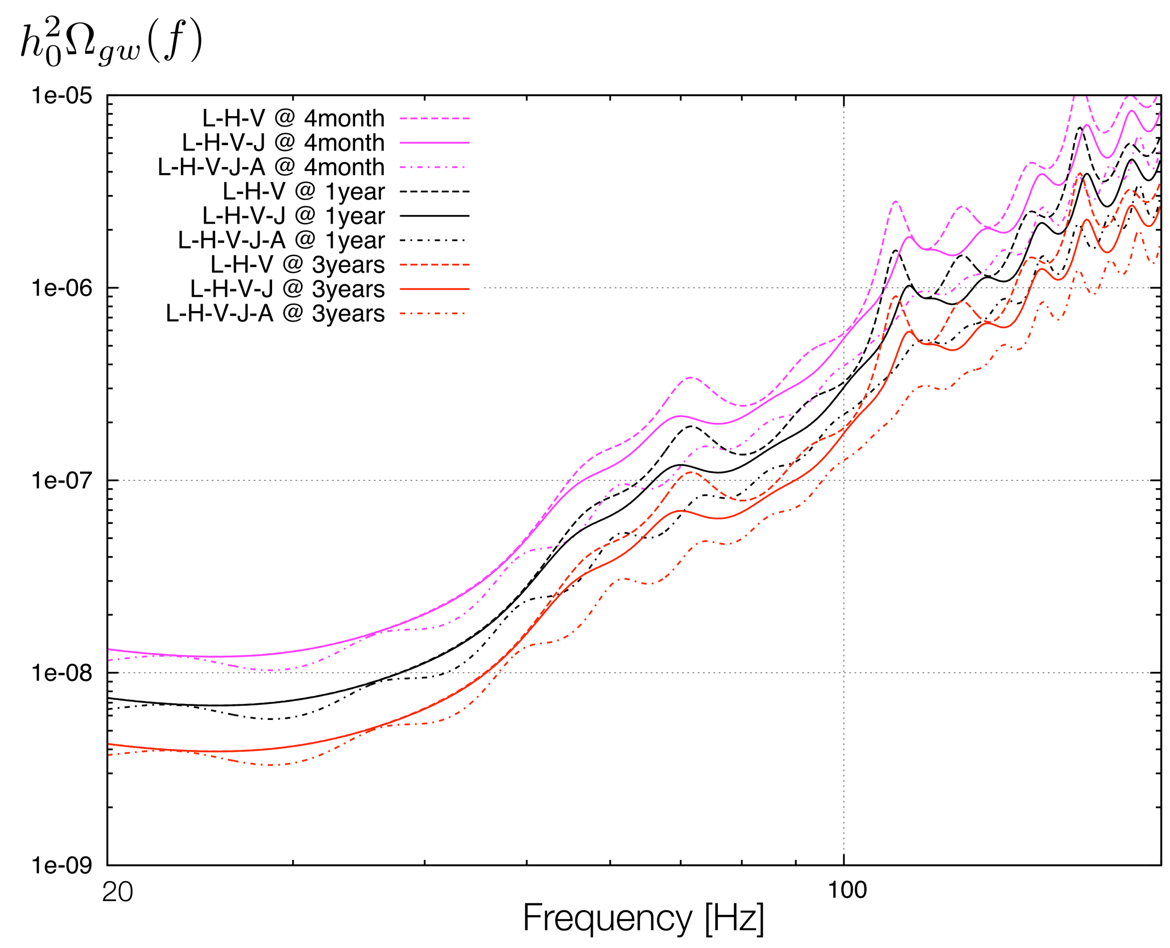


FIG.6 Frequency vs $\Omega_{gw}h_0^2$.

With Network of detectors, the upper limit for isotropic SGWB reach $\Omega_{gw}h_0^2 < 3\times10^{-9}$