Reduced basis method for the templates of the inspiraling compact binaries

Hideyuki Tagoshi (Osaka Univ.)
田越秀行(大阪大学）

2nd　Korea-Japan Workshop on KAGRA, 2012/05/28, ICRR

Matched filtering

Matched filter

$$\rho = (x, h_{\vec{\mu}}) = 4\Re \int_0^\infty \frac{x^*(f) h_{\vec{\mu}}(f)}{S_n(f)}\, df$$

$$\equiv 4\Re \langle x, h_{\vec{\mu}} \rangle$$

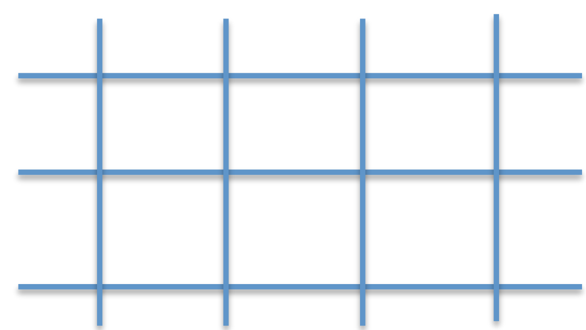
$$\langle x, h_{\vec{\mu}} \rangle \equiv \int_0^\infty \frac{x^*(f) h_{\vec{\mu}}(f)}{S_n(f)}\, df$$

x : data

$h_{\vec{\mu}}$: normalized template $\quad (h_{\vec{\mu}}, h_{\vec{\mu}}) = 1$

We look for μ which maximize ρ=> template bank is introduced $\{\vec{\mu}_i\}$

$\vec{\mu}_i$: discrete parameter points



Computation of $(x, h_{\vec{\mu}_i})$ is the most expensive part of the analysis

Reduced basis

Field et al. PRL 106, 221102 (2011)
Caudill et al. arXiv:1109.5642

From the template bank (N_{temp}), we select N templates. ($N \ll N_{temp}$)
(These templates are called Reduced basis)

Templates in the template bank can be expressed with a linear combination of N Reduced basis.
We can reduce the computation cost for the matched filtering by taking advantage of this property.

In this talk, I will demonstrate that this property is realized.

Below, I will use the following inner product and norm.

$$\langle x, y \rangle \equiv \int_0^\infty \frac{x^*(f) y(f)}{S_n(f)} df$$

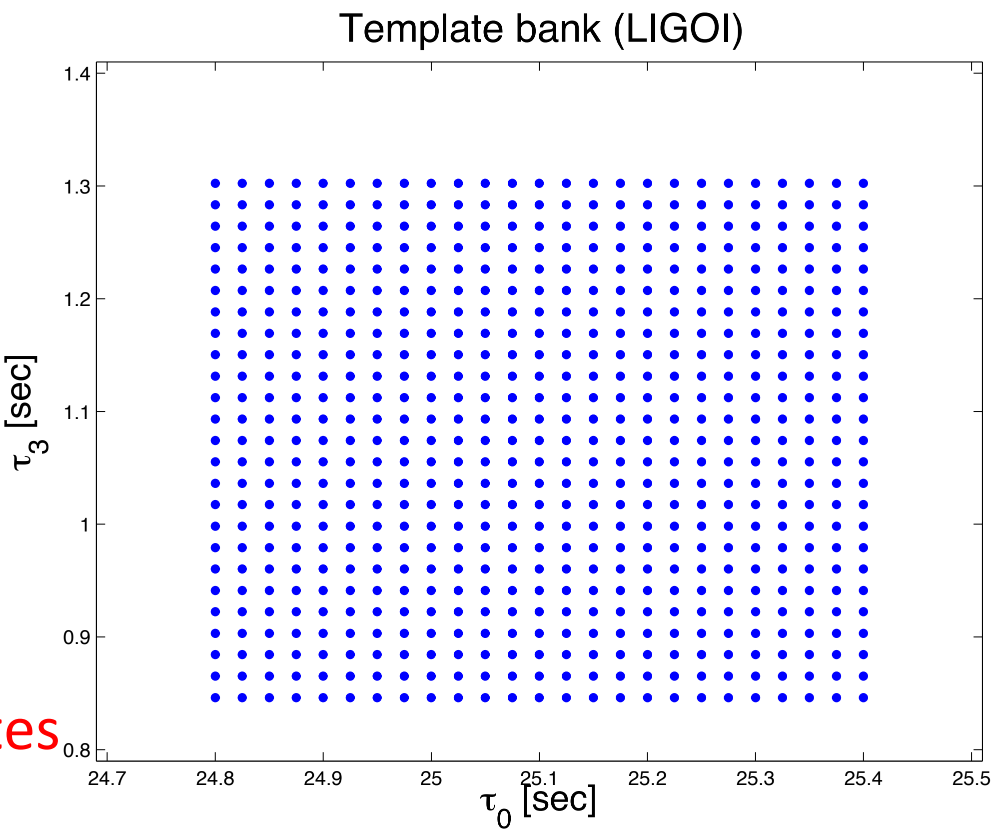
Coalescence times of $x(f), y(f)$ are fixed.
Thus, we do not need to do Fourier transformatio

$$\| x \| = \langle x, x \rangle^{1/2}$$

Today's setting

Initial LIGO noise power spectrum density (f_{min}=40Hz)
Mass parameters : $\tau0$, $\tau3$
Template : 3PN restricted post-Newtonian waveform
Template bank: we set up 25x25=625 grid points uniformly.
This correspond, approximately, maximum loss of S/N is about 3%.
Total mass : 2.7 – 4.2 Msolar, eta= m1 x m2/(m1+m2)^2 : 0.12-0.26

Length of template: 32 seconds
Sampling rate: 2048 Hz
Sampling points: 2^{16}=65536



625 templates

Construction of Reduced basis

Greedy algorithm

$$\langle h_{\vec{\mu}_i}, h_{\vec{\mu}_i} \rangle = 1$$

(1) Select a template $h_{\vec{\mu_1}}$ arbitrary, and set $e_1 = h_{\vec{\mu_1}}$

(2) $\vec{\mu}_2$ is chosen so that the perpendicular component to e1 is maximum.

$$\| h_{\vec{\mu_j}} - P_1(h_{\vec{\mu_j}}) \| \qquad P_1(h_{\vec{\mu_j}}) = e_1 \langle e_1, h_{\vec{\mu_j}} \rangle$$

parallel component to e_1

(3) $e_2 = \lambda_2 (h_{\vec{\mu}_2} - e_1 \langle e_1, h_{\vec{\mu_2}} \rangle)$ λ_2 is the normalization constant

$$\langle e_2, e_2 \rangle = 1$$

: Gram-Schmit orthogonalization

(4) When we select jth basis e_j, we select a template which component perpendicular to e_k (k=1…j-1) is maximum.

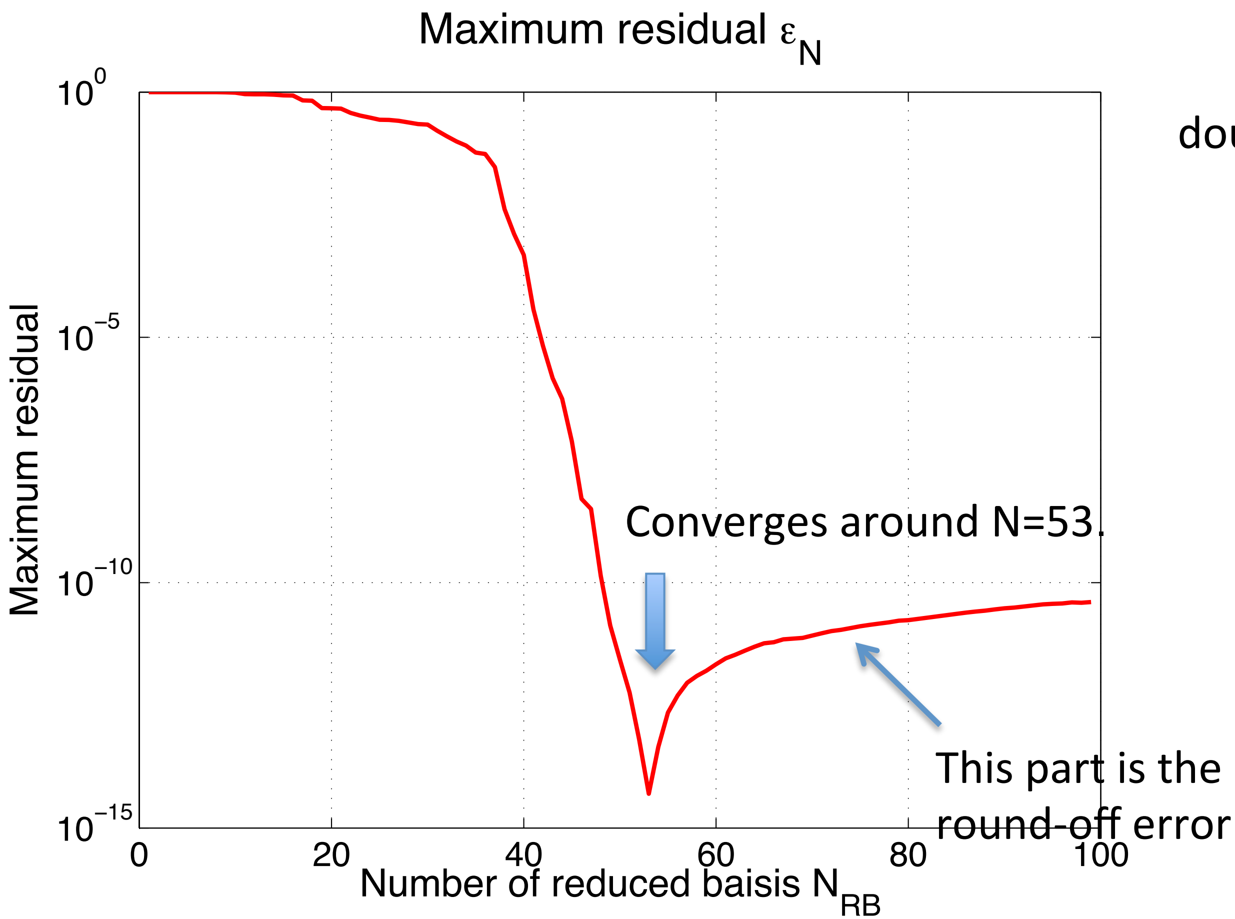
(The norm of this perpendicular component is the residual of these basis.

(5)We repeat above procedure until the residual becomes small.

The residual

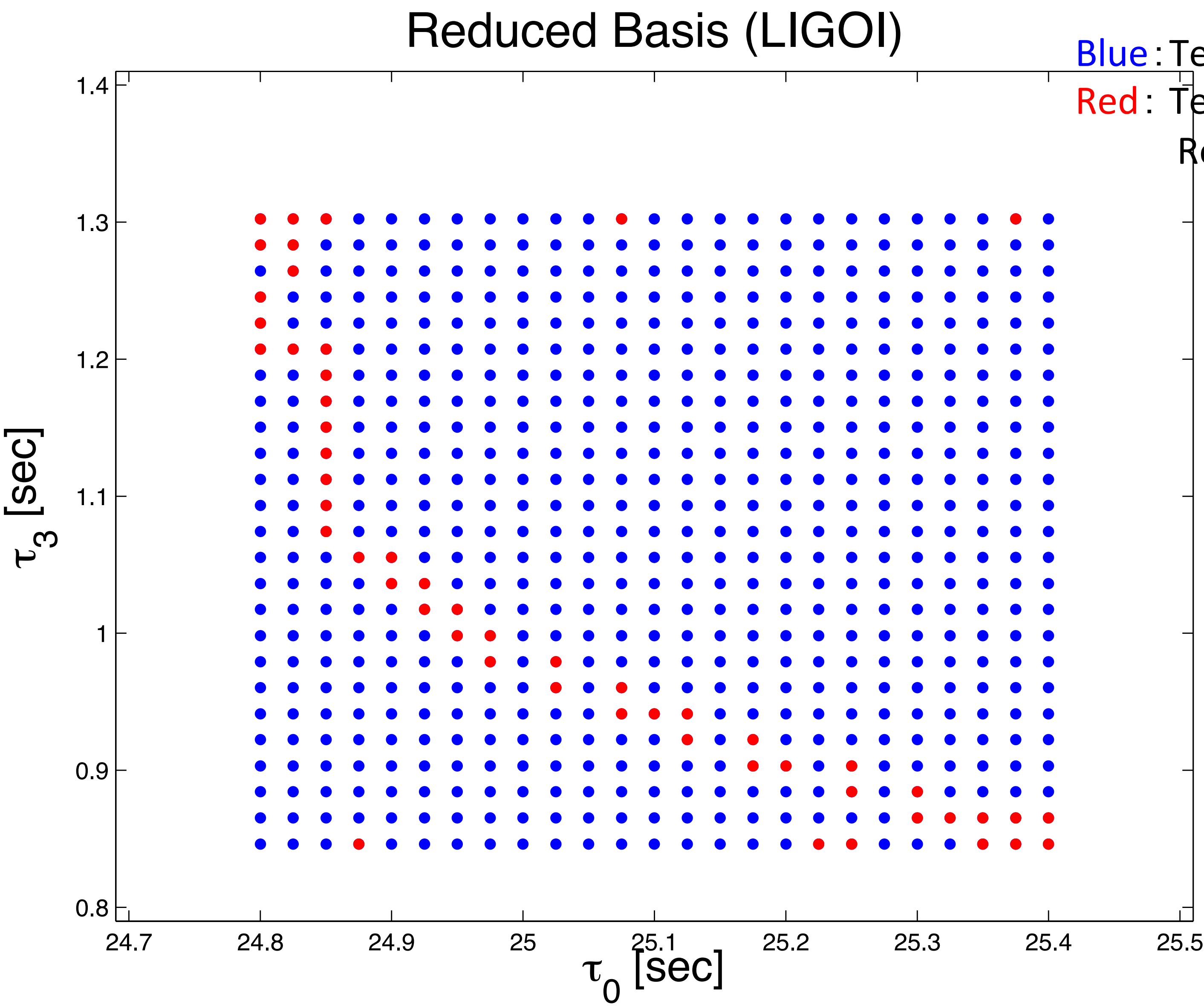
The maximum of the residual as a function of the number of reduced basis.

$$\epsilon_N \equiv \max_{\vec{\mu}_i} \| \, h_{\vec{\mu}_i} - P_N(h_{\vec{\mu}_i}) \, \| \qquad P_N(h_{\vec{\mu}_i}) = \sum_{k=1}^{N} e_k \langle e_k, h_{\vec{\mu}_i} \rangle$$



double precision

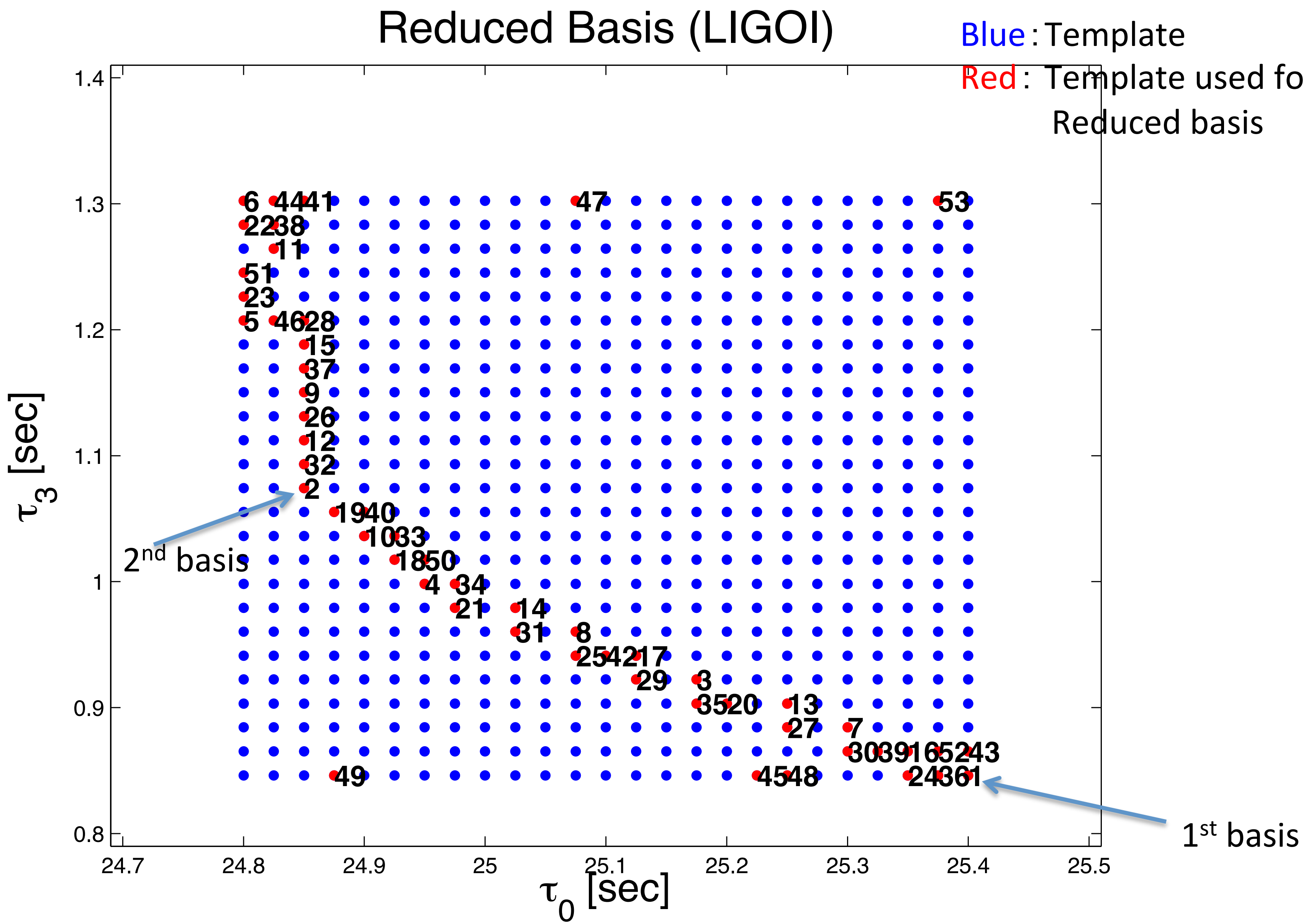
Reduced basis for this template bank



Blue : Template

Red : Template used for the Reduced basis

Reduced basis for this template bank

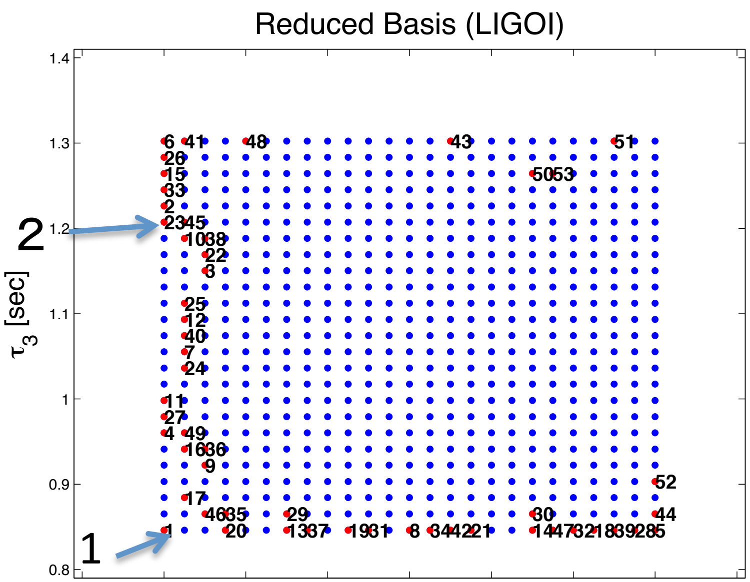


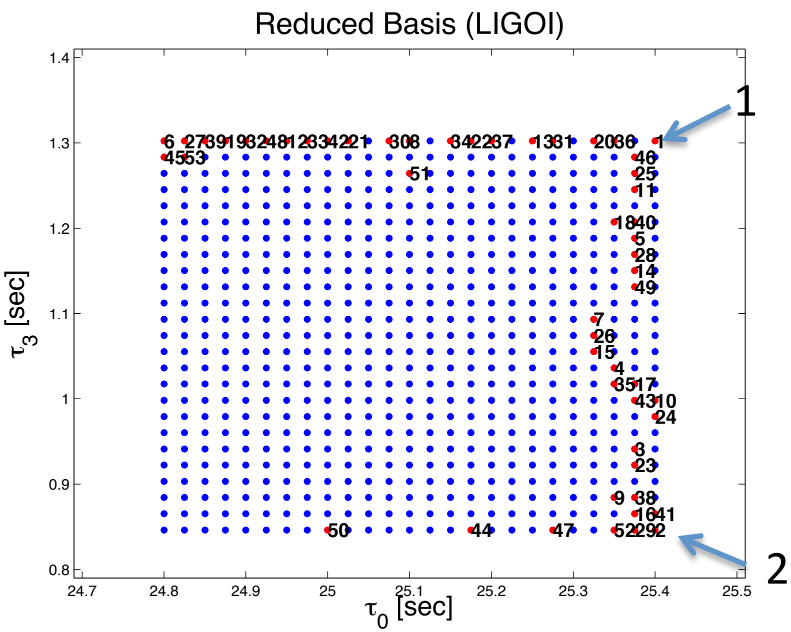
Blue : Template

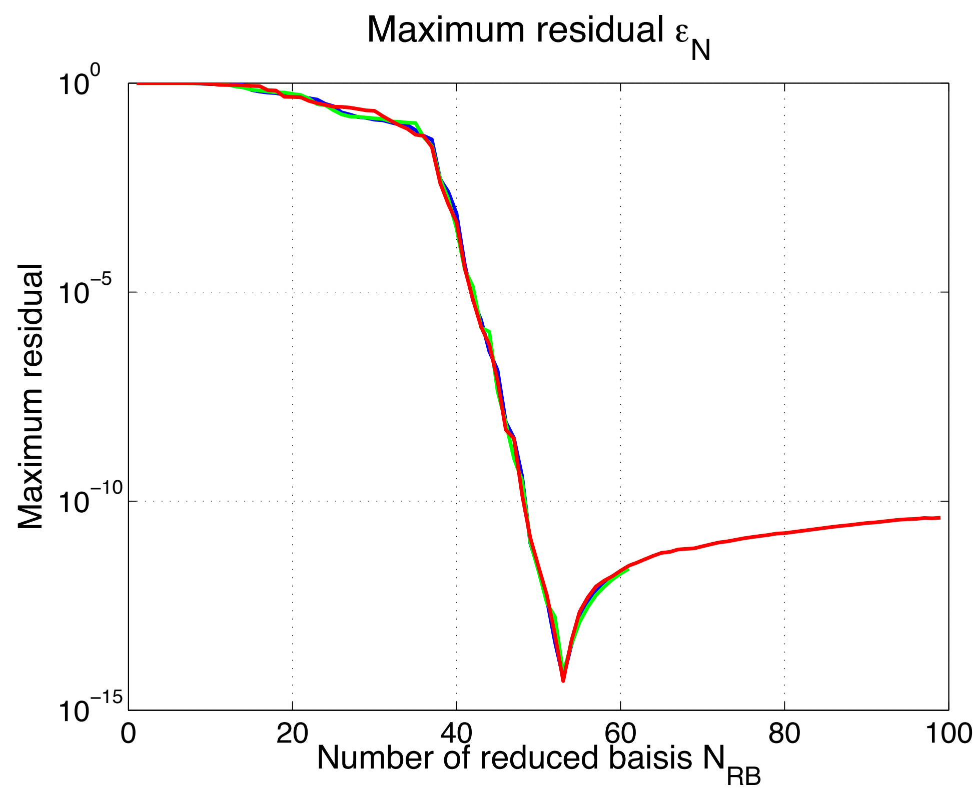
Red : Template used for the Reduced basis

When we change the 1st basis

Position of the reduced basis changes, but the convergence of the residual is the same







What this means?

$$\epsilon_N \equiv \max_{\vec{\mu}_i} \| h_{\vec{\mu}_i} - P_N(h_{\vec{\mu}_i}) \|$$

$$P_N(h_{\vec{\mu}_i}) = \sum_{k=1}^{N} e_k \langle e_k, h_{\vec{\mu}_i} \rangle$$

$$h_{\vec{\mu}_i} = P_N(h_{\vec{\mu}_i}) + \delta h_{\vec{\mu}_i}$$

$$\| \delta h_{\vec{\mu}_i} \| = \epsilon_N \ll 1$$ @N～53

Each template are expressed in terms of the linear combination of the Reduced basis e_i .

Thus, the dimension of this vector space is ～52, within the computation accuracy.

Application to the real Matched filtering analysis

Preprocess

(1) Reduced basis e_i (i=1…N) are constructed. We need computation power for this.

(2) $\alpha_{ij} = \langle e_i, h_{\vec{\mu}_j} \rangle$ (i=1…N, j=1…N_{temp}) are computed in advance.

Although this seems to require a lot of computation time,
This is just a numerical integration, not the Fourier transformation.
Parallel computation is also easy.

(3) When we have data x,

$$
\begin{aligned}
(x, h_{\vec{\mu}_j}) &= 4\mathrm{Re}\langle x, h_{\vec{\mu}_j} \rangle \\
&= 4\mathrm{Re}\langle x, \sum_{k=1}^{N} e_k \langle e_k, h_{\vec{\mu}_j} \rangle \rangle \\
&= 4\mathrm{Re} \sum_{k=1}^{N} \langle e_k, h_{\vec{\mu}_j} \rangle \langle x, e_k \rangle \\
&= 4\mathrm{Re} \sum_{k=1}^{N} \alpha_{k,j} \underline{\langle x, e_k \rangle}
\end{aligned}
$$

If $N \ll N_{temp}$ a lot of computation time is reduced.

Only N times computation are requied

Summary

We select N template from the template bank of N_{temp} ($N \ll N_{temp}$)
and orthogonalize them (they are called Reduced basis)

We confirm that arbitrary template within the region of the template bank can be expressed as a linear combination of the N reduced basis.

Using this property, there is a possibility that we can reduce the computation time of the matched filtering.
In the case of the example of today, we can expect 1/10 reduction of the computation time.

Discussion

(1)Reduced basis depend on the shape of the noise spectrum.
But it's dependence is not large (Field et al. (2011))
Thus, it may be good enough for the low latency analysis aiming the event alert system.

(2)Similar method : Singular Value Decomposition

Singular Value Decomposition (SVD)

Prepare a matrix from M templates

$$A = (\overrightarrow{h}_1, \overrightarrow{h}_2, \ldots, \overrightarrow{h}_M)$$

each $\overrightarrow{h}_i$ is a N dim column vector

N × M matrix (N>M is assumed)

Any complex matrix can be decomposed "Singular Value Decomposition"

$$A = U \, \Sigma \, V^\dagger$$

U: N×N Unitary matrix
V: M×M unitary matrix († is the Hermite conjugation)

$$\Sigma = \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_M \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_M$$

:Singular value

Singular Value Decomposition (SVD)

$$A = (\overrightarrow{h}_1, \overrightarrow{h}_2, \ldots, \overrightarrow{h}_M) \qquad A = U \Sigma V^\dagger$$

Ignore small σ_k , and approximate A as follows

$$A_{ij} = \sum_{k=1}^{M} U_{ik} \sigma_k V^*_{jk} \simeq \sum_{k=1}^{M'} U_{ik} \sigma_k V^*_{jk} \qquad M' < M$$

In the waveform,

$$h_j(f_i) = A_{ij} = \sum_{k=1}^{M'} U_{ik} \sigma_k V^*_{jk}$$

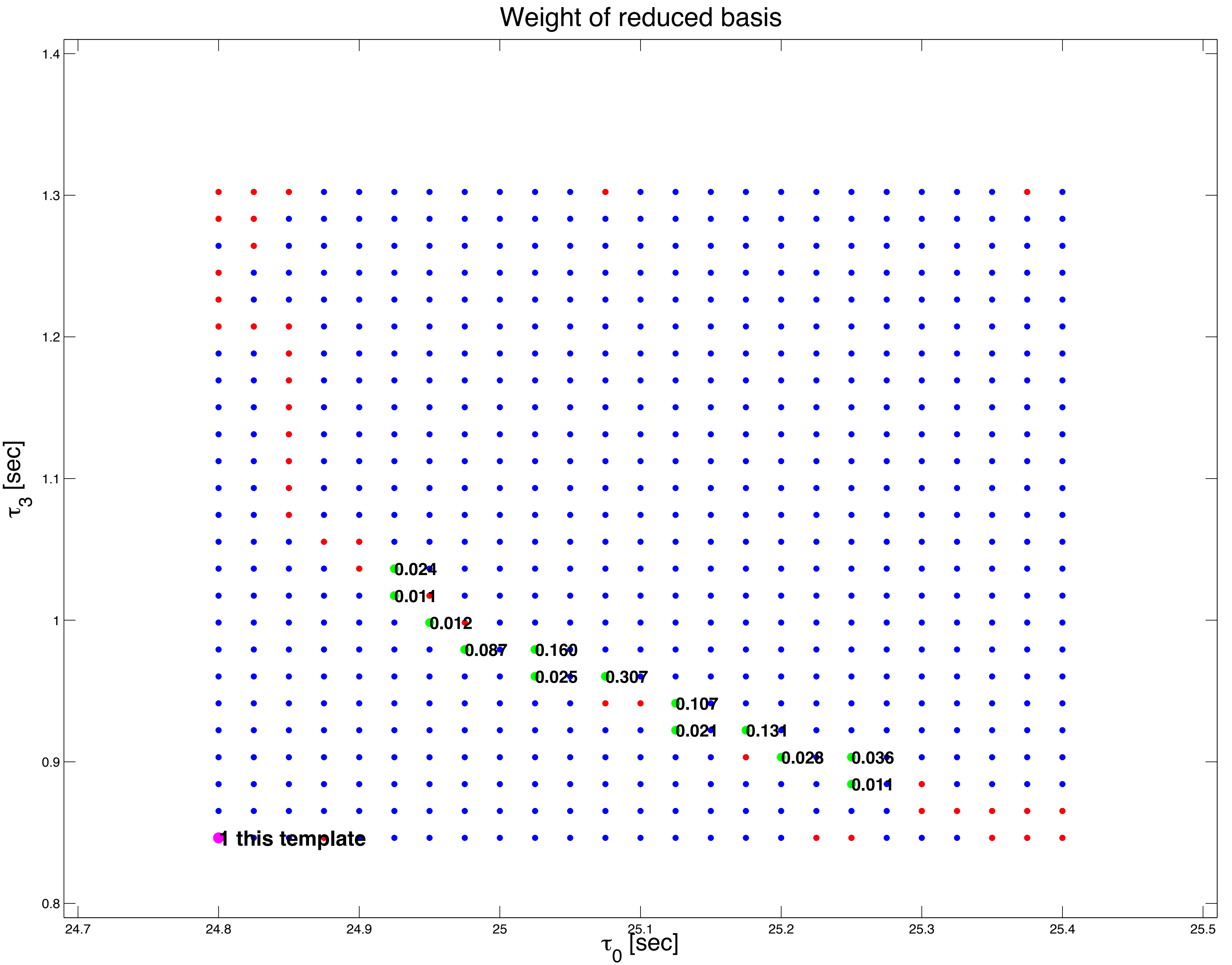
Each column vector of U_{ik} is

$$\overrightarrow{u}_k(f_i) = U_{ik}$$

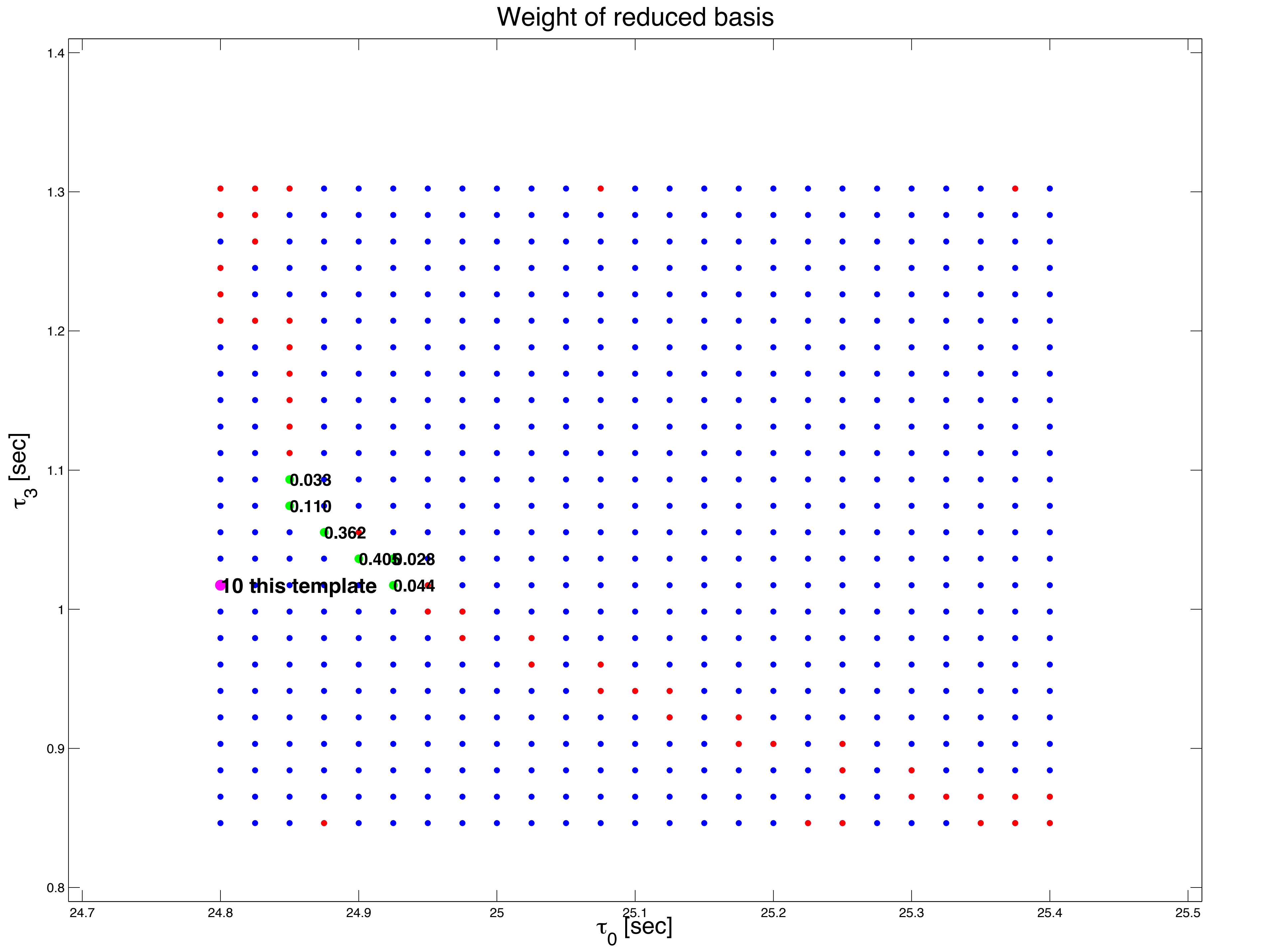
$$\boxed{\begin{aligned} \overrightarrow{h}_j &= \sum_{k=1}^{M'} \overrightarrow{u}_k \sigma_k V^*_{jk} \\ &= \sum_{k=1}^{M'} \alpha_{jk} \overrightarrow{u}_k \end{aligned}}$$

Any template can be expressed with a linear combination of smaller number M’ of other templates

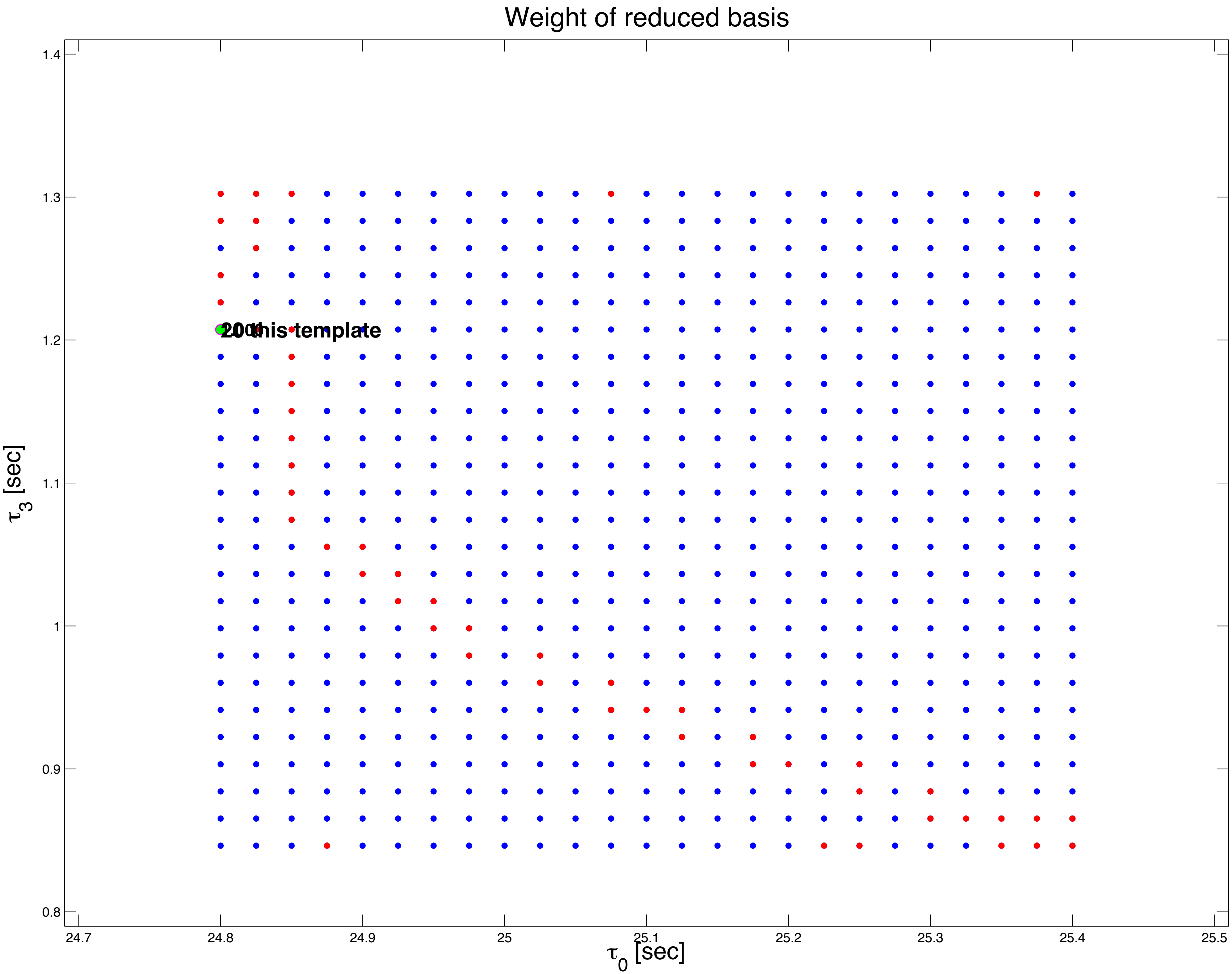
$$\alpha_{jk} = \sigma_k V^*_{jk}$$

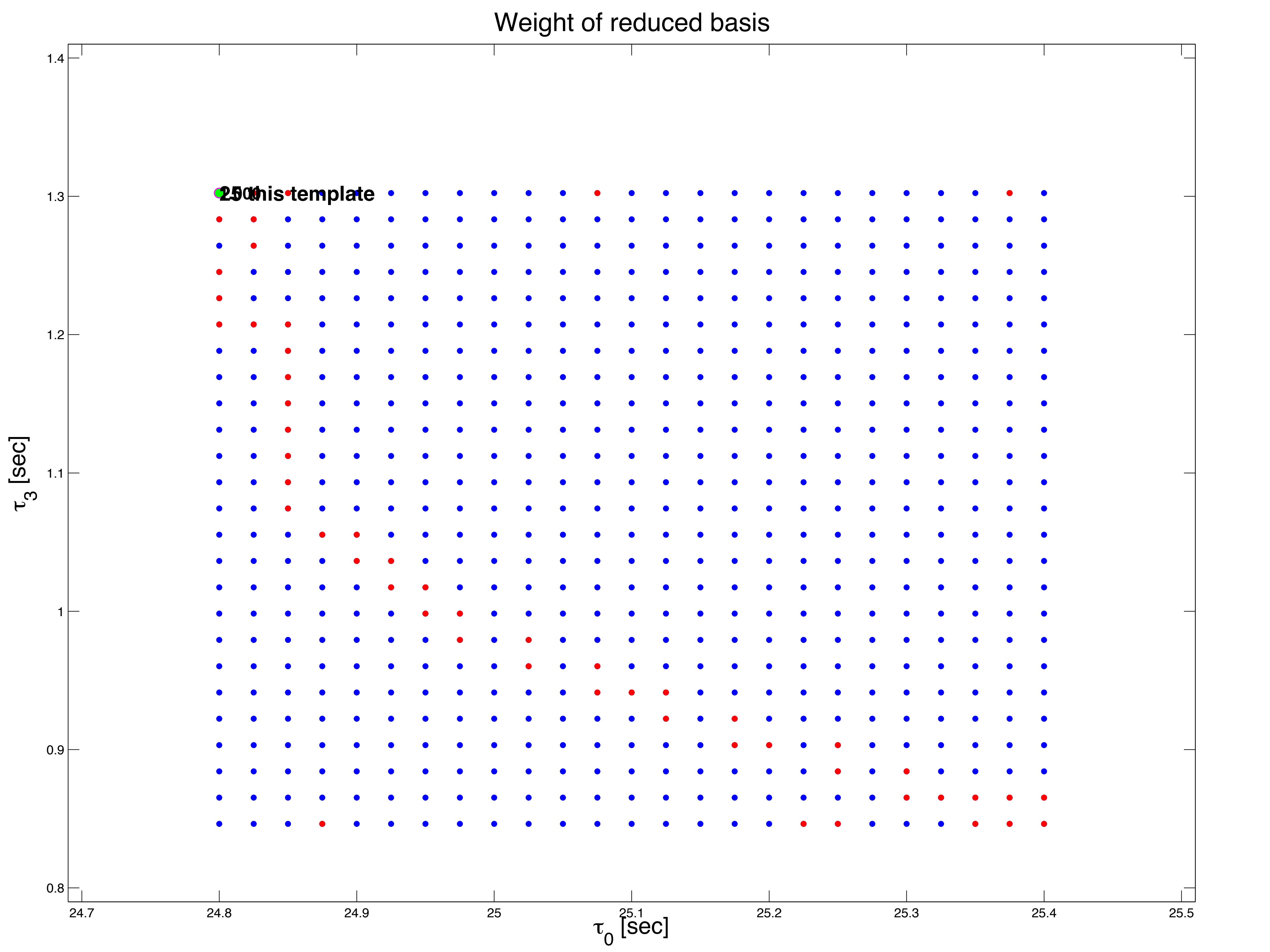


Weight of reduced basis
1.4
1.3
1.2
1.1
1
0.9
0.8
τ_3 [sec]
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5
τ_0 [sec]
0.024
0.011
0.012
0.087
0.160
0.025
0.307
0.107
0.021
0.131
0.028
0.036
0.011
1 this template

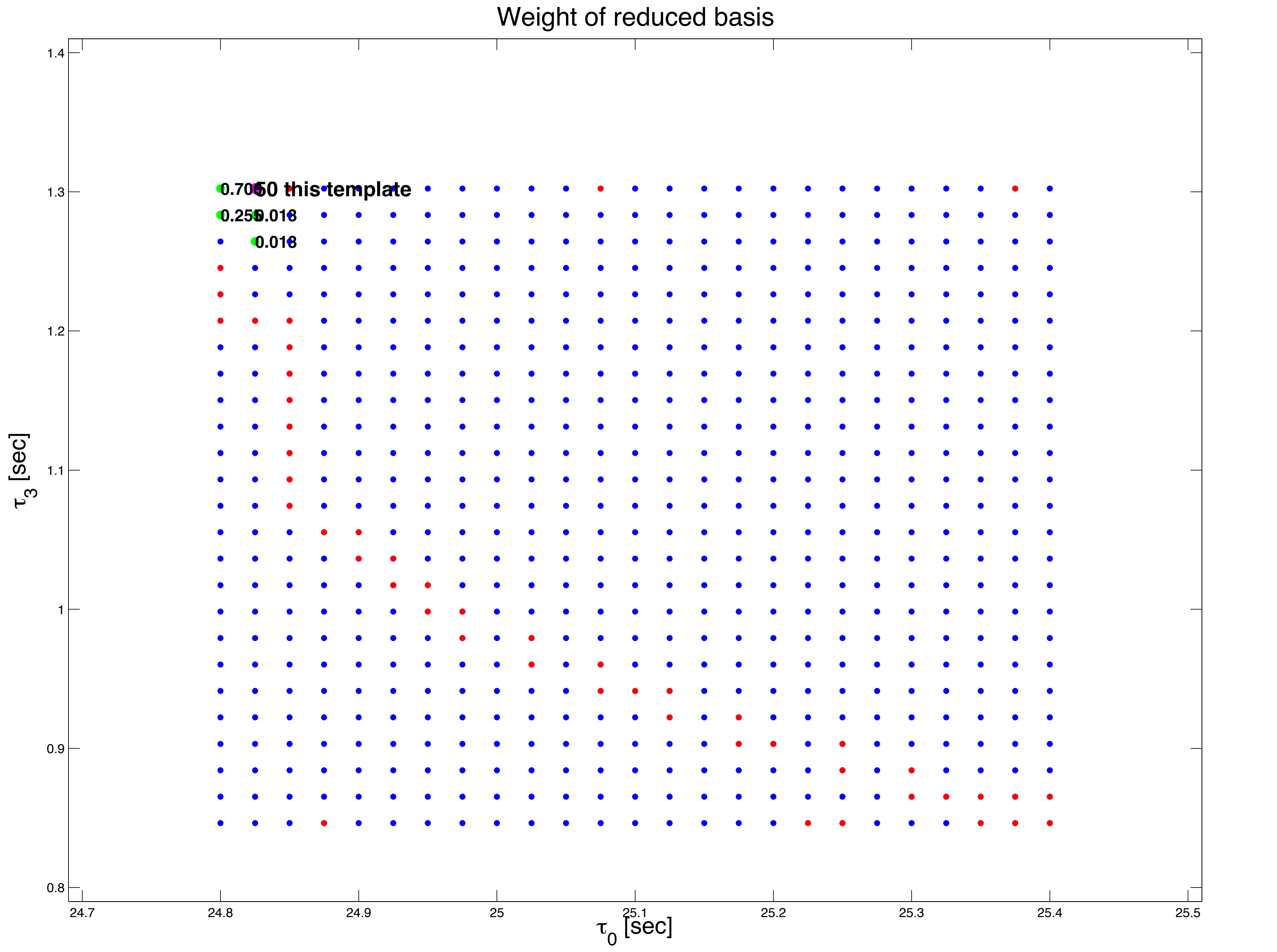


Weight of reduced basis
0.036
0.110
0.362
0.405
0.028
10 this template
0.044
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5
τ3 [sec]
τ0 [sec]

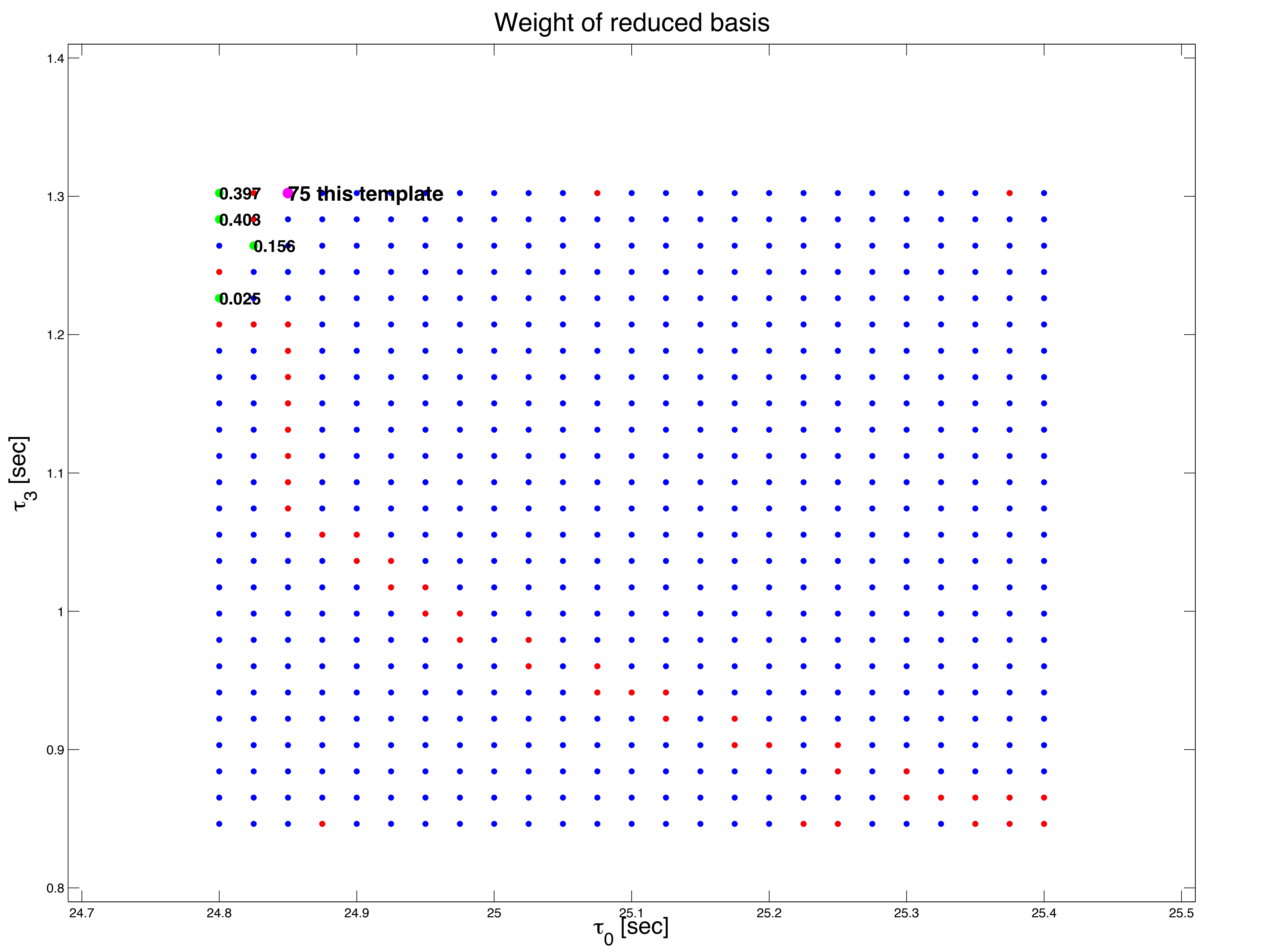

Weight of reduced basis
20 this template
τ3 [sec]
τ0 [sec]
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5



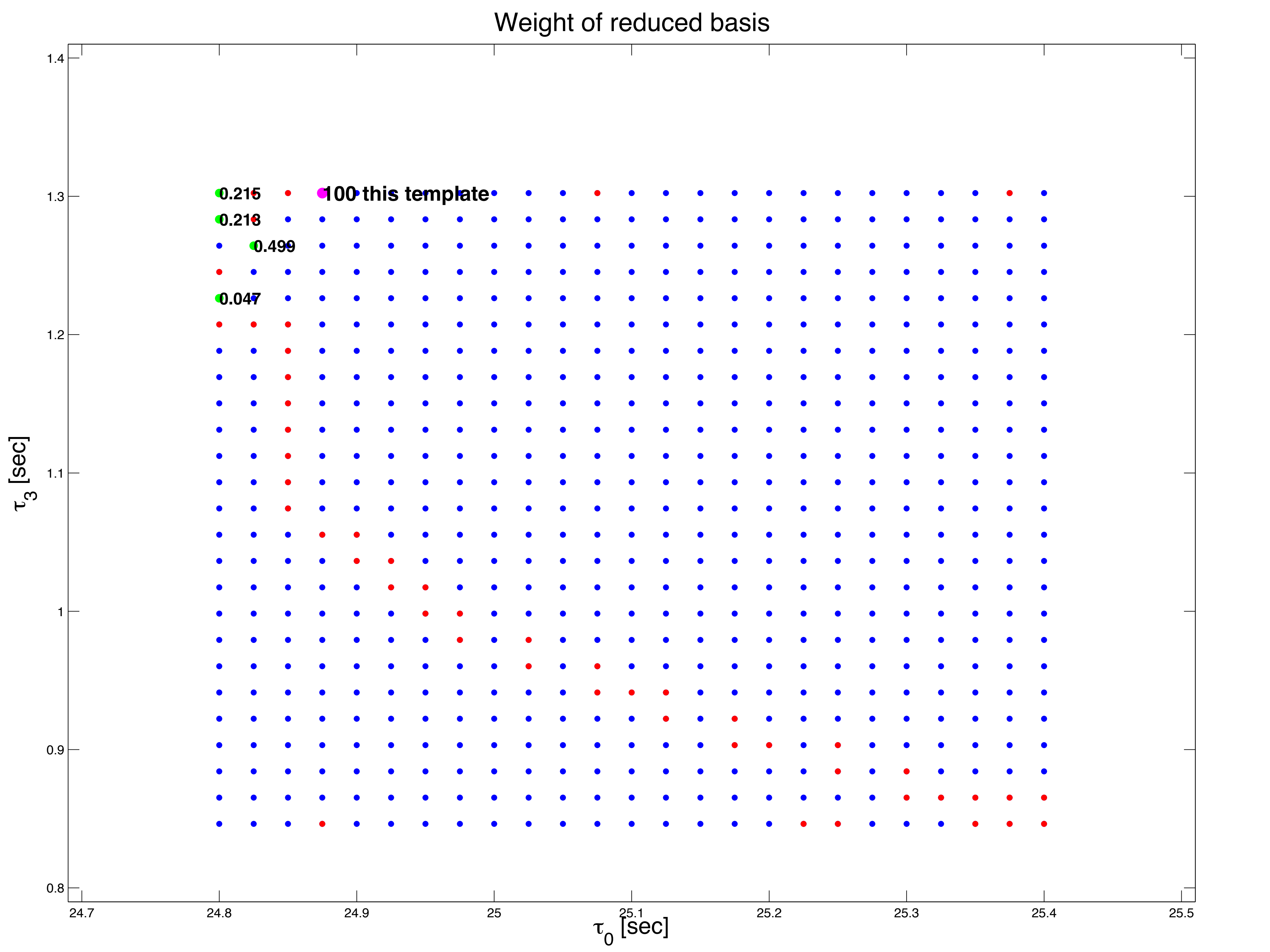
Weight of reduced basis
25 this template
1.4
1.3
1.2
1.1
1
0.9
0.8
τ_3 [sec]
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5
τ_0 [sec]



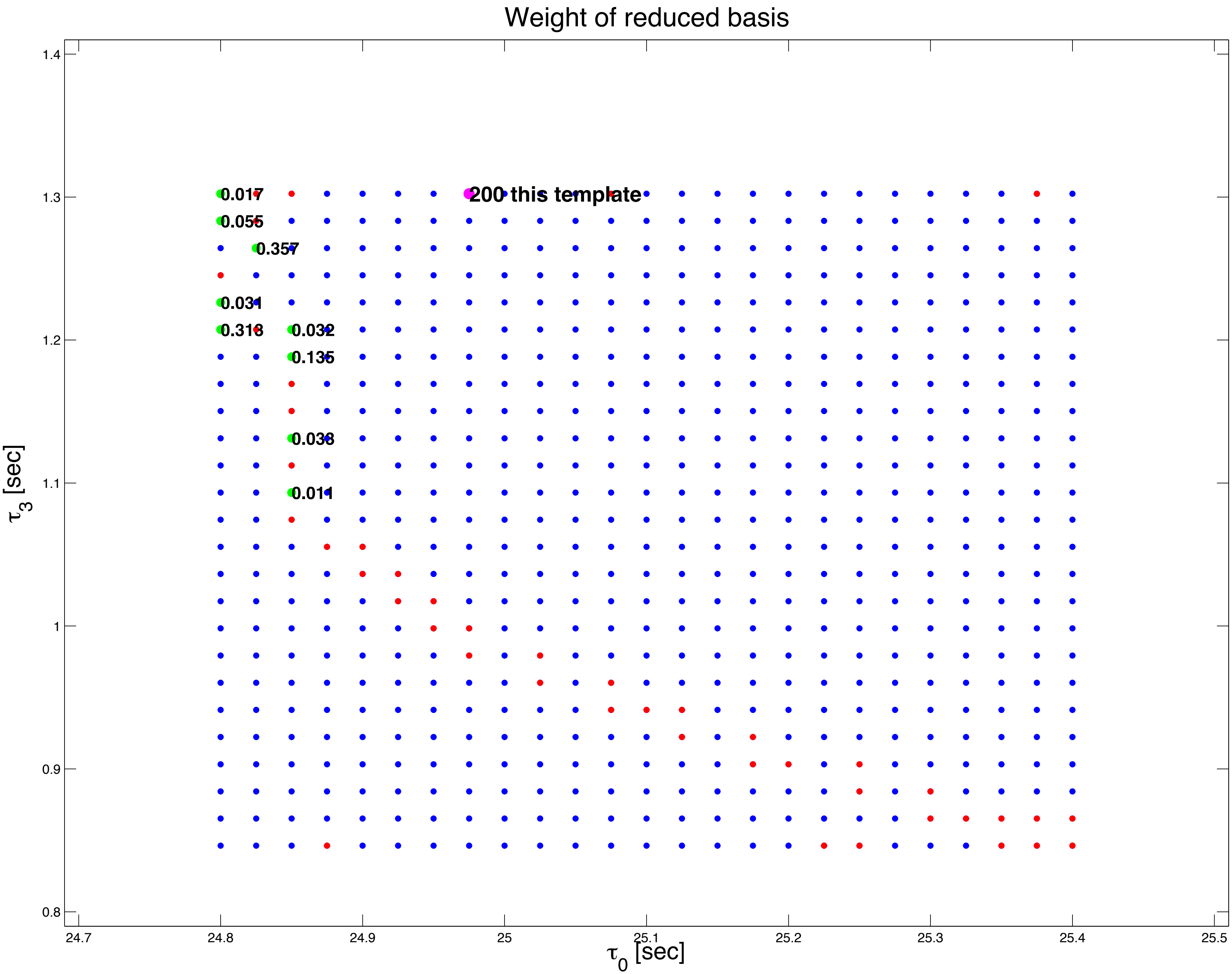
Weight of reduced basis
0.705
0.250
0.018
0.018
50 this template
τ_3 [sec]
τ_0 [sec]
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5



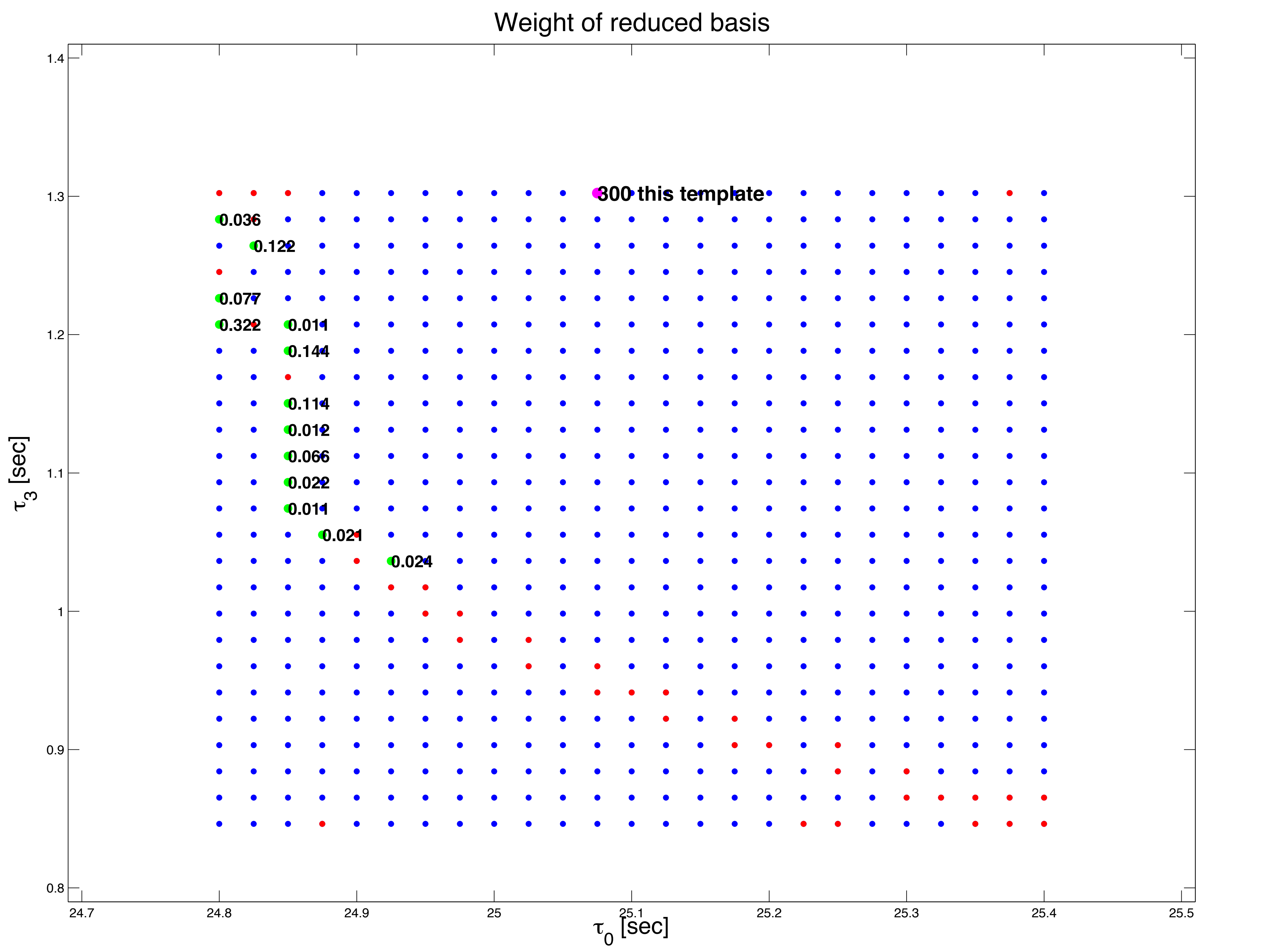
Weight of reduced basis
0.397
75 this template
0.408
0.156
0.025
τ3 [sec]
τ0 [sec]
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5



Weight of reduced basis
0.215
0.218
0.499
0.047
100 this template
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5
τ_3 [sec]
τ_0 [sec]

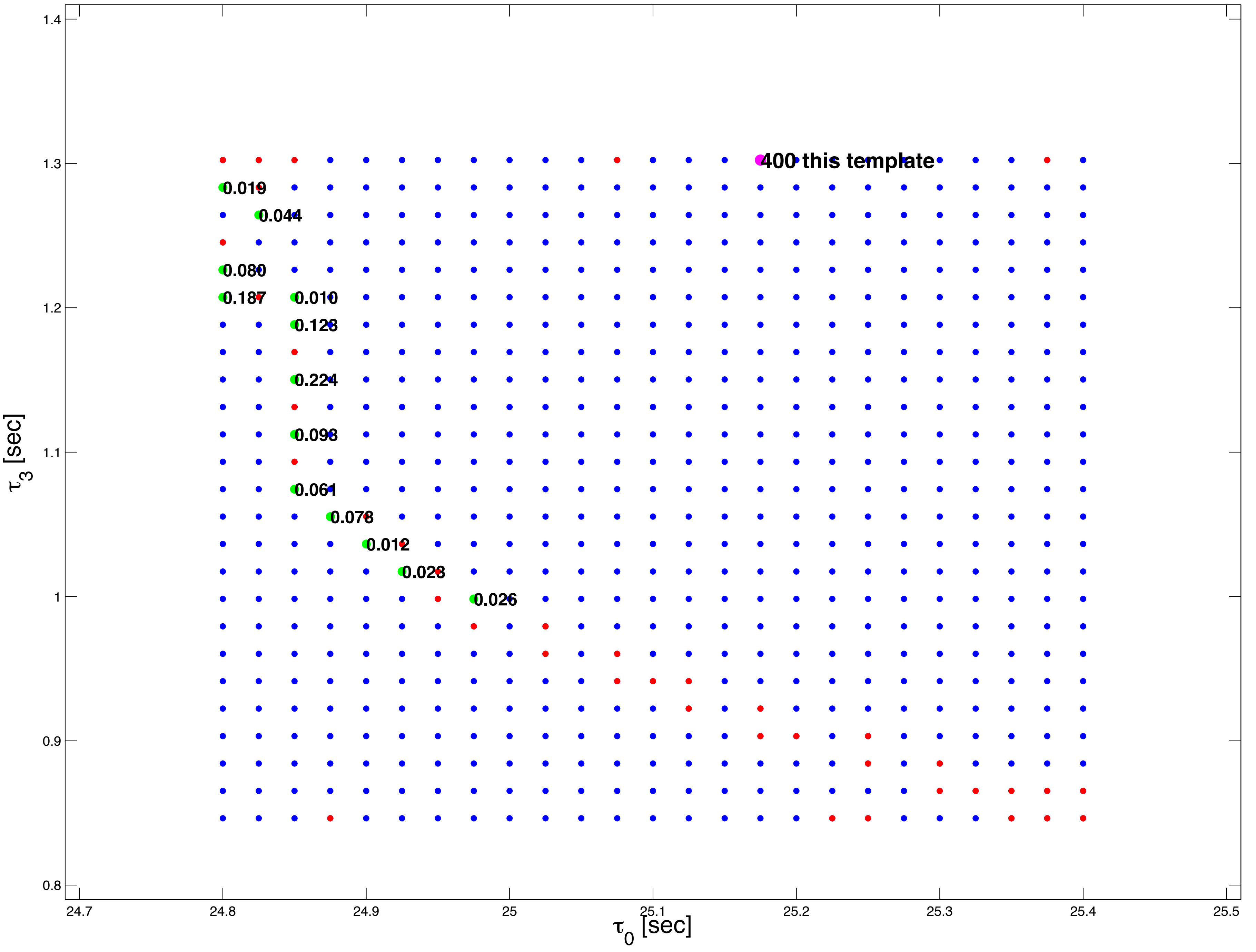


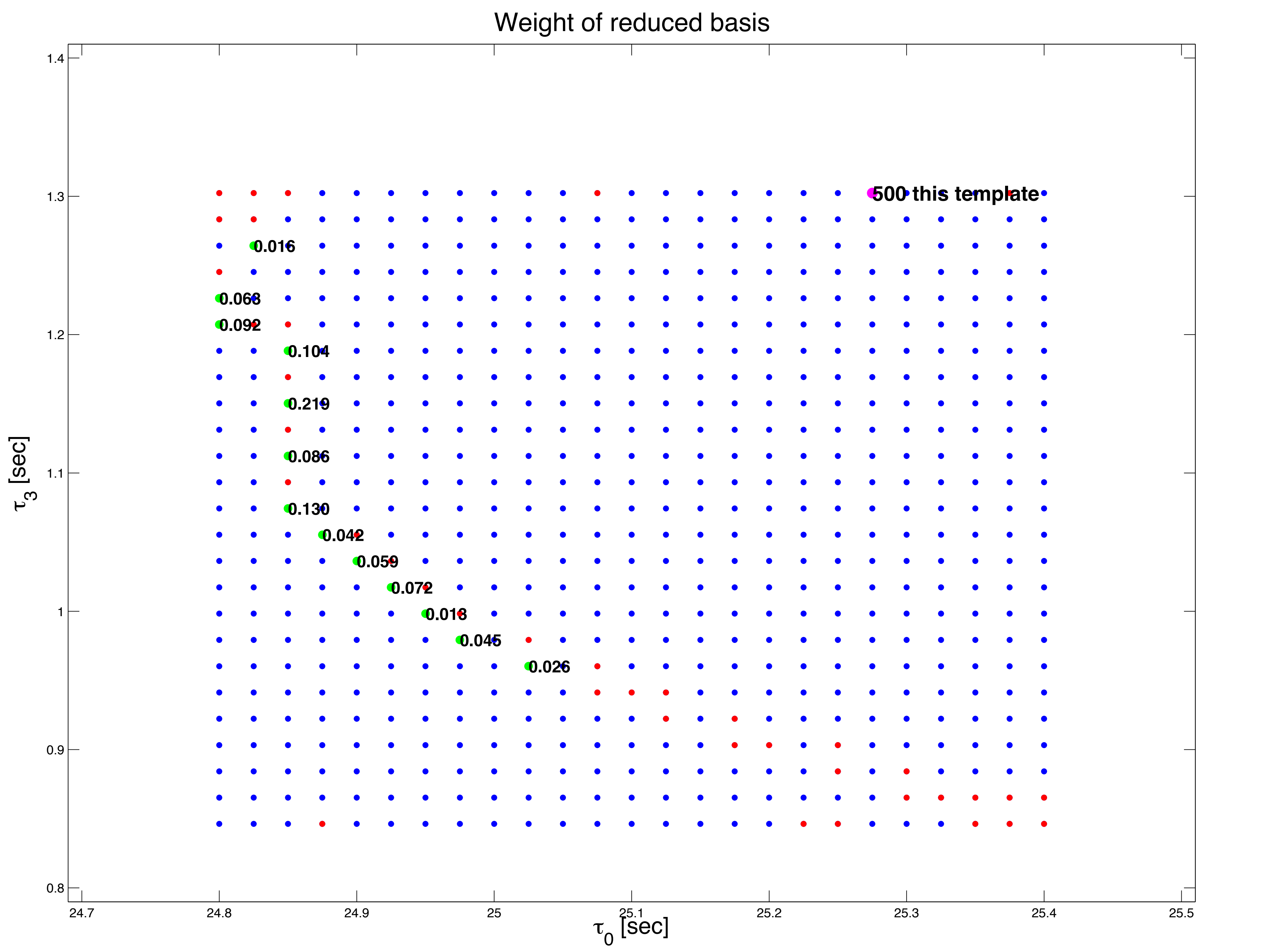
Weight of reduced basis
0.017
0.055
0.357
0.031
0.318
0.032
0.135
0.038
0.011
200 this template
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5
τ_3 [sec]
τ_0 [sec]

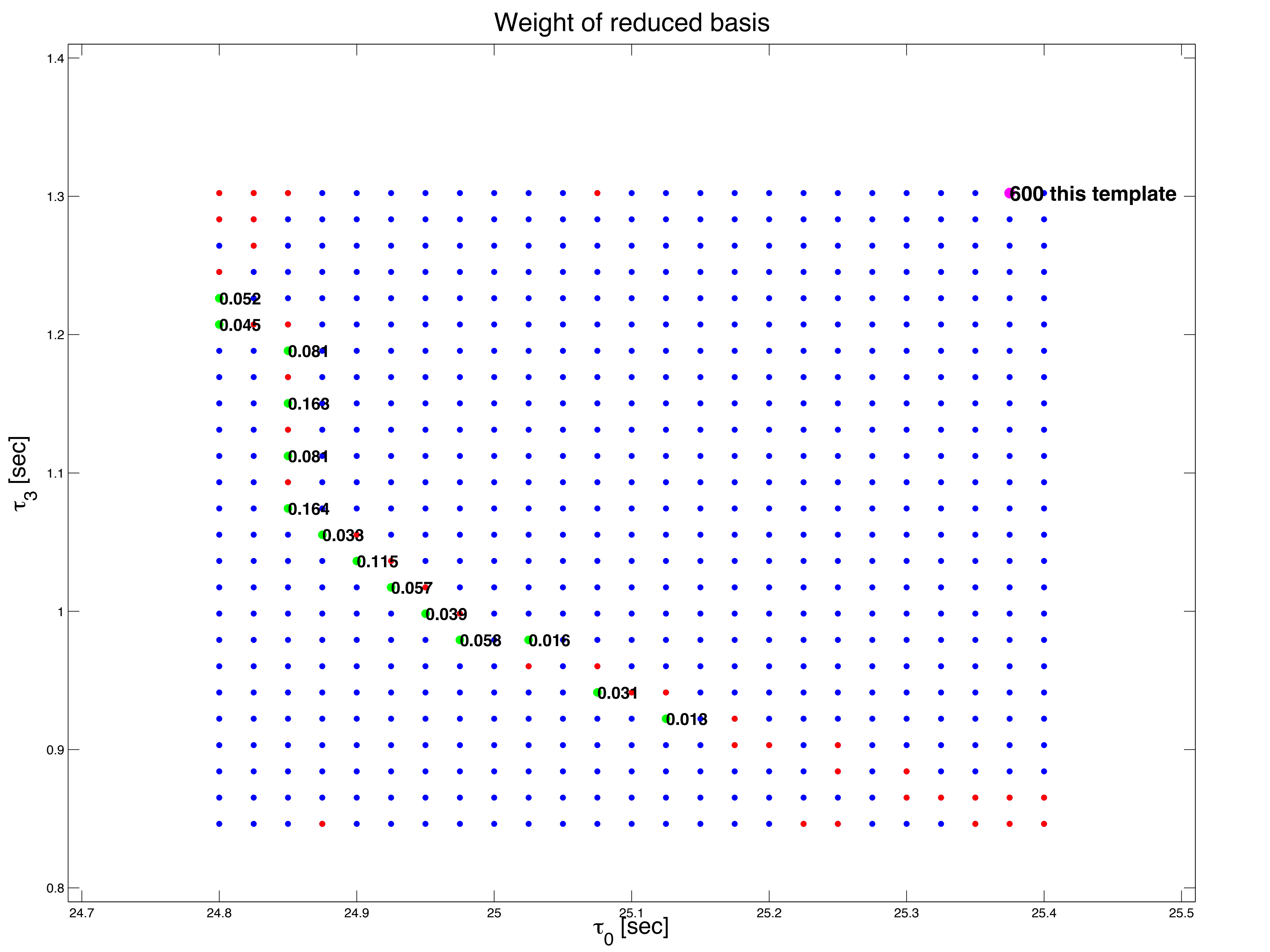


Weight of reduced basis
300 this template
0.036
0.122
0.077
0.322
0.011
0.144
0.114
0.012
0.066
0.022
0.011
0.021
0.024
τ3 [sec]
τ0 [sec]
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5

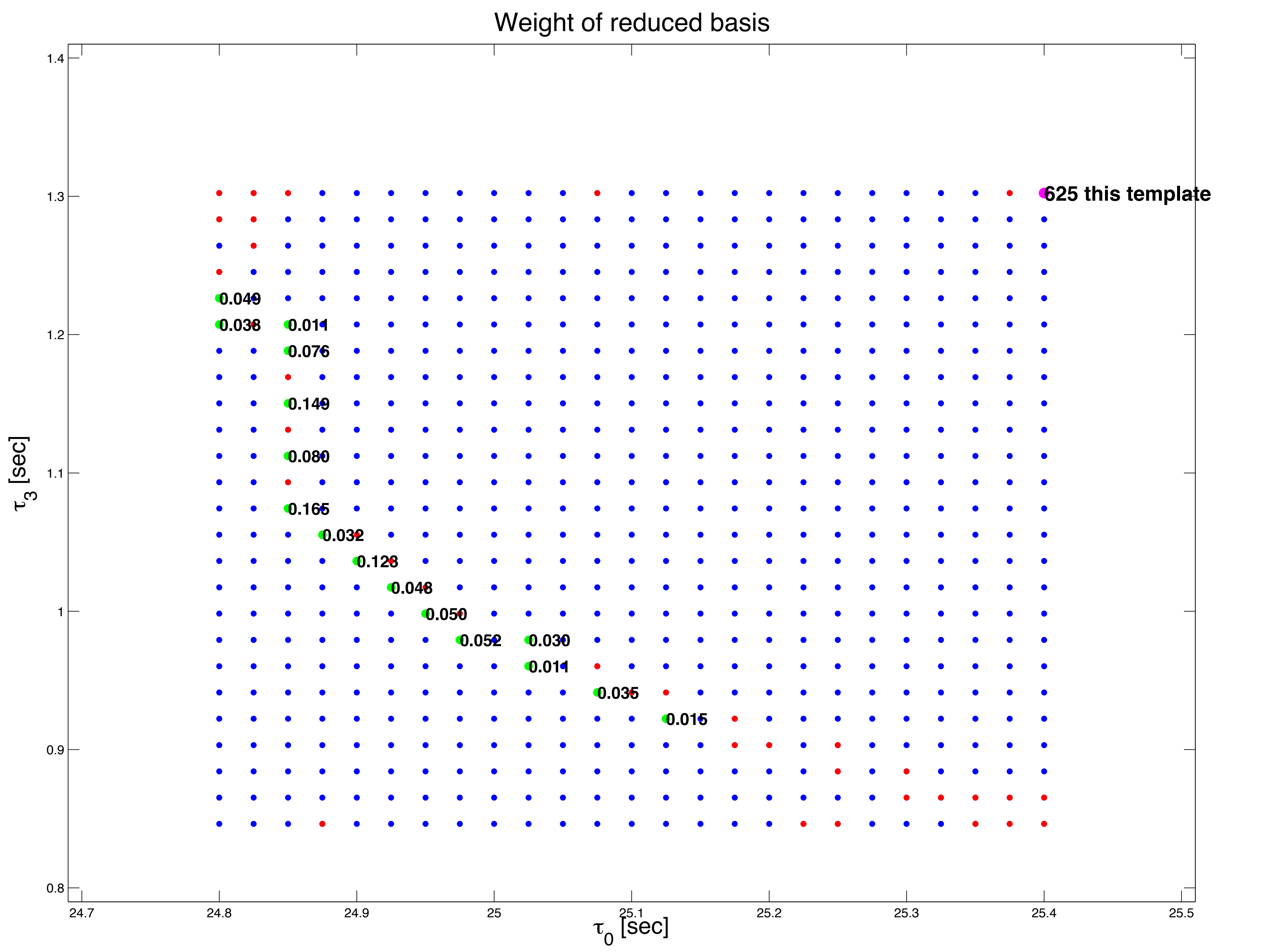
Weight of reduced basis




Weight of reduced basis
500 this template
0.016
0.068
0.092
0.104
0.219
0.086
0.130
0.042
0.059
0.072
0.018
0.045
0.026
τ_3 [sec]
τ_0 [sec]
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5



Weight of reduced basis
τ_3 [sec]
τ_0 [sec]
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5
600 this template
0.052
0.045
0.081
0.168
0.081
0.164
0.038
0.115
0.057
0.039
0.058
0.016
0.031
0.018



Weight of reduced basis
625 this template
0.049
0.038
0.011
0.076
0.149
0.080
0.165
0.032
0.128
0.048
0.050
0.052
0.030
0.011
0.035
0.015
τ_3 [sec]
τ_0 [sec]
1.4
1.3
1.2
1.1
1
0.9
0.8
24.7
24.8
24.9
25
25.1
25.2
25.3
25.4
25.5