Stochastic gravitational wave and parameter estimation

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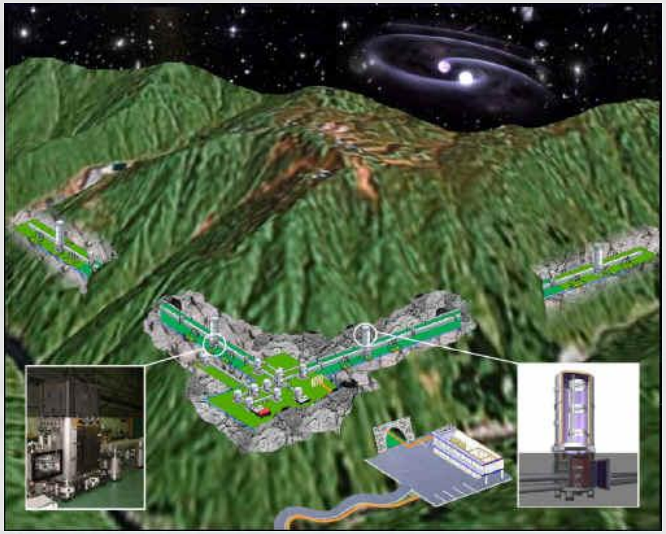
Contents

Introduction

- Large scale Laser interferometer detector for Gravitational Wave(GW)
  - LIGO, EGO-Virgo, GEO600, KAGRA
  - Detecting known astrophysical sources
  - Expect real detection around 2020
- Possibly detect stochastic GW of cosmological origin
  - A new window to study very early universe

KAGRA

GW Observatories



Characterization of stochastic backgrounds of GWs

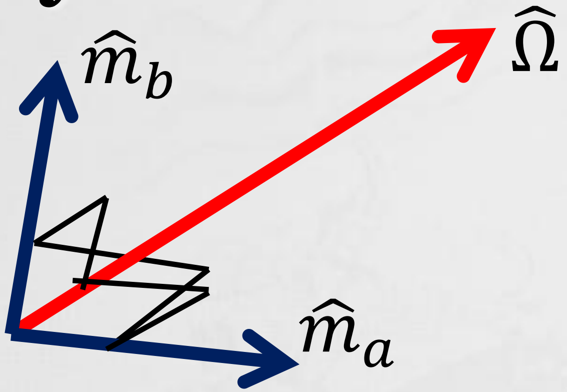
- Isotropic, stationary, unpolarized
- Frequency spectrum is main property
  - Energy density per unit log frequency, $h_0^2\Omega_{GW}(f)$
  - Spectral density of ensemble average the Fourier component of the metric, $S_h(f)$
  - Characteristic amplitude of the stochastic background, $h_c(f)$

Energy density parameter

- Definition $\Omega_{GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln f}$
- $\rho_c = \frac{3H_0^2}{8\pi G}$, $H^2 = \frac{8\pi G}{3}\rho$ (1st Hubble equation)
- $H_0 = h_0 \times 100\mathrm{km/s} \cdot \mathrm{Mpc}$, $0.5 < h_0 < 0.65$
- $h_0^2\Omega_{GW}(f)$ independent of h_0
- Total density parameter for GW

$$\Omega_{GW} = \int_0^\infty \Omega_{GW}(f)d\ln f$$

Spectral density



- Fourier transform

$$h_{ab}(t) = \sum_{A=+,-} \int_{-\infty}^{\infty} df \int d\widehat{\Omega} \tilde{h}_A(f,\widehat{\Omega}) e^{-2\pi i f t} \mathrm{e}_{ab}^{A}(\widehat{\Omega})$$

$$\mathrm{e}_{ab}^{+}(\hat{\Omega}) = \hat{m}_a \hat{m}_b - \hat{n}_a \hat{n}_b, \qquad \mathrm{e}_{ab}^{\times}(\hat{\Omega}) = \hat{m}_a \hat{n}_b + \hat{n}_a \hat{m}_b \ ,$$

$$\mathrm{e}_{ab}^{A}(\hat{\Omega}) \mathrm{e}^{A',ab}(\hat{\Omega}) = 2\delta^{AA'}$$

$$\langle \tilde{h}_A^*(f,\hat{\Omega}) \tilde{h}_{A'}(f',\hat{\Omega}') \rangle = \delta(f-f')(1/4\pi)\delta^2(\hat{\Omega},\hat{\Omega}')\delta_{AA'} \tfrac{1}{2} S_h(f)$$

Characteristic amplitude

$$\langle h_{ab}(t)h^{ab}(t)\rangle = 2\int_{-\infty}^{\infty} \mathrm{d}f\, S_h(f) = 4\int_{f=0}^{f=\infty} \mathrm{d}(\log f)\, f\, S_h(f)$$

$$\langle h_{ab}(t)h^{ab}(t)\rangle = 2\int_{f=0}^{f=\infty} \mathrm{d}(\log f)\, h_{\mathrm{c}}^2(f)$$

$$h_{\mathrm{c}}^2(f) = 2f\, S_h(f)$$

- Energy density

$$\rho_{\mathrm{gw}} = (1/32\pi G)\langle \dot{h}_{ab}\dot{h}^{ab}\rangle$$

Relationships

$$\rho_{\rm gw} = \frac{4}{32\pi G}\int_{f=0}^{f=\infty} \mathrm{d}(\log f)\, f\,(2\pi f)^2 S_h(f)$$

$$\mathrm{d}\rho_{\rm gw}/\mathrm{d}\log f = (\pi/2G) f^3 S_h(f) = (\pi/4G) f^2 h_{\rm c}^2(f)$$

$$\Omega_{\rm gw}(f) = (2\pi^2/3H_0^2) f^2 h_{\rm c}^2(f)$$

$$\Omega_{\rm gw}(f) = (4\pi^2/3H_0^2) f^3 S_h(f)$$

$$h_{\rm c}(f) \simeq 1.263\times 10^{-18}(1\,\mathrm{Hz}/f)\sqrt{h_0^2\Omega_{\rm gw}(f)}$$

$$\int_f^{f+\Delta f} \mathrm{d}(\log f) h_0^2 \Omega_{\rm gw}(f) \simeq \frac{\Delta f}{f} h_0^2 \Omega_{\rm gw}(f) \quad h_{\rm c}(f,\Delta f) = h_{\rm c}(f)(\Delta f/f)^{1/2}$$

Detector responses

- Detector signal $S(t) = s(t) + n(t)$

$$s(t) = (\delta L_x(t) - \delta L_y(t))/L = D^{ab} h_{ab}(t)$$

$$s(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}\hat{\Omega} \tilde{h}_A(f, \hat{\Omega}) \mathrm{e}^{-2\pi \mathrm{i} f t} D^{ab} \mathrm{e}^A_{ab}(\hat{\Omega})$$

$$s(t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}\hat{\Omega} \tilde{h}_A(f, \hat{\Omega}) F_A(\hat{\Omega}) \mathrm{e}^{-2\pi \mathrm{i} f t}$$

$$\tilde{s}(f) = \sum_{A=+,\times} \int \mathrm{d}\hat{\Omega} \tilde{h}_A(f, \hat{\Omega}) F_A(\hat{\Omega}) \quad F_A(\hat{\Omega}) = D^{ab} \mathrm{e}^A_{ab}(\hat{\Omega})$$

$$\langle s^2(t) \rangle = F \int_{-\infty}^{\infty} \mathrm{d}f \tfrac{1}{2} S_h(f) = F \int_0^{\infty} \mathrm{d}f\, S_h(f)$$

Detectability

- Noise spectrum

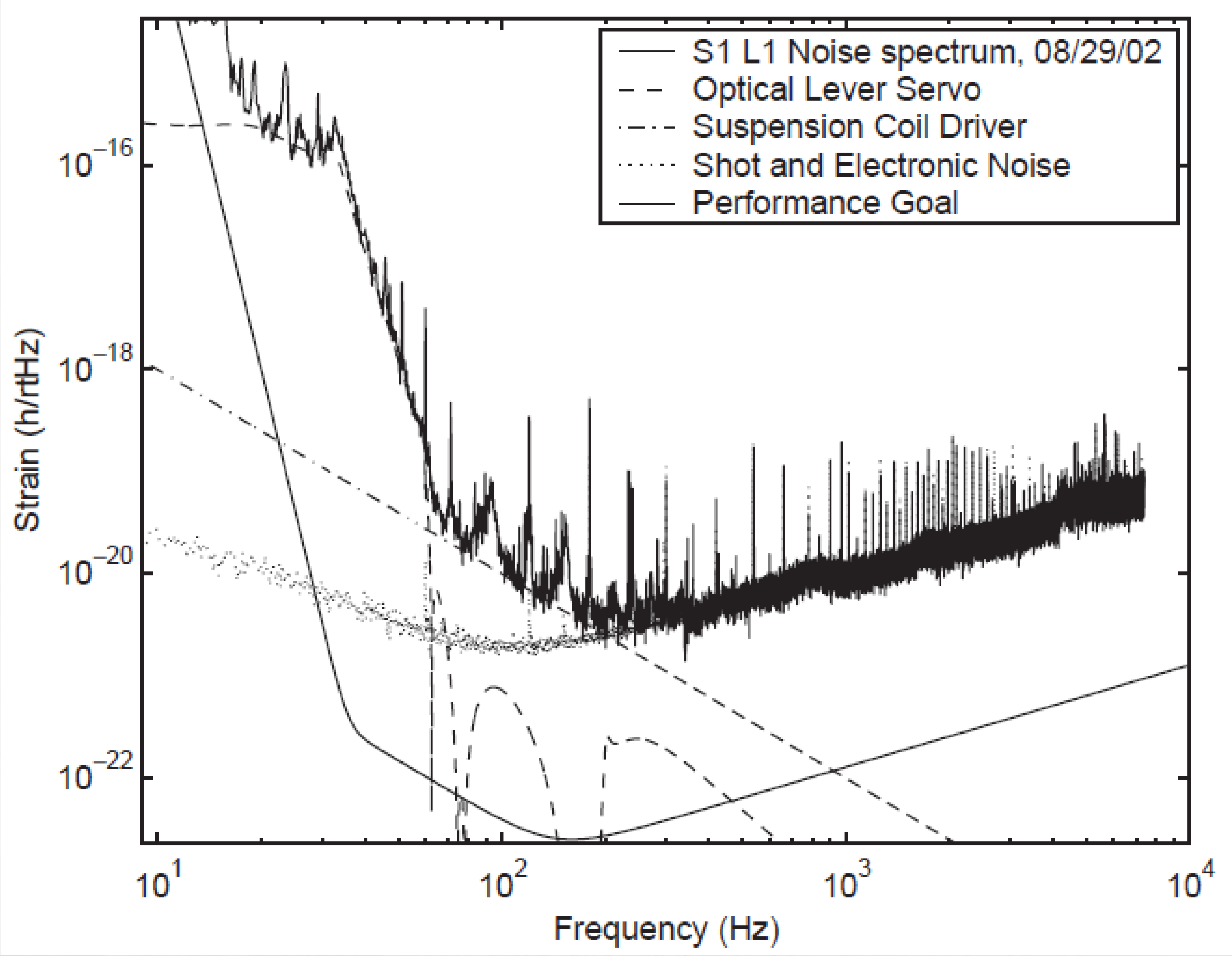
$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \delta(f-f')\tfrac{1}{2}S_n(f)$$

$$\langle n^2(t)\rangle = \int_0^\infty \mathrm{d}f\, S_n(f) \quad \tilde{h}_f \equiv \sqrt{S_n(f)}$$

- Detectable if

$$S_h(f) > (1/F)S_n(f)$$

Sensitivity curve



Production of relic GWs

- Frequency of GW (characteristic)

$$f_0 = f_* a(t_*)/a(t_0) \simeq 8.0 \times 10^{-14} f_* (100/g_S(T_*))^{1/3} (1\,\mathrm{GeV}/T_*)$$

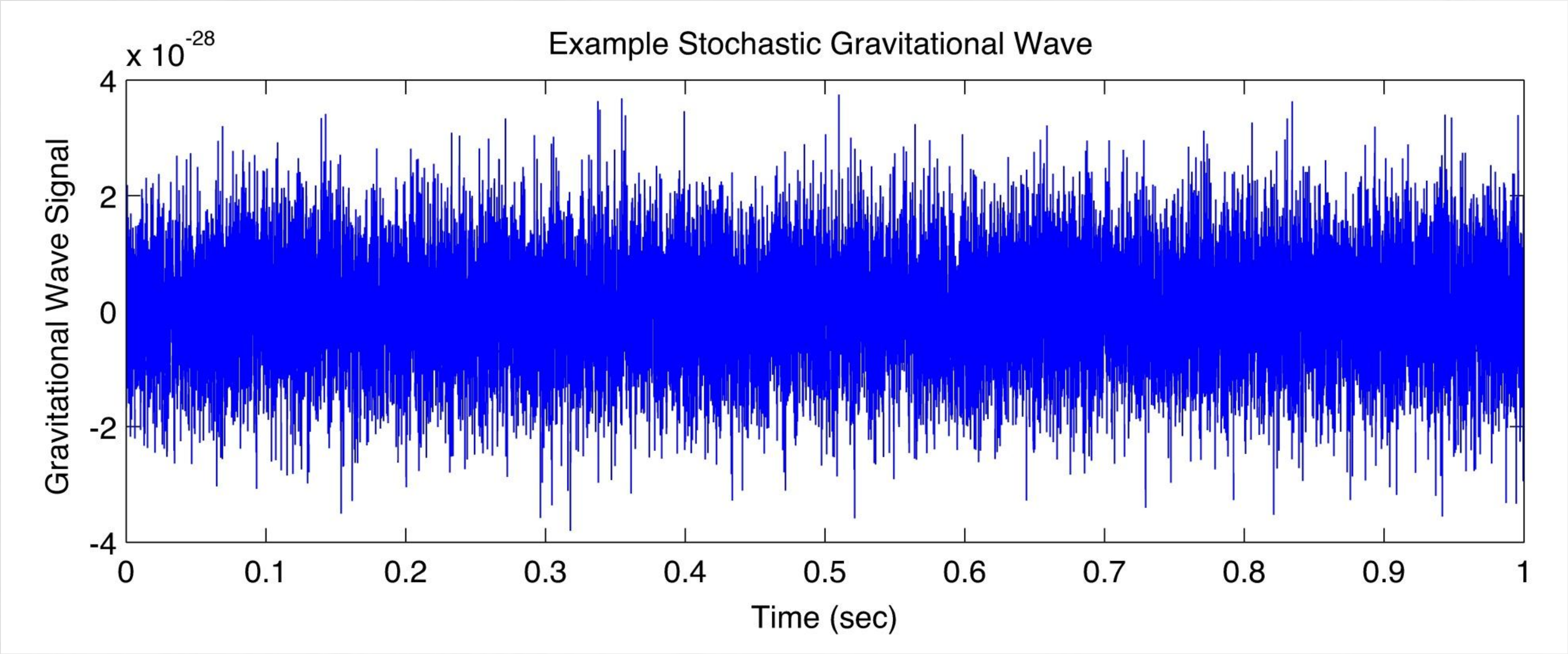
The production time t_* and the production temperature T_* for GWs observed today at frequency f_0, if at time of production they had a wavelength of the order of the horizon length

Detection frequency (Hz)	Production time (s)	Production temperature (GeV)
10^{-4}	7×10^{-13}	600
1	7×10^{-21}	6×10^{6}
100	7×10^{-25}	6×10^{8}
10^{3}	7×10^{-27}	6×10^{9}

$$f_0 \sim 400 (g_*/100)^{1/6}\,\mathrm{GHz}, \quad (T_* = M_{\mathrm{Pl}}/\sqrt{8\pi})$$

$$f_0 \sim 2\,\mathrm{GHz}, \quad (T_* = M_{\mathrm{GUT}})$$

Stochastic signal



From LIGO homepage

Parameter estimation

- Observed stochastic GW will depends on
  - Creation mechanism
  - Cosmological evolution model
  - Various interactions with environments
- Bayesian method should be applied
  - $\wp(\alpha|x) = \frac{\wp(x|\alpha)\wp(\alpha)}{\int \wp(x|\alpha)\wp(\alpha)d\alpha}$
  - $\wp(\alpha, x) = \wp(x|\alpha)\wp(\alpha) = \wp(\alpha|x)\wp(x)$
  - α : unobservable model parameters
  - x : observable data

Remarks

- Scattering GW by Galaxies?
- Interference between GWs?
- Effect of cosmological evolution?
- Is it possible to detect high frequency in future?