2012/02/04 KAGRA F2F MEETING JGW-G1200859

Suspension Thermal Noise of a Non-Uniform Fiber

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Sapphire Fibers for **KAGRA**

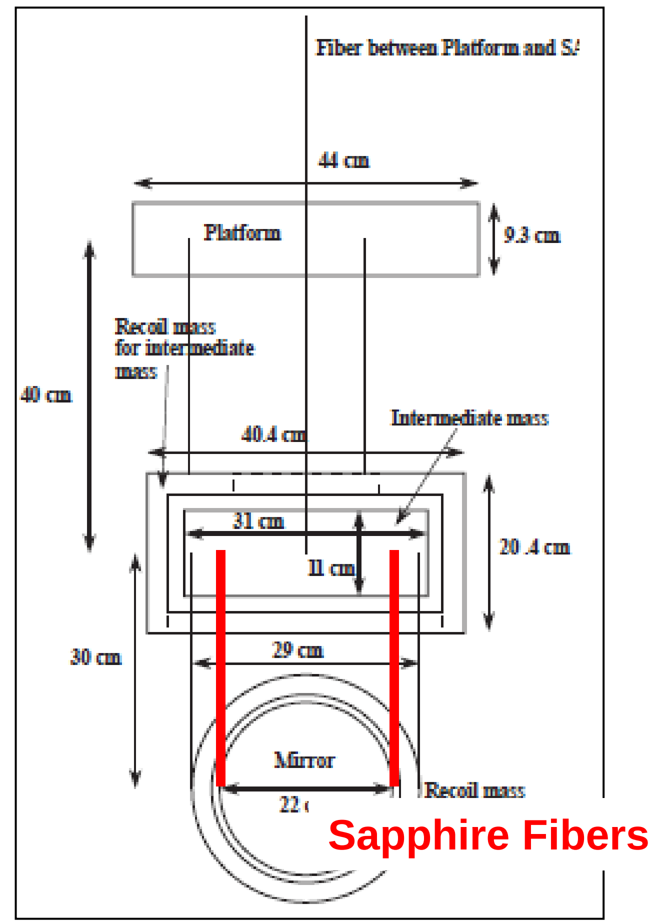
- **Short** and **thick** sapphire fibers to transfer enough amount of heat (~1 W)

- Current Parameters:
  Φ1.6 mm, L30 cm

- Problems of thick & short fibers:
  - Low dilution factor and large thermal noise
  - High resonant frequency of vertical mode
  - Heavy violin modes

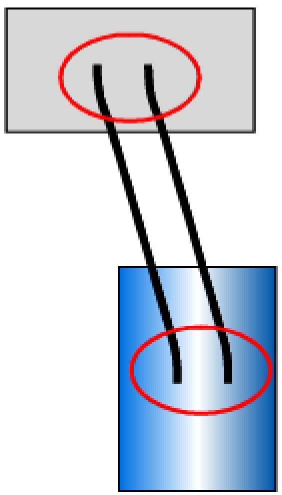
Degrade the sensitivity @ ~ 100 Hz

Cryogenic Payload for KAGRA



Non-Uniform Fiber

- **<u>Thin</u>** fiber at both ends to lower the suspension thermal noise.
- **<u>Thick</u>** fiber in the middle to obtain large thermal transmissivity.

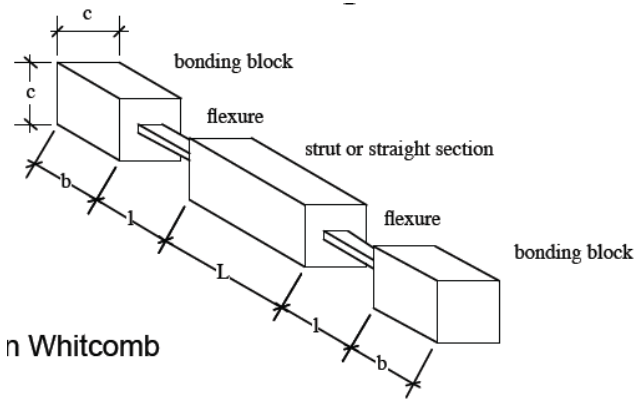


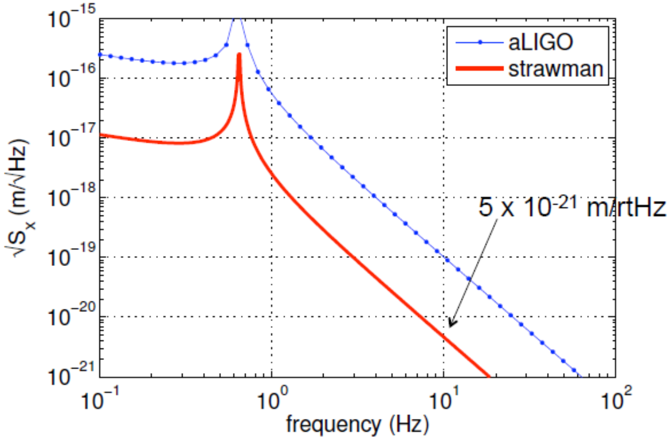
Ue: Potential energy stored in the bending of fibers
Ug: Gravitational potential energy (lossless)

$$\phi_{\mathrm{pendulum}} = \phi_{\mathrm{fiber}} \times \frac{U_e}{U_g + U_e}$$

About This Work

- “Flexure + Strut” fiber was suggested by Warren Johnson.
  [W. Johnson, KAGRA Collaboration Meeting, Nov. 8th, 2011]

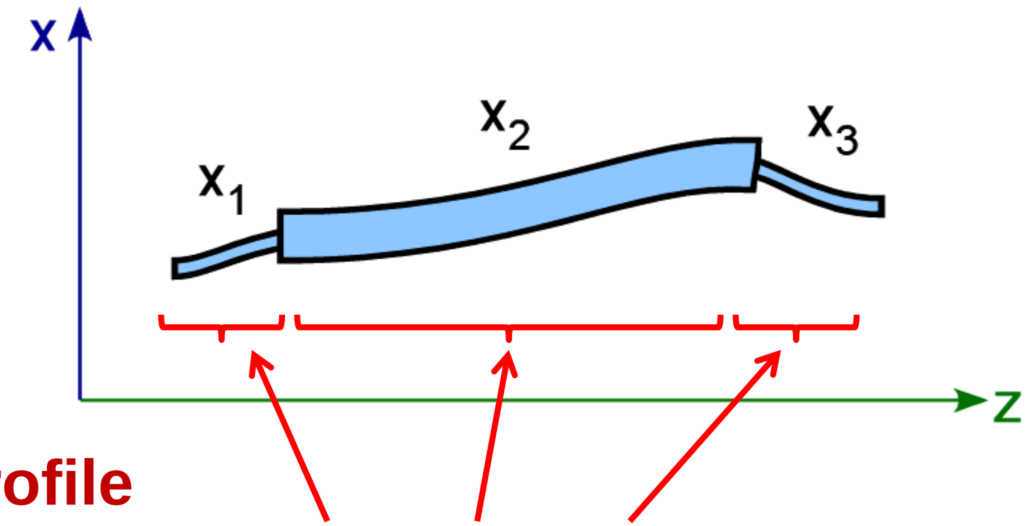




- We have not known its suspension thermal noise **<u>at high frequencies</u>** (>100 Hz), including the **violin modes** and the **vertical modes**.

Model & Calculation Method

- Analytical model in the frequency domain.
- 12 coefficients (A1~D3) must be calculated from boundary conditions.



Beam Profile

$$x_n(z) = A_n \cos(k_n z) + B_n \sin(k_n z) + C_n \exp(k_{e,n} z) + D_n \exp(-k_{e,n} z) \qquad (n = 1,2,3)$$

Boundary Conditions

Phil Willems, Physics Letters A 300 (2002) 162–168

- The displacement and gradient are 0 at the top of the fiber.

$$x_1 = 0, \quad x_1' = 0 \qquad (\text{at } z = 0)$$

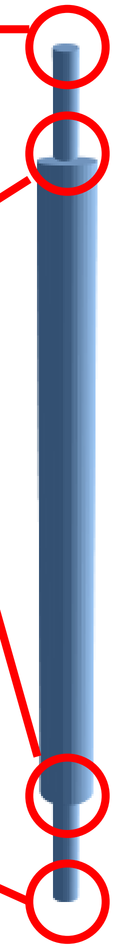
- Two beam profiles are smoothly connected at the boundary.

Continuous displacement	$x_1 = x_2,$
Continuous gradient	$x_1' = x_2',$
Equilibrium of moment	$I_1 x_1'' = I_2 x_2''$
Equilibrium of force	$EI_1 x_1''' - T x_1' = EI_2 x_1''' - T x_2'$

$$(\text{at } z = L_e)$$

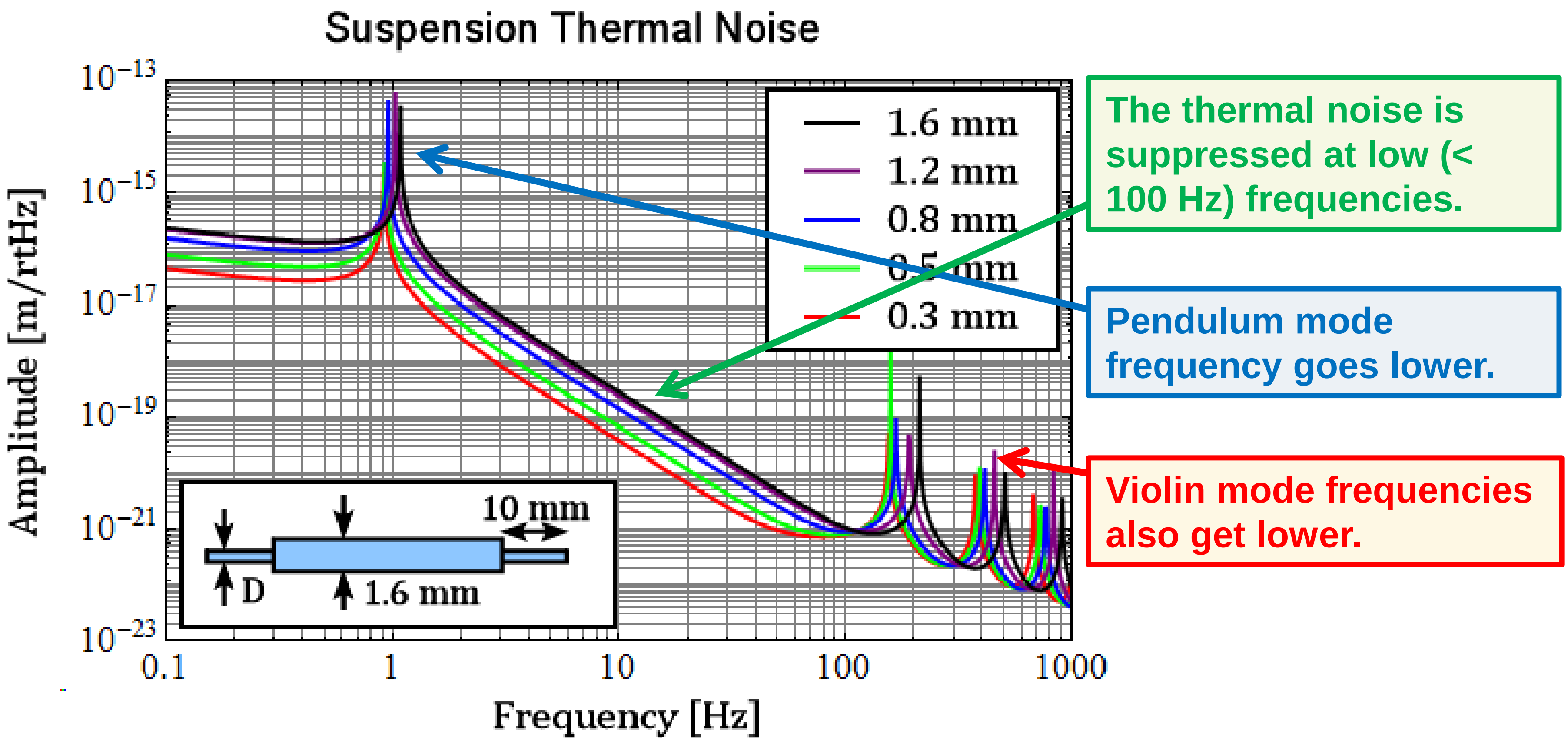
- Equation of motion about the suspended mirror.

$$x_3' = 0\,, \quad EI_1 x_3''' - T x_3' + M_{\mathrm{TM}} \omega^2 x_3 = -\tilde{F} \qquad (\text{at } z = L)$$

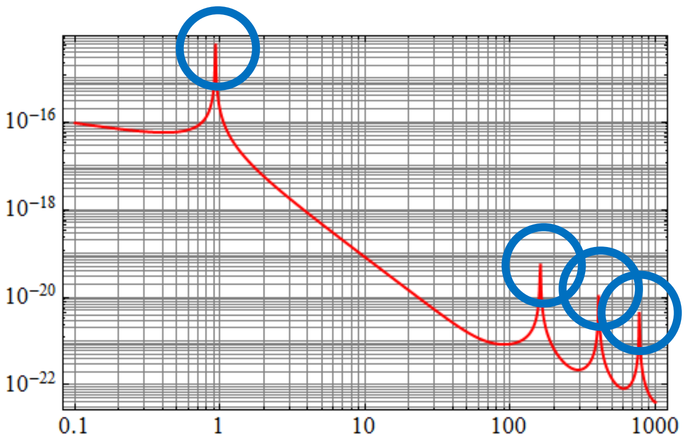


Calculation Result (1)

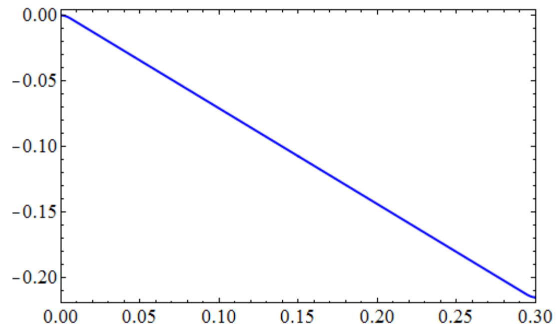
- Decreasing the fiber thickness in the first 1 cm.



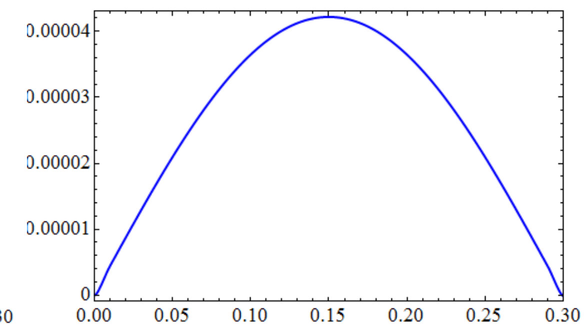
Mode Shape (Beam Profile)

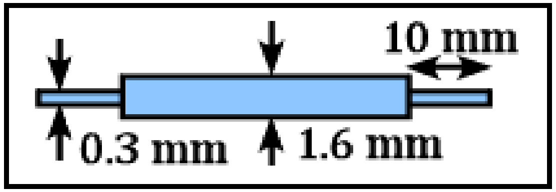


Pendulum Mode

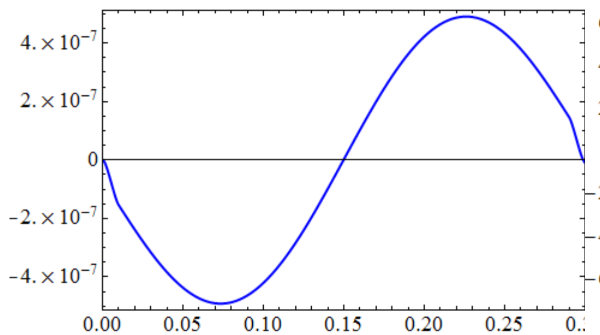


1st Violin Mode

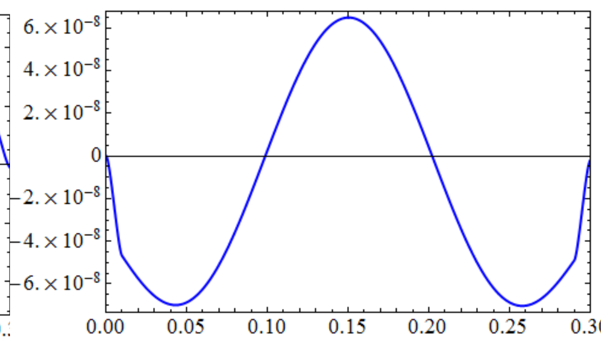




2nd Violin Mode

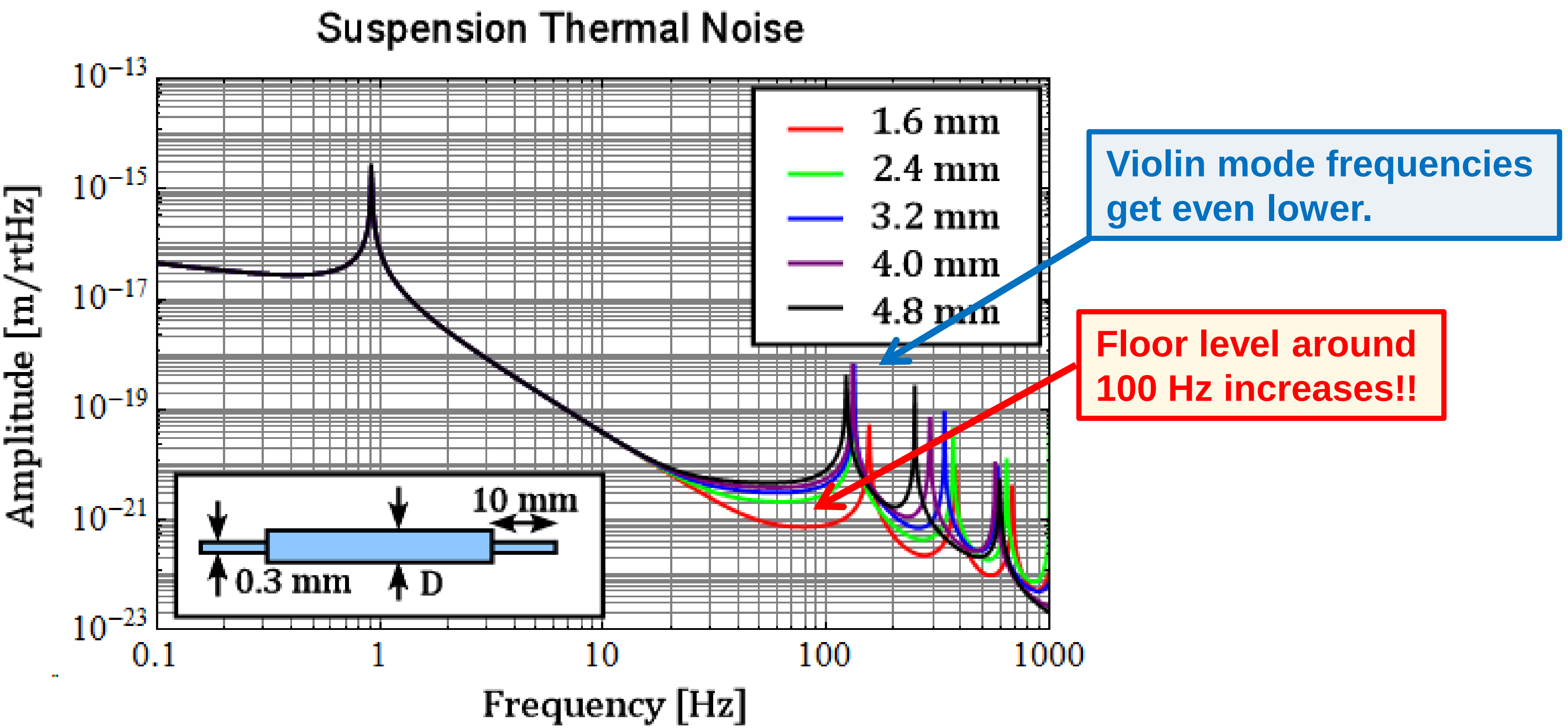


3rd Violin Mode



Calculation Result (2)

- Increasing the thickness in the middle

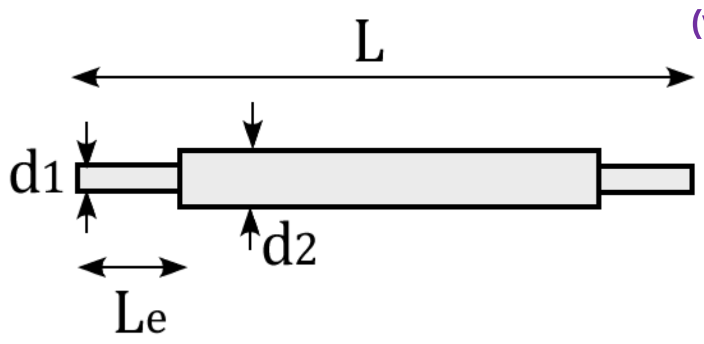


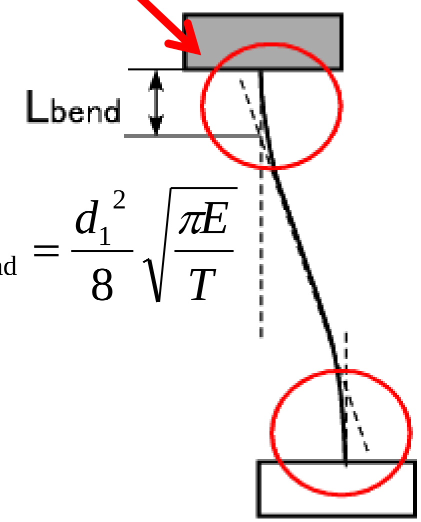
Parameters for KAGRA

1. Decrease the wire thickness in the ends.
2. Make the thin part long enough to cover **the bending part** of the fiber.
3. Increase the wire thickness in the middle to keep the heat transmissivity.
   (taking into account the size effect of the thermal conductivity)

d1 [mm]	d2 [mm]	Le [mm]	f [Hz]
0.60	2.67	6.3	127
0.80	2.11	11.2	111
1.00	1.90	17.4	106
1.30	1.73	29.5	102
1.60	1.60	44.7	102

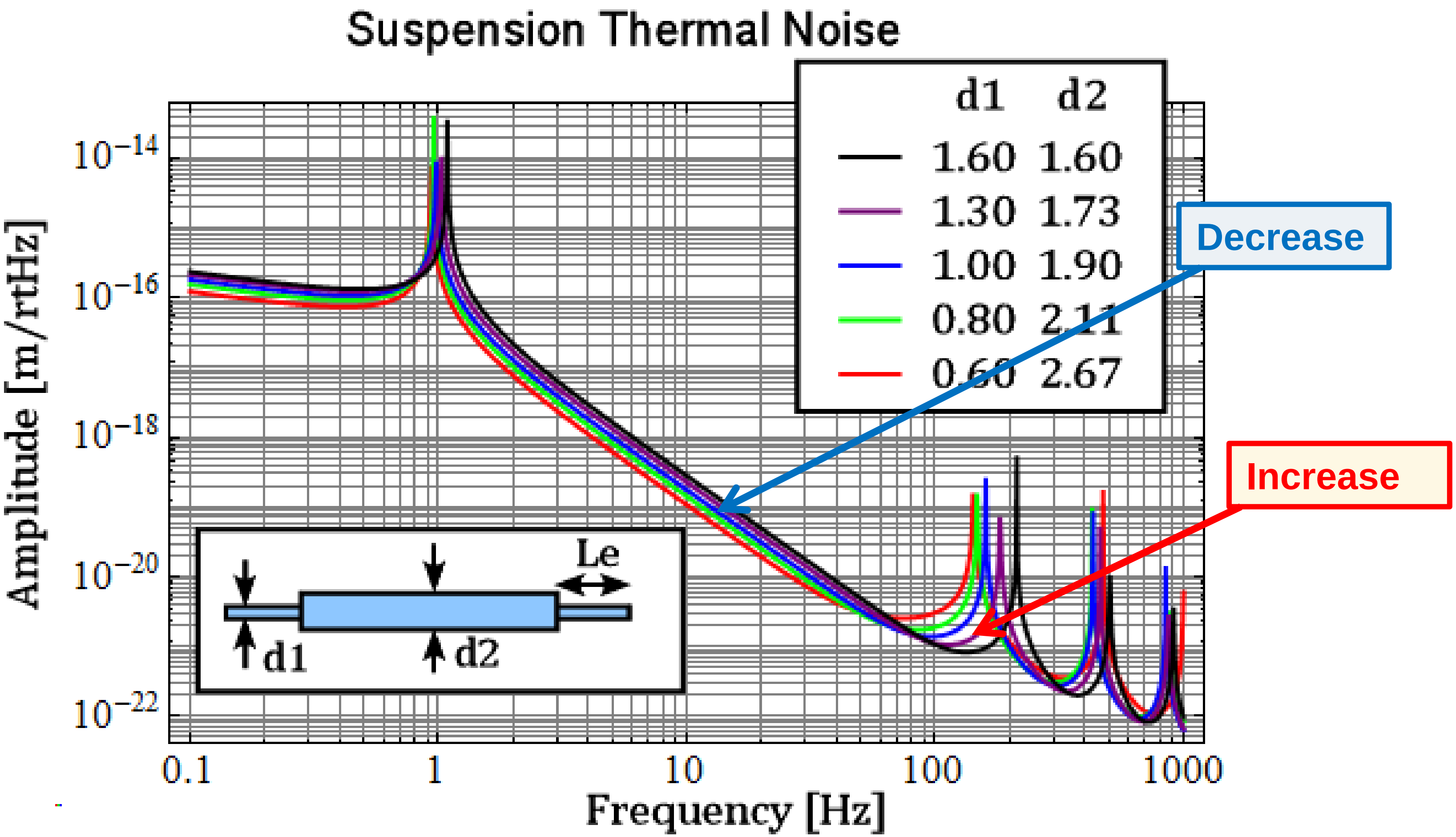
(vertical resonance)





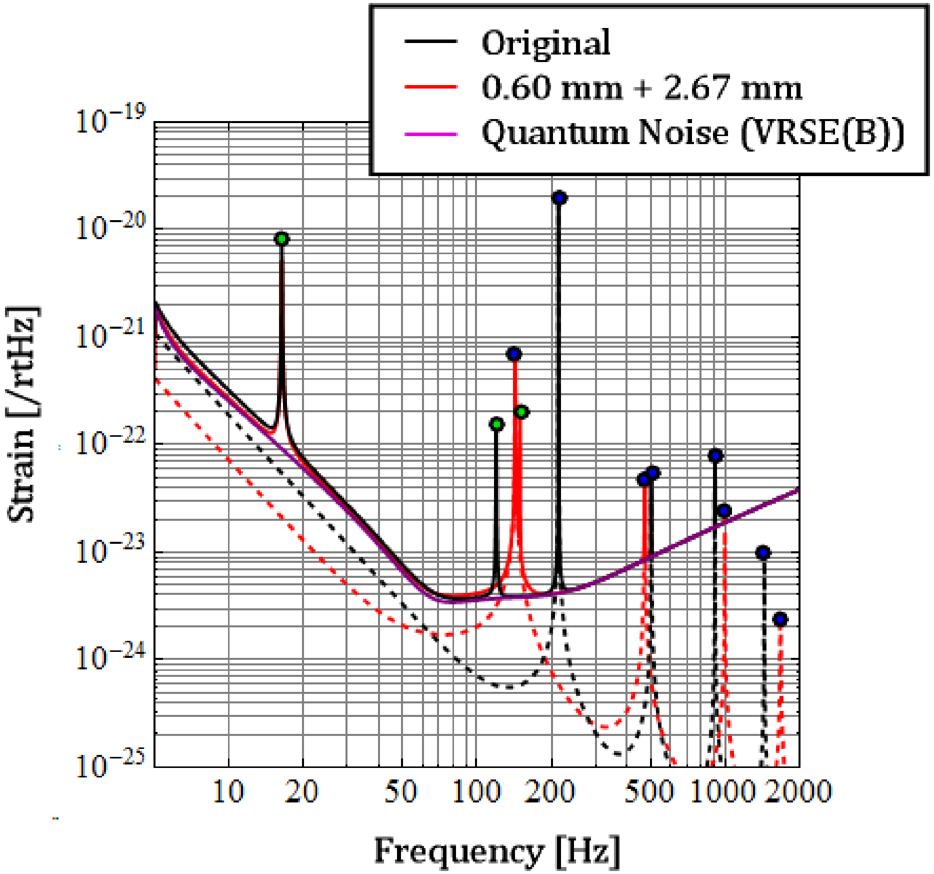
$$L_{\text{bend}} = \frac{d_1^{\,2}}{8}\sqrt{\frac{\pi E}{T}}$$

Calculation Result (3)



Total Sensitivity

- Including **vertical thermal noise** and **quantum noise**



- **Vertical Mode**
- **Violin Mode**

<50 Hz:

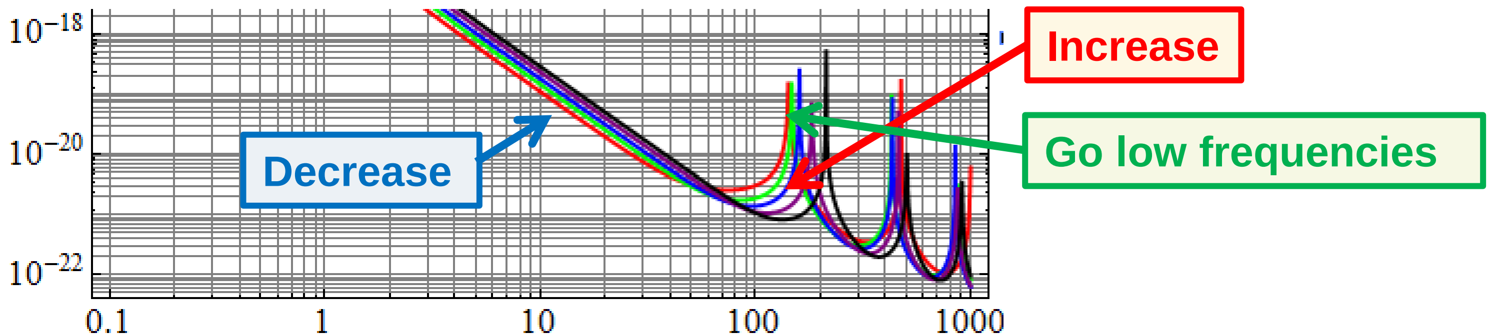
Thermal noise decreases, but the sensitivity is not improved.
(due to the existence of the quantum noise)

>50 Hz:

In both cases, the sensitivity around 100 Hz is degraded by the violin and the vertical mode resonances.

Conclusion

- There is a **trade-off** of using non-uniform fibers :
  - The suspension thermal noise at low frequencies (<50 Hz) decreases, by the "dissipation dilution" effect.
  - However, the suspension thermal noise at high frequencies (~100 Hz) increases and the resonant frequencies of the violin modes get lower.



- Resonances of the violin modes and vertical mode still remain at ~100 Hz.

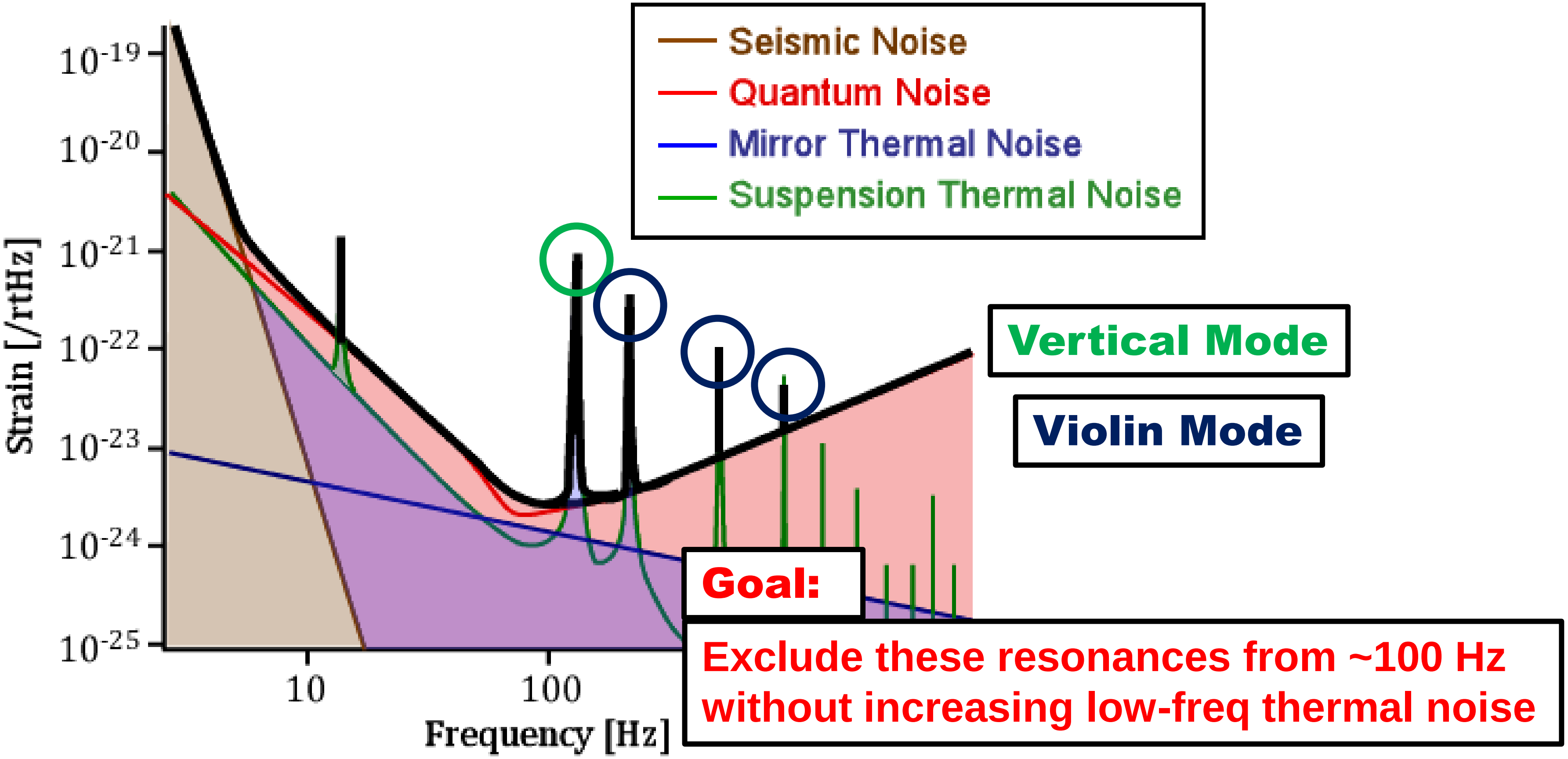
END

Appendix

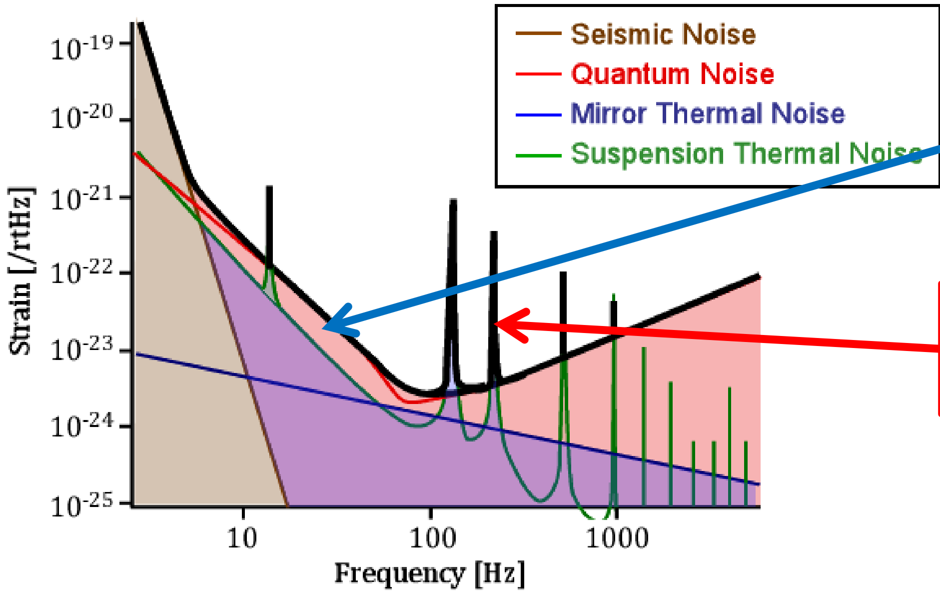
KAGRA Sensitivity

* 1/300 Coupling from vertical noise
* Loss angle of the sapphire fiber: 2 x 10-7
* Quantum Noise: VRSE (Detuned)

Sapphire Fibers (Φ1.6 mm, L30 cm)



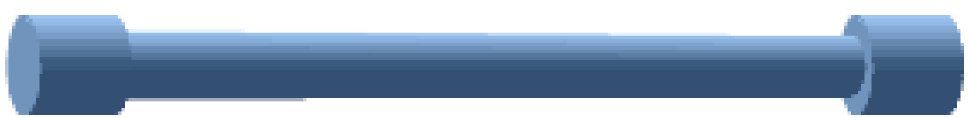
Dumbbell Fiber



The sensitivity is not dominated by the thermal noise at < 50 Hz

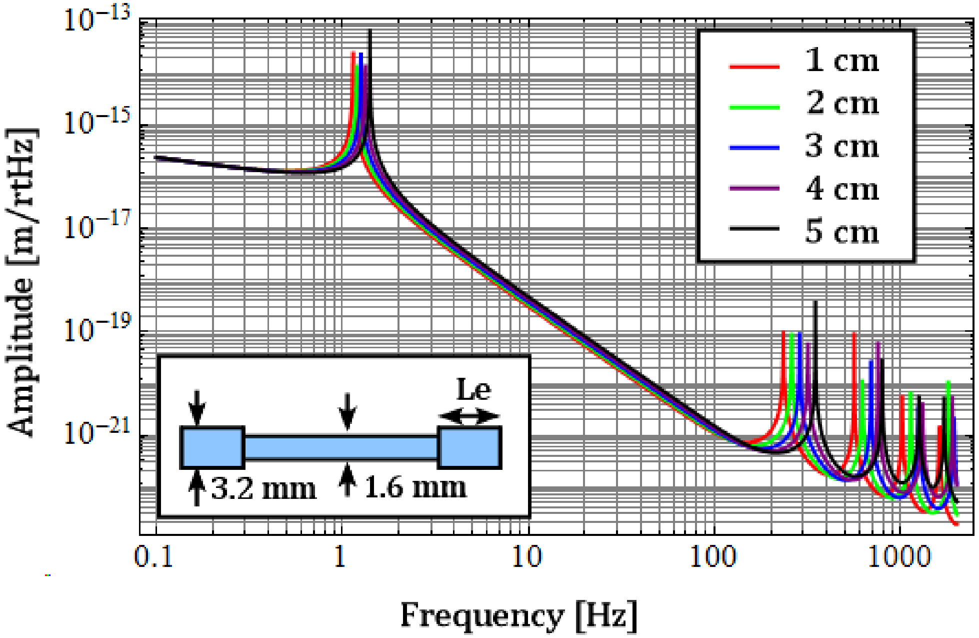
We want to exclude these resonances away from ~100 Hz

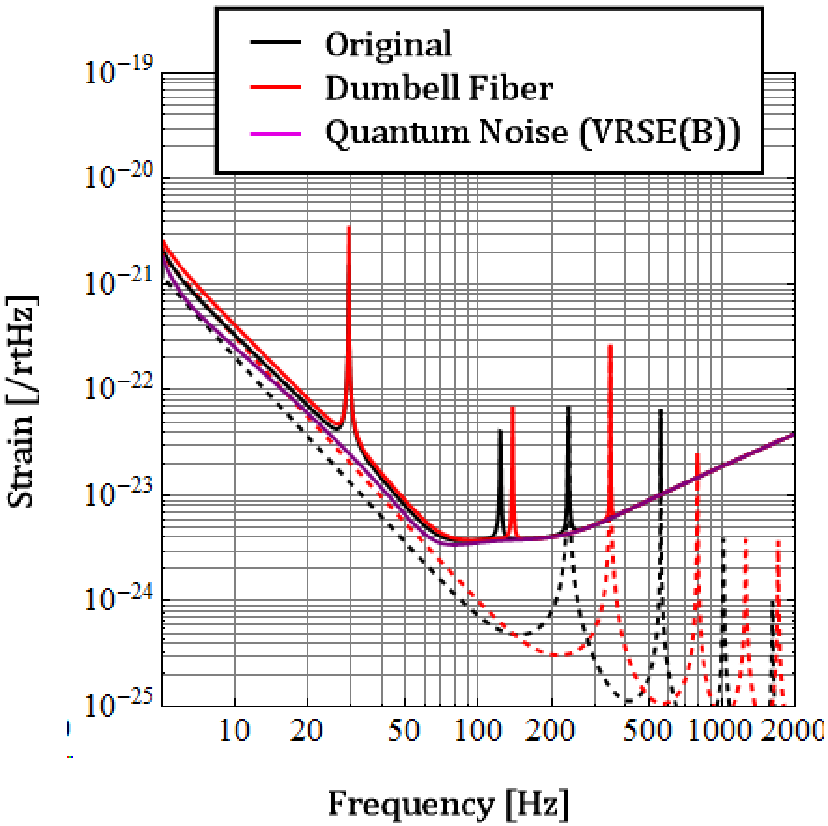
- Reverse idea: **dumbbell fibers**



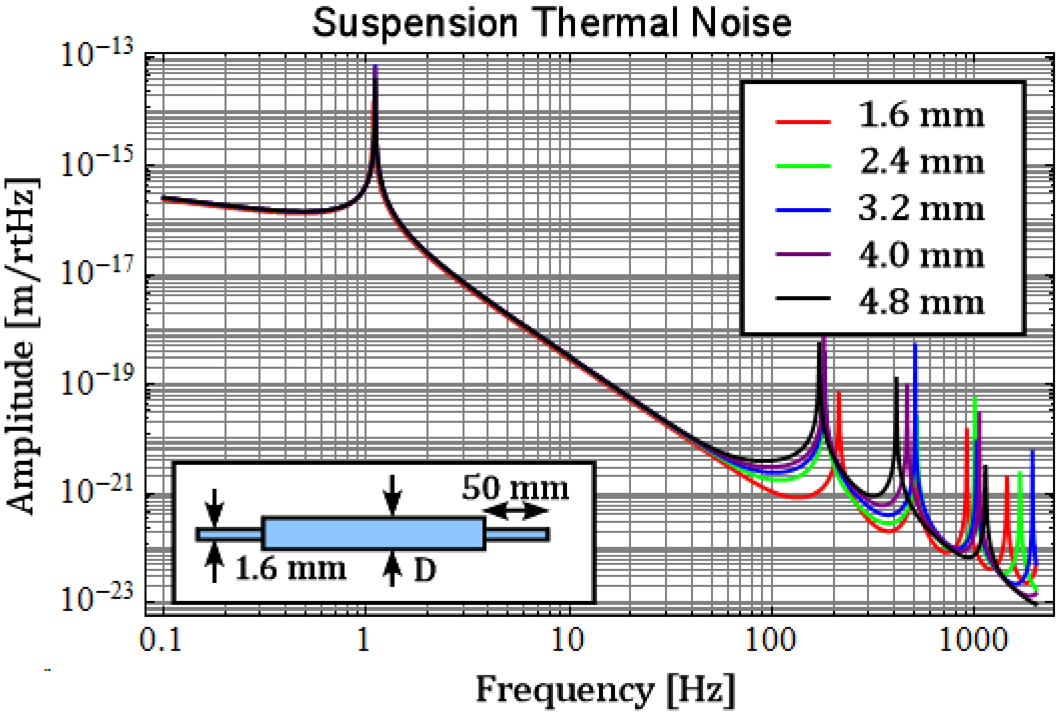
(Just making the effective length of the wire short …)

Dumbbell Fiber Results

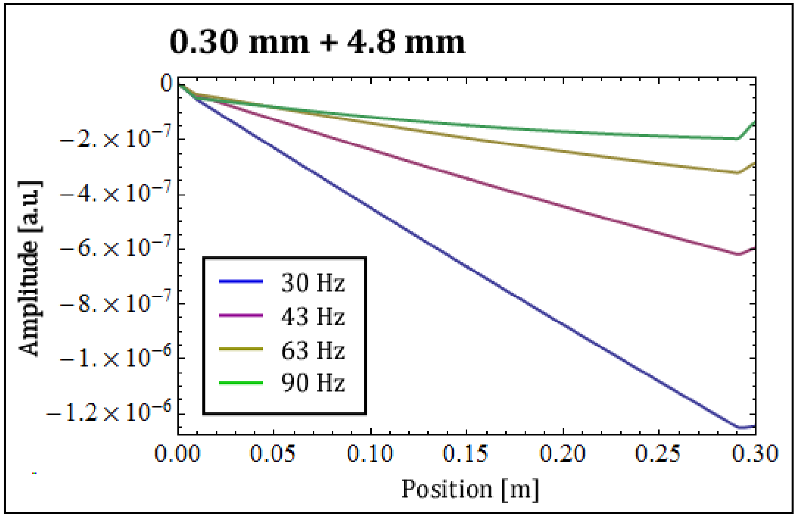


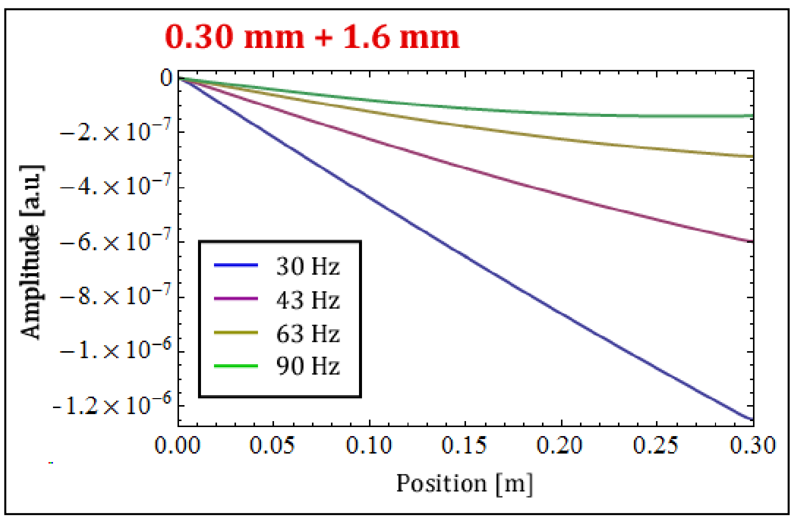


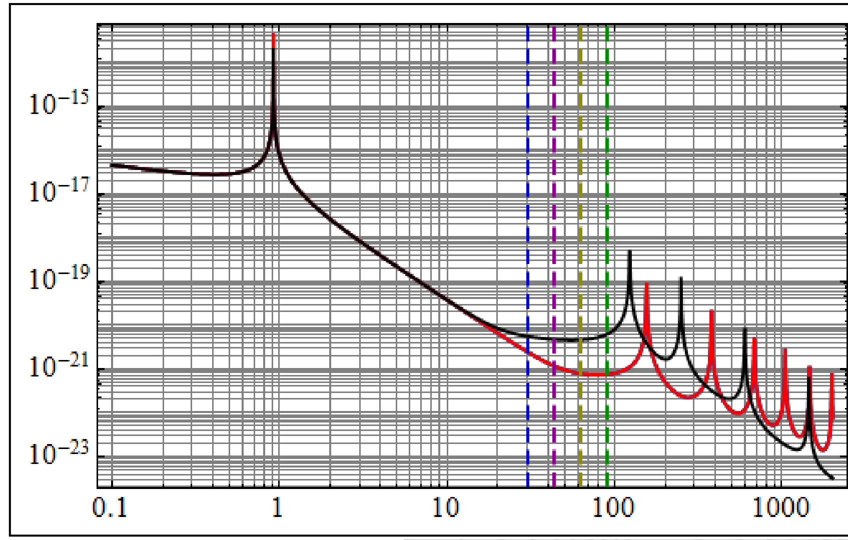
Increasing Middle Thickness

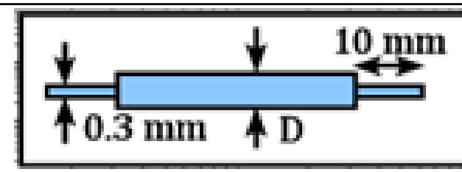


Beam Profile

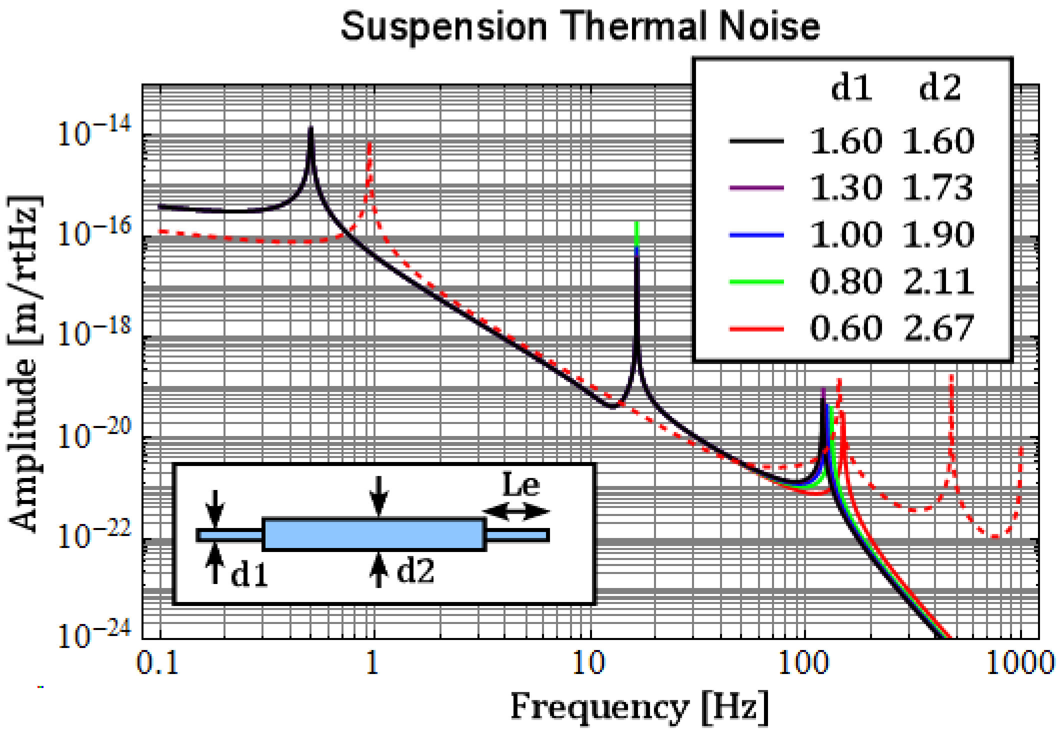


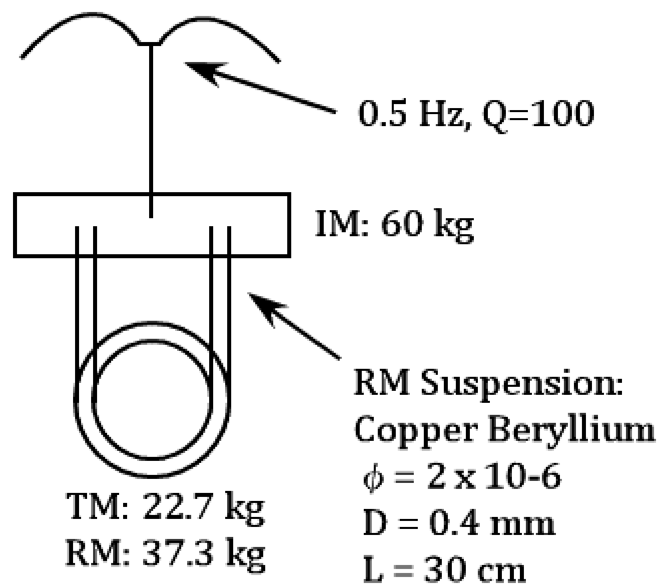




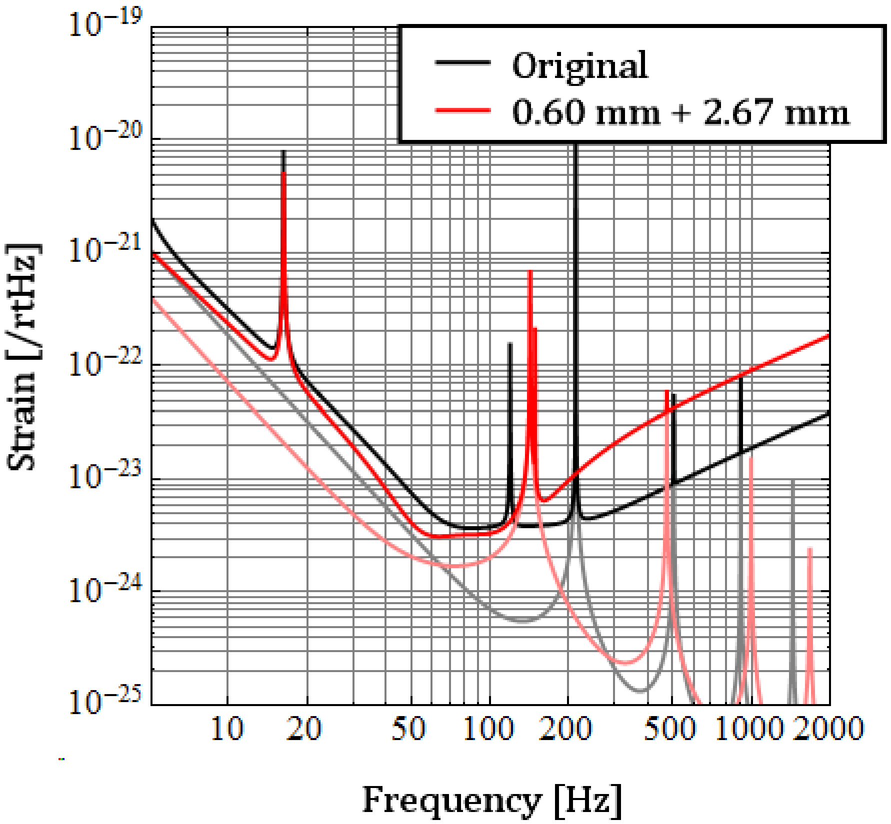


Vertical Thermal Noise





Optimize Quantum Noise



Laser Power→ 1/2
Finesse → 2/3

IR: 248 Mpc → 257 Mpc