Parametric Instability in the bKAGRA Arm Cavity

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Outline

1. Background: Why consider the PI?
2. Introduction: What is the PI?
3. Strategy: What and how to calculate
4. Result
5. Discussion
6. Conclusion

Detail report: JGW-T1200787

Background: Why consider the PI? (1)

- **Curvatures(g-factors) was not determined yet.**
- There are 2 candidates for g-factor(a set of ITM/ETM curvatures)

Negative g-factor set
ITM curvature: $R_1 = 1.68\ \mathrm{km}.$
ETM curvature: $R_2 = 1.87\ \mathrm{km}$

Positive g-factor set
ITM curvature: $R_1 = 14\ \mathrm{km}$
ETM curvature: $R_2 = 7.5\ \mathrm{km}$

PI=Parametric Instability

Background: Why consider the PI? (2)

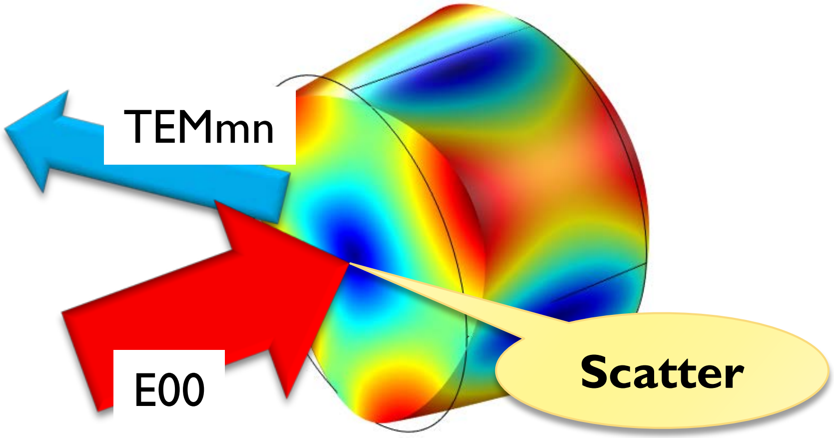
- The negative g-factor set is said to be better in the ASC report [1].

- But it is known that for the negative g-factor set, the parametric instability (PI) might become a serious problem.

[1] Michimura's talk (last f2f in Aug. 2011): JGW-G1100533

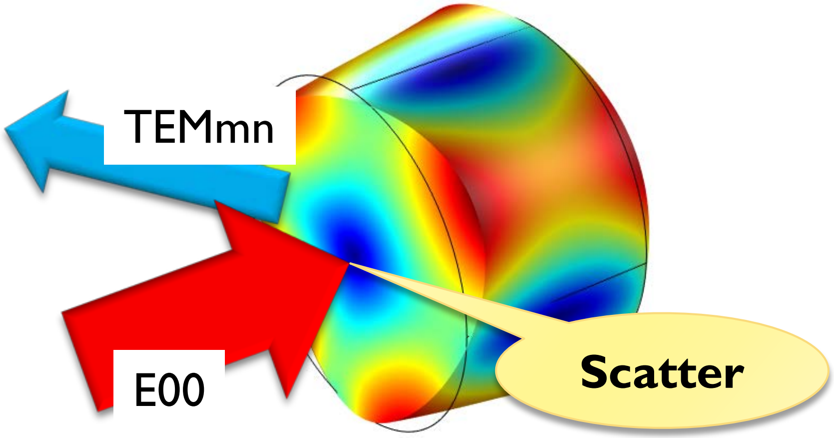
Introduction: What is the PI? (1)

- In the FP cavity, the optical power in the cavity becomes enormous.
- Elastic modes scatter the fundamental mode E_{00} to the higher TEM modes TEM_{mn}.
- TEM_{mn} also excite elastic modes by pumping mirrors.



Introduction: What is the PI? (2)

- By this cycle, some elastic modes are so excited that it make trouble over locking the interferometer.



Introduction: What is the PI? (3)

- The PI condition is represented as follows [2]:

$$\mathcal{R} > 1,$$

$$\mathcal{R} = \sum \frac{2P}{cLM} \frac{Q_\mathrm{m}}{{\omega_\mathrm{m}}^2} \left(\frac{Q_\mathrm{PR}}{1+\left(\frac{\Delta\omega}{\delta\omega_\mathrm{PR}}\right)^2} + \frac{Q_\mathrm{RSE}}{1+\left(\frac{\Delta\omega-\Delta_\mathrm{RSE}}{\delta\omega_\mathrm{RSE}}\right)^2} \right) \Lambda,$$

Λ: overlap factor between optical and elastic mode,

$\Delta\omega = \omega_{00} - \omega_\mathrm{TEM} - \omega_\mathrm{m}$,

ω_m: angular frequency of the elastic mode.

$\mathcal{R}$ is called a “parametric gain,” and the summation is taken over all optical modes.

- PI occurs when shapes and frequencies of the optical and elastic modes are same.

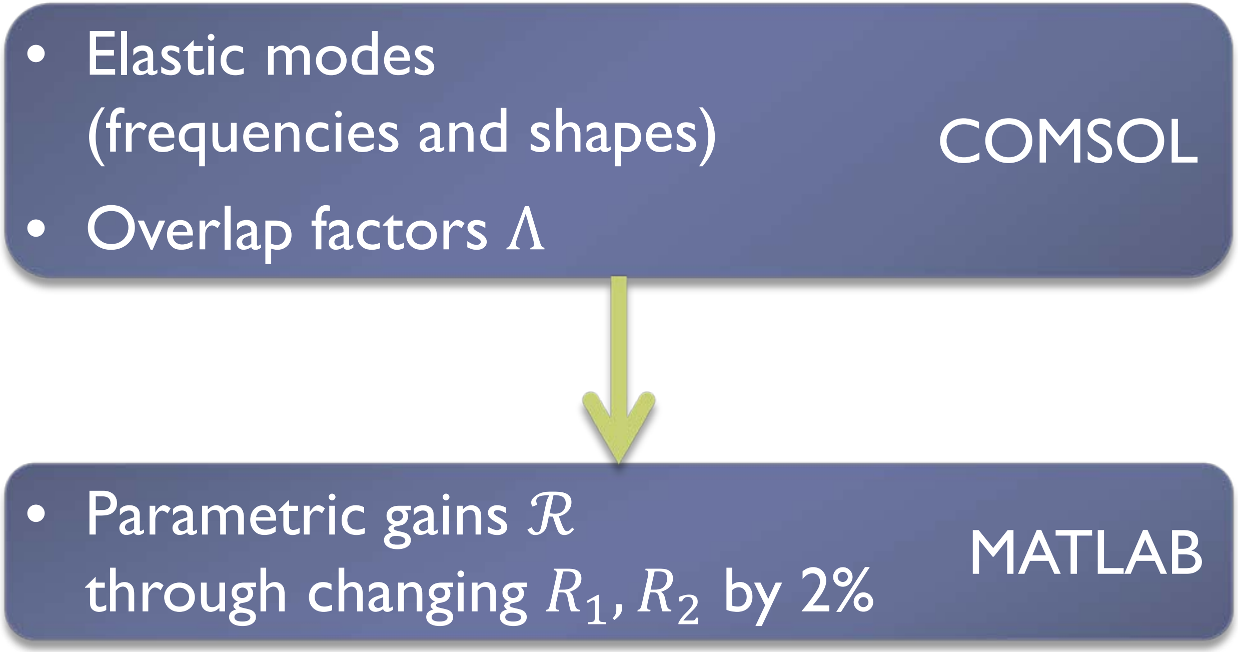
Strategy: What to calculate (1)

$$\mathcal{R} = \sum \frac{2P}{cLM} \frac{Q_{\mathrm{m}}}{{\omega_{\mathrm{m}}}^2} \left(\frac{Q_{\mathrm{PR}}}{1+\left(\frac{\Delta\omega}{\delta\omega_{\mathrm{PR}}}\right)^2} + \frac{Q_{\mathrm{RSE}}}{1+\left(\frac{\Delta\omega-\Delta_{\mathrm{RSE}}}{\delta\omega_{\mathrm{RSE}}}\right)^2} \right) \Lambda$$

- When R_1, R_2 changes within their errors, the transverse mode frequencies vary.
  →In some area of the (R_1, R_2) plane, parametric gain might be large.
- Calculated by COMSOL with MATLAB

Strategy: How to calculate (2)

$$\mathcal{R} = \sum \frac{2P}{cLM} \frac{Q_{\mathrm{m}}}{{\omega_{\mathrm{m}}}^2} \left(\frac{Q_{\mathrm{PR}}}{1+\left(\frac{\Delta\omega}{\delta\omega_{\mathrm{PR}}}\right)^2} + \frac{Q_{\mathrm{RSE}}}{1+\left(\frac{\Delta\omega-\Delta_{\mathrm{RSE}}}{\delta\omega_{\mathrm{RSE}}}\right)^2} \right) \Lambda$$



- Elastic modes (frequencies and shapes)
- Overlap factors Λ

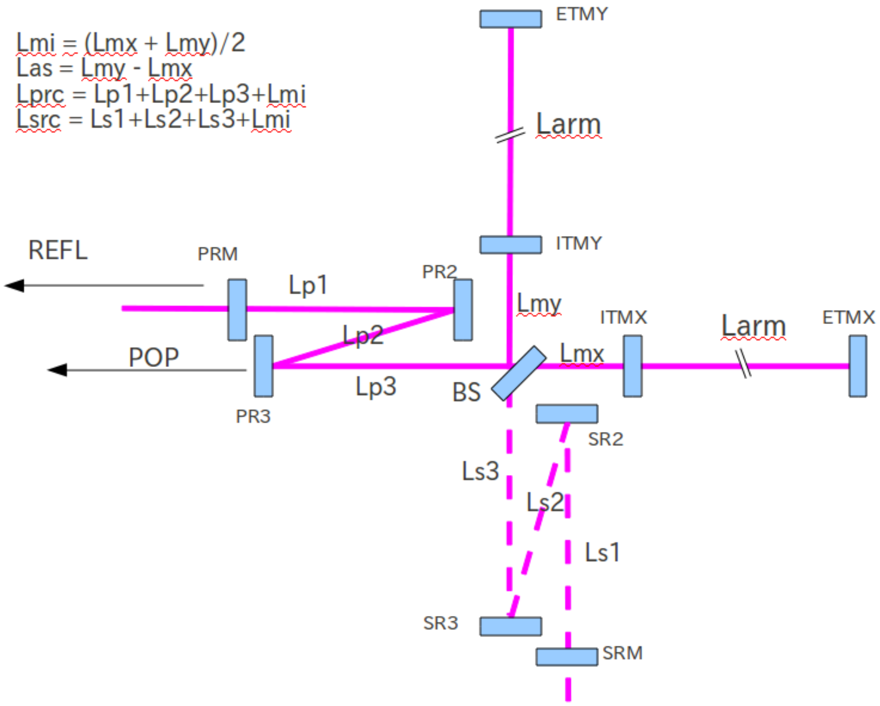
COMSOL

- Parametric gains $\mathcal{R}$ through changing R_1, R_2 by 2%

MATLAB

Parameters

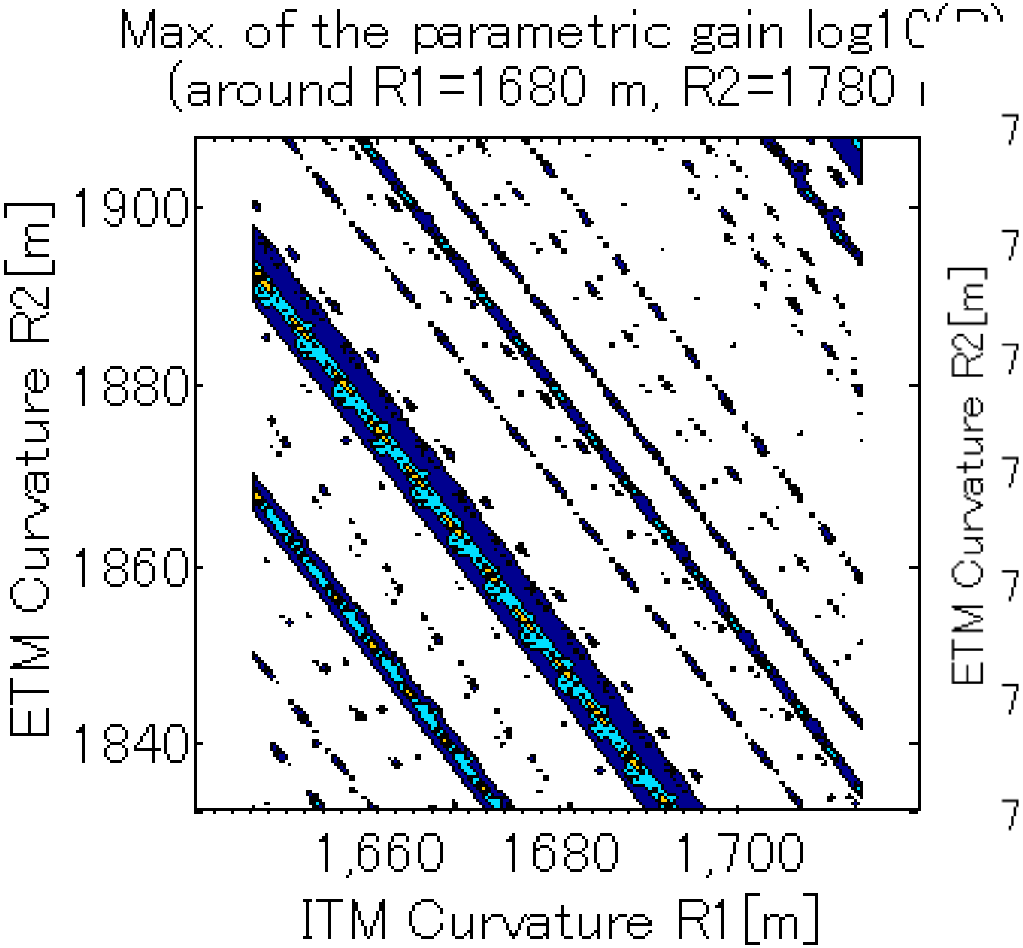
- Mirror radius $r = 110\ \mathrm{mm}$
- Mirror depth $d = 150\ \mathrm{mm}$
- Mirror direction c-axis
- Power in the main cavity $P = 0.41\ \mathrm{MW}$
- Wavelength $\lambda = 1064\ \mathrm{nm}$
- Main cavity length $L = 3000\ \mathrm{m}$
- SR cavity length $L_{\mathrm{SRC}} = 66.591\ \mathrm{m}$
- Q-value of mechanical modes $Q_{\mathrm{m}} = 10^8$
- Finesse $\mathcal{F} = 1550$
- Power transmittance of the PR mirror $T_{\mathrm{PR}} = 0.10$
- Power transmittance of the RSE mirror $T_{\mathrm{RSE}} = 0.37$



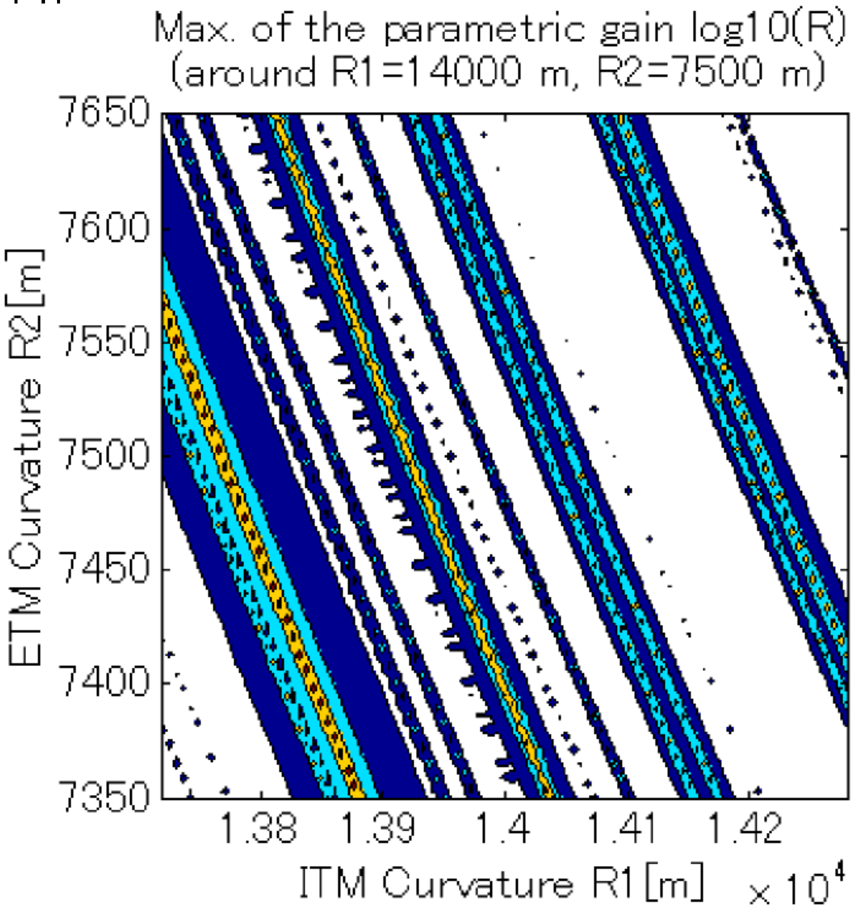
Result (with PR and detuned RSE)

The colored areas show where $\mathcal{R} > 1$.

Negative g



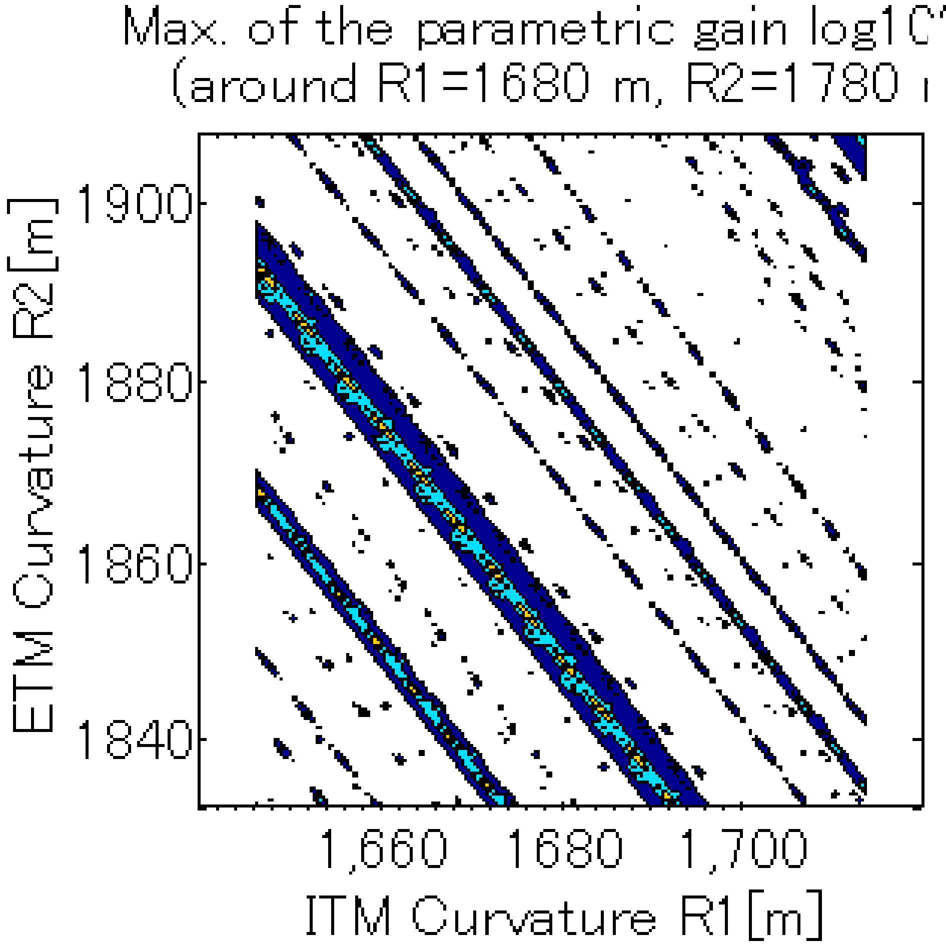
Positive g



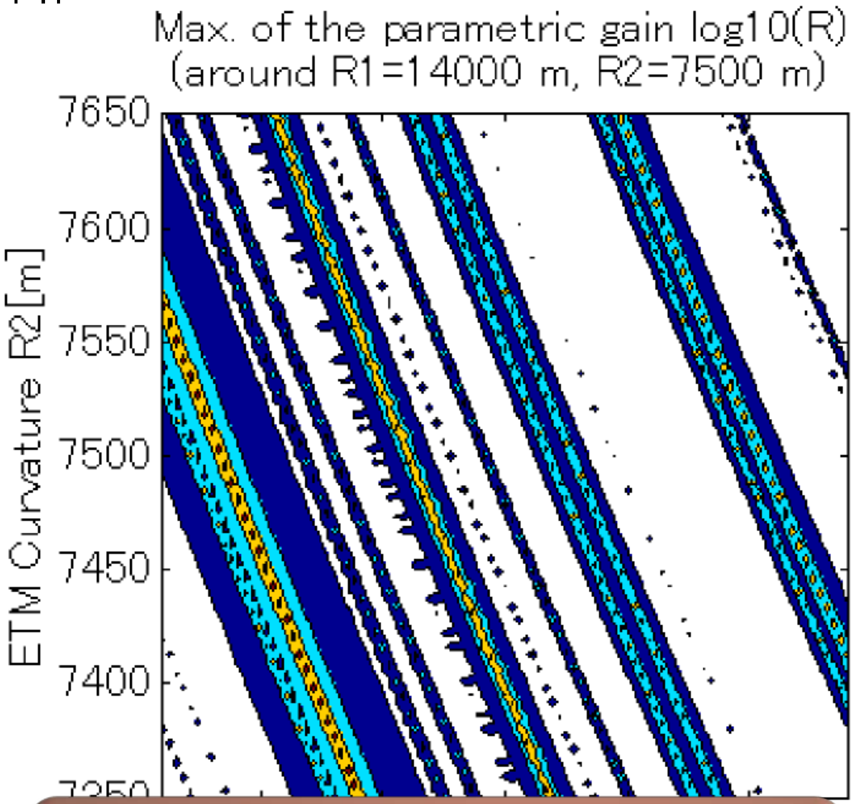
Result (with PR and detuned RSE)

The colored areas show where $\mathcal{R} > 1$.

Negative g



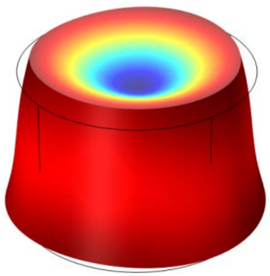
Positive g



Each case has the domain $\mathcal{R} > 1$,
but it does not seems so bad
(**not always $\mathcal{R} > 1$**).

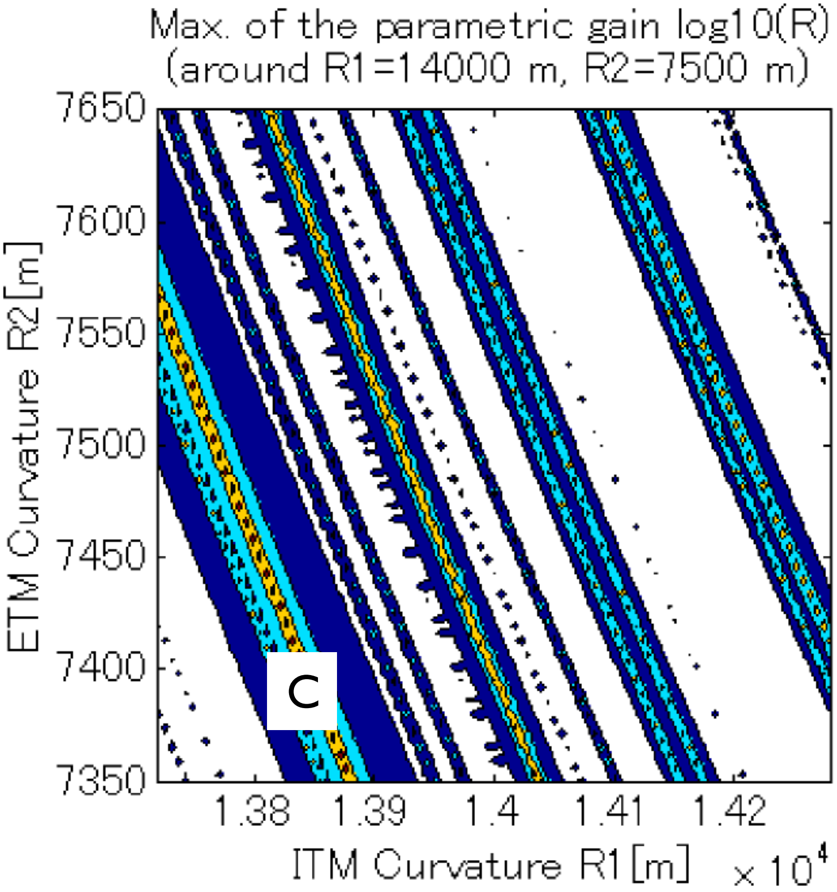
Discussion (positive g)

- $\max(\mathcal{R}) = 4000$
- Only 1 elastic mode C is so unstable in the sense that $\mathcal{R} \gg 1$:



C

23.9 kHz



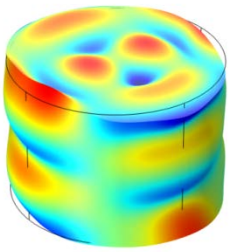
- The overlap is largest with the **TEM20** mode.
- We should care about only the elastic mode C.

Discussion (negative g)

- $\max(\mathcal{R}) = 16000$
  (4 times larger than the positive g)

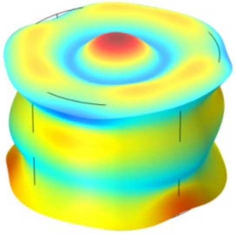
- In addition to C, the below 2 modes are also unstable :

A



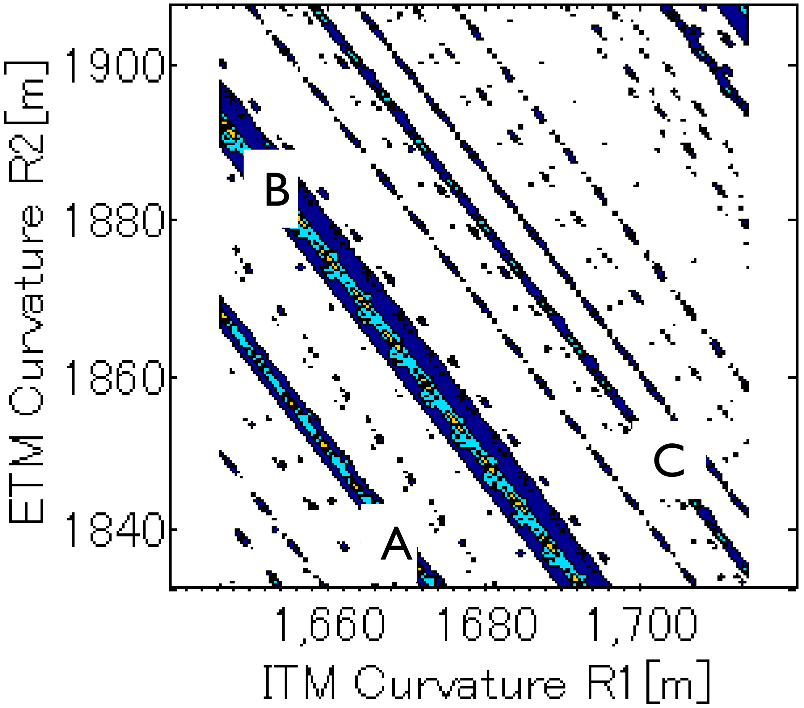
74.9 kHz

B



75.4 kHz

- For both A and B, their overlaps are largest with **TEM02**.



Max. of the parametric gain log10(R)
(around R1=1680 m, R2=1780 m)

Conclusion

- In the previous figs., we see more colored region in the positive case.

- The negative g case has more elastic mode to care than the positive case.
  (Since when curvature R changes with 2%, $g = 1 - L/R$ varies more drastic for the negative g than for the positive g.)

- **But the PI does not seem to be a serious problem in both cases(Not Always $\mathcal{R} > 1$).**

- With the ASC report by Michimura,

  the negative g is finally supported.

  (@ the MIF meeting 1/19)

Fin.

Appendix A: Calculation error

- **Q.** By setting the curvature requirement like +1%/-0% (not $\pm 0.5\%$ as usual), can we get rid of the PI risk in advance?

- **A.** Since the error of the FEM is $1 \sim 5\%$, the error of $\max(\mathcal{R})$ should be $> 2\%$. So we cannot get rid of the risk completely.

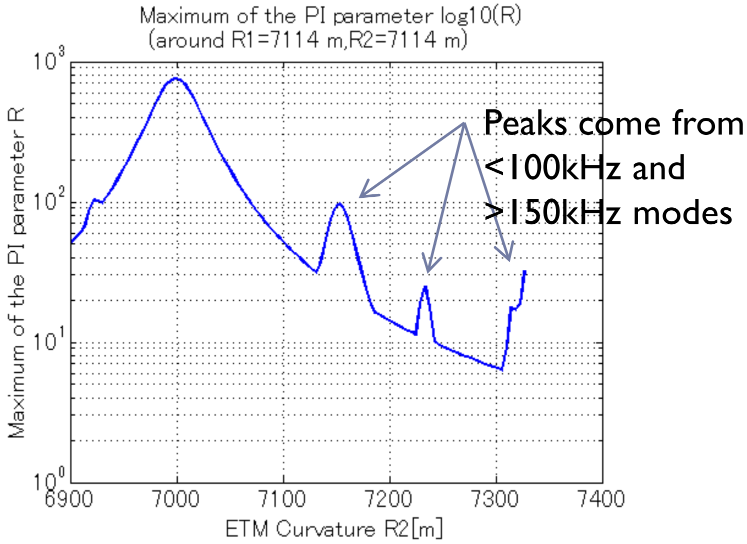
(To get rid of the PI in advance, the error of the FEM should be $\ll 0.5\%$. And there is no such a method.)

Appendix B: Check with the old result

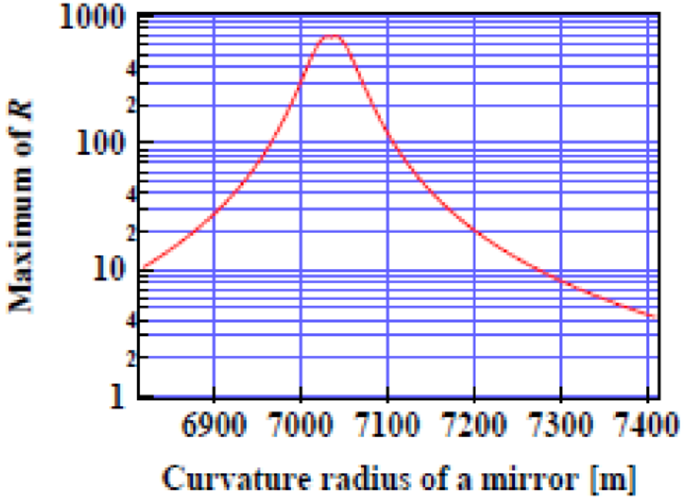
- Often reported that the FEM calculations return with > 1% errors.

- Check with the Yamamoto's result(old parameters) [2]
  →My result agree with it by < 2% errors for the peak position and value.
  →OK

[2]H. Yamamoto et al., *Amaldi proceedings*(2009)



Peaks come from <100kHz and >150kHz modes

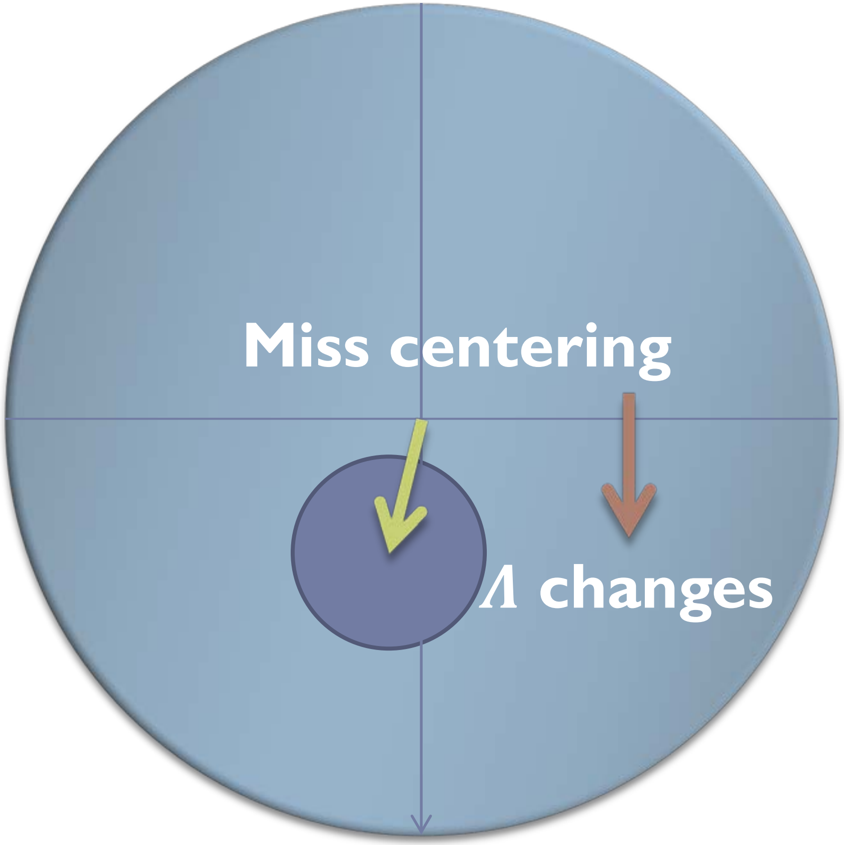


Appendix C: Loss of higher TEM modes

- For higher TEM modes, the power loss at mirrors is large.
- So, the Finesse becomes smaller than $\mathcal{F} = 1550$ for them.
- **Q.** How much is that effect?
- **A.** The highest peak comes from TEM20&02, and their power loss are about $3\ \mathrm{ppm}$. So that effect doesn't make role.

Appendix D: Beam miss-centering

- A beam miss-centering might be occurs with 1mm error.
- **Q.** By this error, the PI occurs?
- **A.** No. (By miss-centering, $\mathcal{R}$ changes only 1%.)

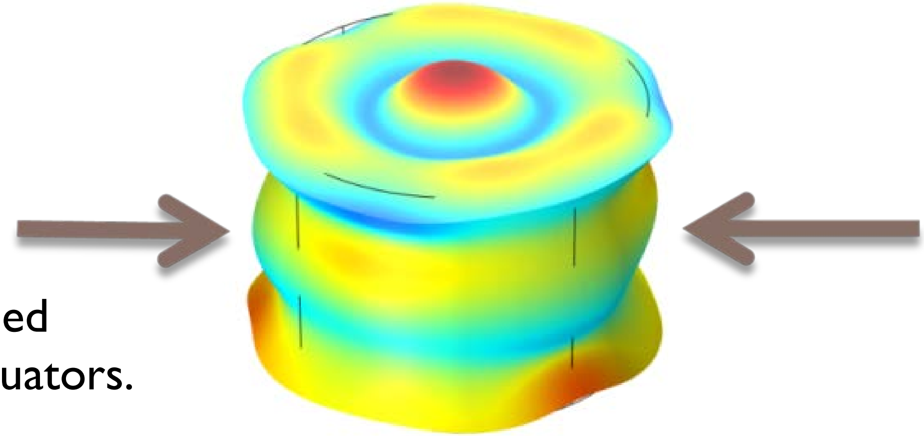


Appendix E: When we face with the PI

- Because of the error of the FEM by COMSOL, the risk of the PI is not excluded completely.

- We should prepare for the PI by some method.
(e.g. Suppress the quality value of the elastic mode by some actuators [3].)

[3] Zhang et al., 2011



Damped
by actuators.