

Parametric Instability in the bKAGRA Arm Cavity

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Outline

1. Background: Why consider the PI?
2. Introduction: What is the PI?
3. Strategy: What and how to calculate
4. Result
5. Discussion
6. Conclusion

Detail report: JGW-TI200787

Background: Why consider the PI? (1)

- ▶ **Curvatures(g-factors) have not been determined yet.**
- ▶ There are 2 candidates for g-factor(a set of ITM/ETM curvatures)

Negative g-factor set

ITM curvature: $R_1 = 1.68$ km.

ETM curvature: $R_2 = 1.87$ km

Positive g-factor set

ITM curvature: $R_1 = 14$ km

ETM curvature: $R_2 = 7.5$ km

PI=Parametric Instability

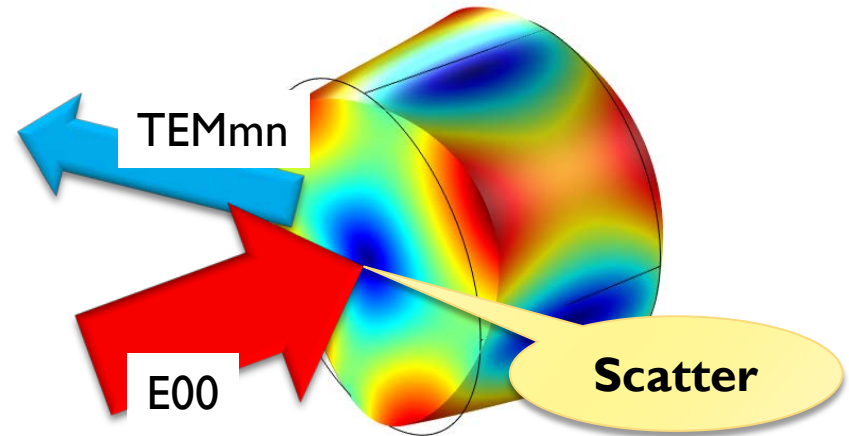
Background: Why consider the PI? (2)

- ▶ The negative g-factor set is said to be better in the ASC report [1].
- ▶ But it is known that for the negative g-factor set, the parametric instability (PI) might become a serious problem.

[1] Michimura's talk (last f2f in Aug. 2011)

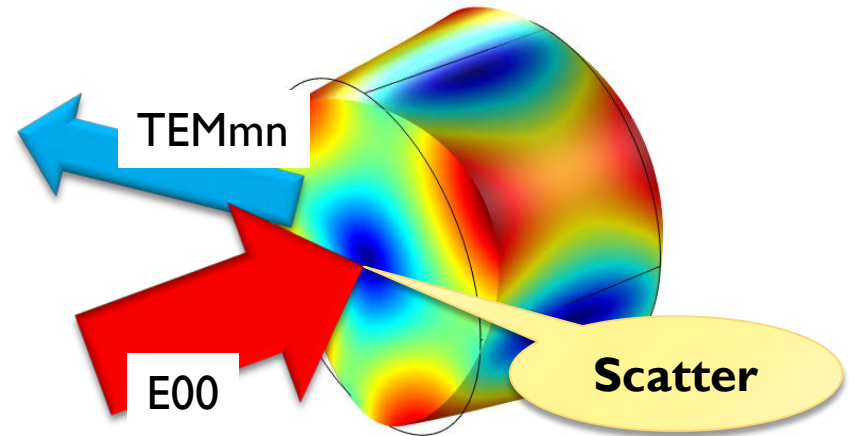
Introduction: What is the PI? (1)

- ▶ In the FP cavity, the optical power in the cavity becomes enormous.
- ▶ Elastic modes scatter the fundamental mode E_{00} to the higher TEM modes TEM_{mn} .
- ▶ TEM_{mn} also excite elastic modes by pumping mirrors.



Introduction: What is the PI? (2)

- ▶ By this cycle, some elastic modes are so excited that it make trouble over locking the interferometer.



Introduction: What is the PI? (3)

- ▶ The PI condition is represented as follows [2]:

$$\mathcal{R} > 1,$$

$$\mathcal{R} = \sum \frac{2P}{cLM} \frac{Q_m}{\omega_m^2} \left(\frac{Q_{PR}}{1 + \left(\frac{\Delta\omega}{\delta\omega_{PR}} \right)^2} + \frac{Q_{RSE}}{1 + \left(\frac{\Delta\omega - \Delta_{RSE}}{\delta\omega_{RSE}} \right)^2} \right) \Lambda,$$

Λ : overlap factor between optical and elastic mode,

$$\Delta\omega = \omega_{00} - \omega_{\text{TEM}} - \omega_m,$$

ω_m : angular frequency of the elastic mode.

\mathcal{R} is called a “parametric gain,” and the summation is taken over all optical modes.

- ▶ PI occurs when shapes and frequencies of the optical and elastic modes are same.

Strategy: What to calculate (1)

$$\mathcal{R} = \sum \frac{2P}{cLM} \frac{Q_m}{\omega_m^2} \left(\frac{Q_{PR}}{1 + \left(\frac{\Delta\omega}{\delta\omega_{PR}} \right)^2} + \frac{Q_{RSE}}{1 + \left(\frac{\Delta\omega - \Delta_{RSE}}{\delta\omega_{RSE}} \right)^2} \right) \Lambda$$

- ▶ When R_1, R_2 changes within their errors, the transverse mode frequencies vary.
→ In some area of the (R_1, R_2) plane, parametric gain might be large.
- ▶ Calculated by COMSOL with MATLAB

Strategy: How to calculate (2)

$$\mathcal{R} = \sum \frac{2P}{cLM} \frac{Q_m}{\omega_m^2} \left(\frac{Q_{PR}}{1 + \left(\frac{\Delta\omega}{\delta\omega_{PR}} \right)^2} + \frac{Q_{RSE}}{1 + \left(\frac{\Delta\omega - \Delta_{RSE}}{\delta\omega_{RSE}} \right)^2} \right) \Lambda$$

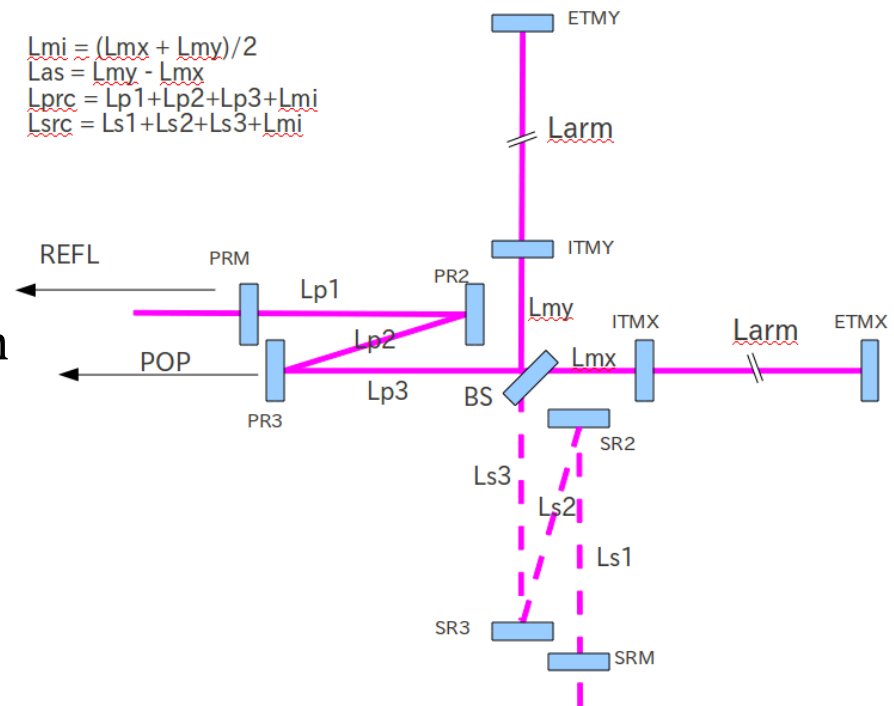
- Elastic modes
(frequencies and shapes) COMSOL
- Overlap factors Λ



- Parametric gains \mathcal{R}
through changing R_1, R_2 by 2% MATLAB

Parameters

- ▶ Mirror radius $r = 110$ mm
- ▶ Mirror depth $d = 150$ mm
- ▶ Mirror direction c-axis
- ▶ Power in the main cavity
 $P = 0.41$ MW
- ▶ Wavelength $\lambda = 1064$ nm
- ▶ Main cavity length $L = 3000$ m
- ▶ SR cavity length $L_{\text{SRC}} = 66.591$ m
- ▶ Q-value of mechanical modes
 $Q_{\text{m}} = 10^8$
- ▶ Finesse $\mathcal{F} = 1550$
- ▶ Power transmittance of the PR mirror $T_{\text{PR}} = 0.10$
- ▶ Power transmittance of the RSE mirror $T_{\text{RSE}} = 0.37$

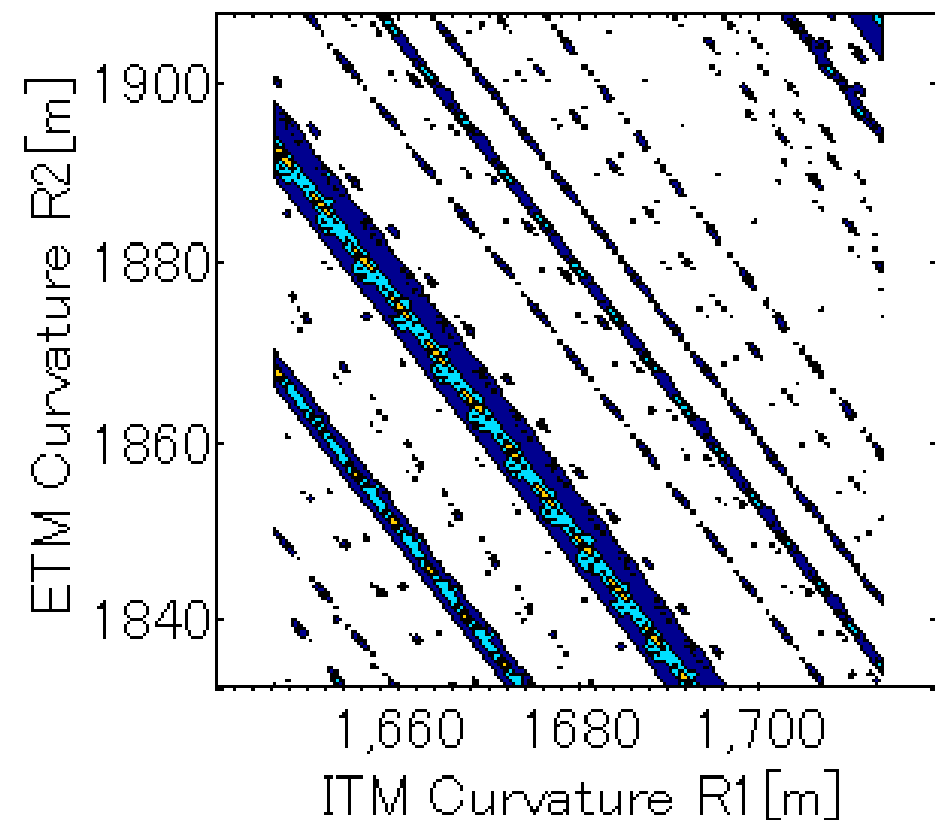


Result (with PR and detuned RSE)

The colored areas show where $\mathcal{R} > 1$.

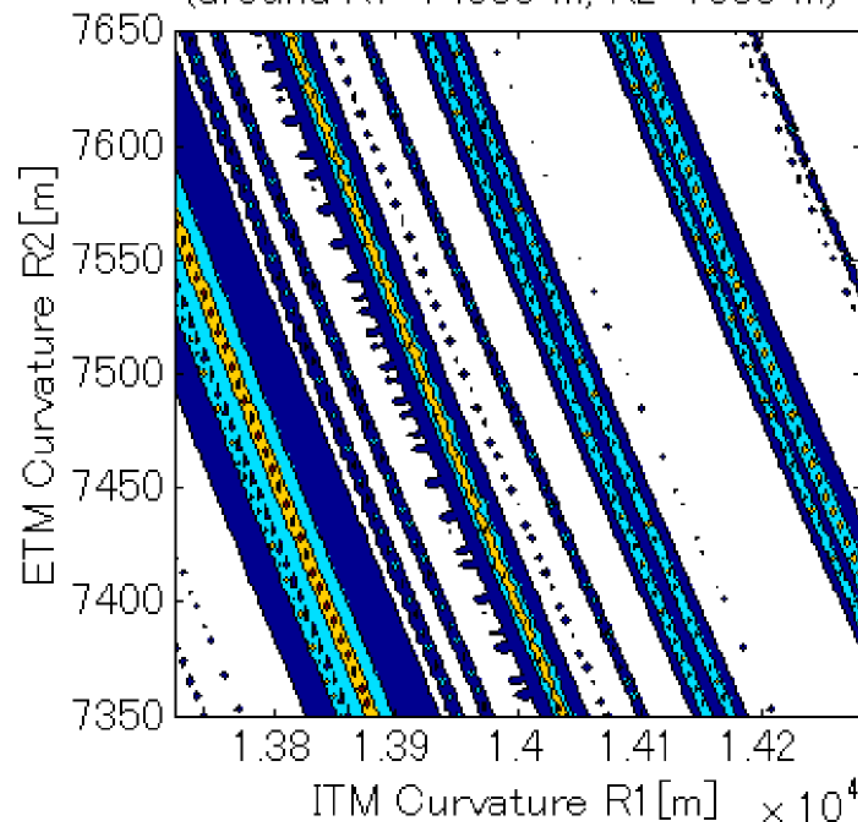
Negative g

Max. of the parametric gain $\log_{10}(R)$
(around $R_1=1680$ m, $R_2=1780$ m)



Positive g

Max. of the parametric gain $\log_{10}(R)$
(around $R_1=14000$ m, $R_2=7500$ m)

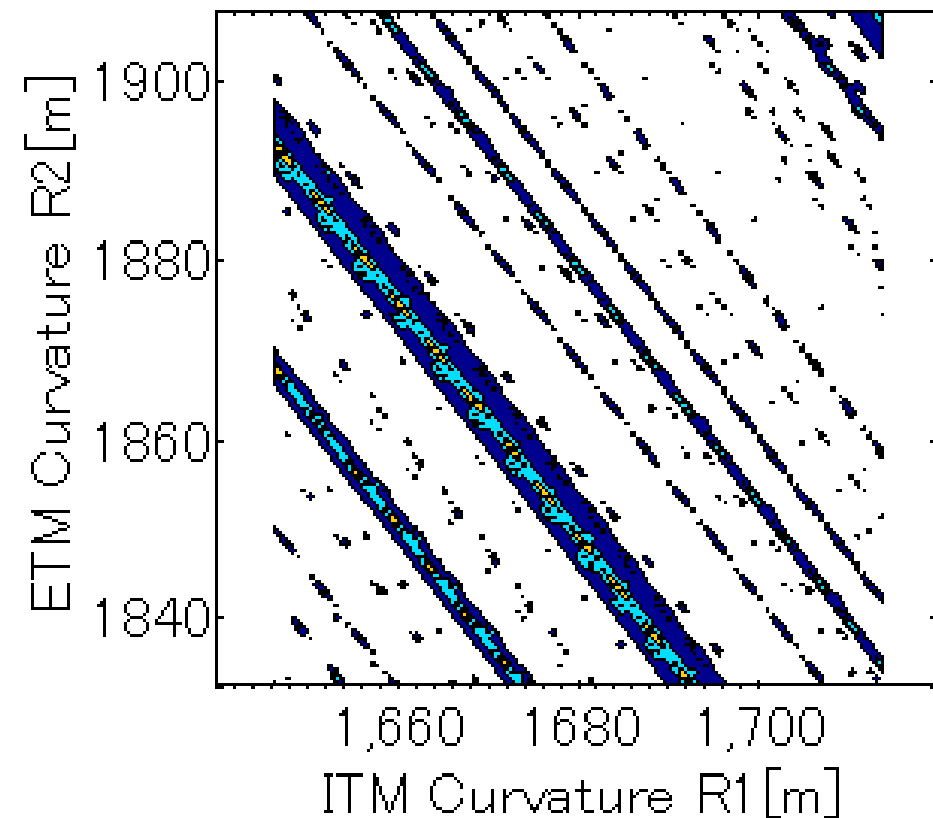


Result (with PR and detuned RSE)

The colored areas show where $\mathcal{R} > 1$.

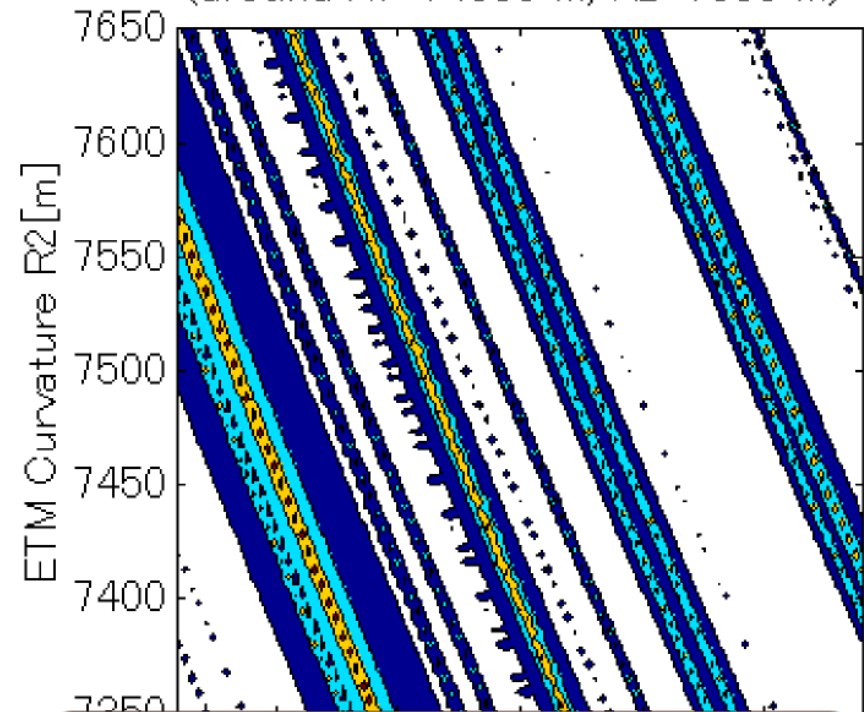
Negative g

Max. of the parametric gain $\log_{10}(R)$
(around $R_1=1680$ m, $R_2=1780$ m)



Positive g

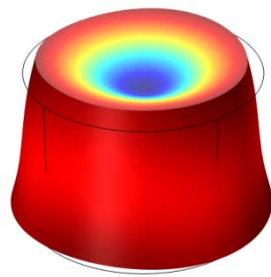
Max. of the parametric gain $\log_{10}(R)$
(around $R_1=14000$ m, $R_2=7500$ m)



Each case has the domain $\mathcal{R} > 1$,
but it does not seem so bad
(not always $\mathcal{R} > 1$).

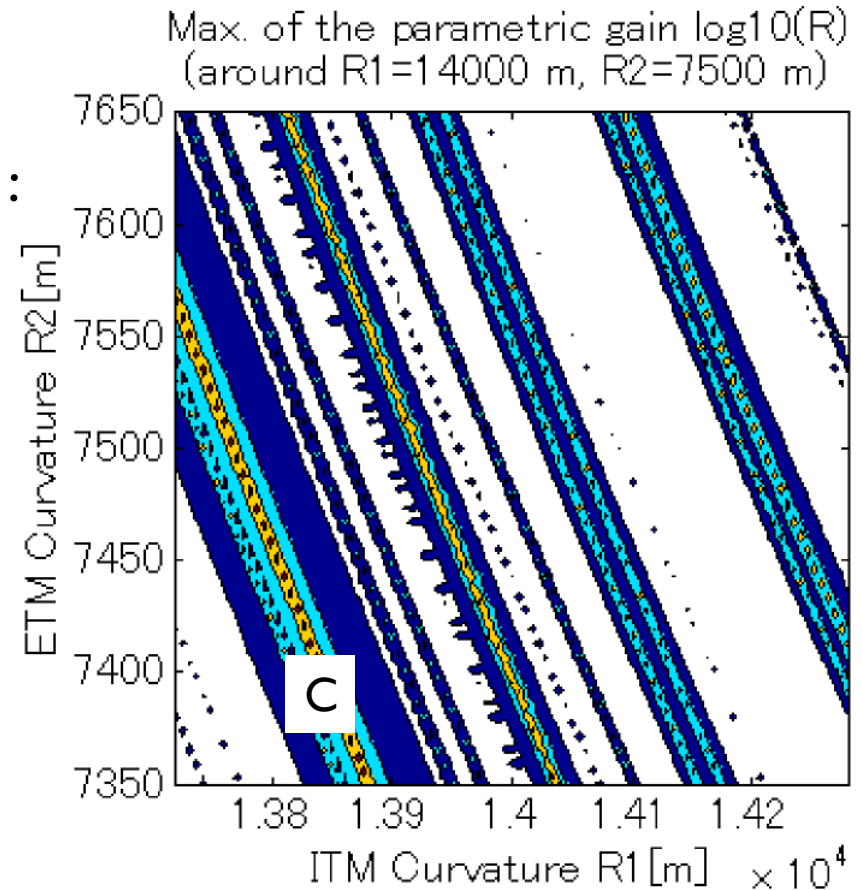
Discussion (positive g)

- ▶ $\max(\mathcal{R}) = 4000$
- ▶ Only 1 elastic mode C is so unstable in the sense that $\mathcal{R} \gg 1$:



C
23.9 kHz

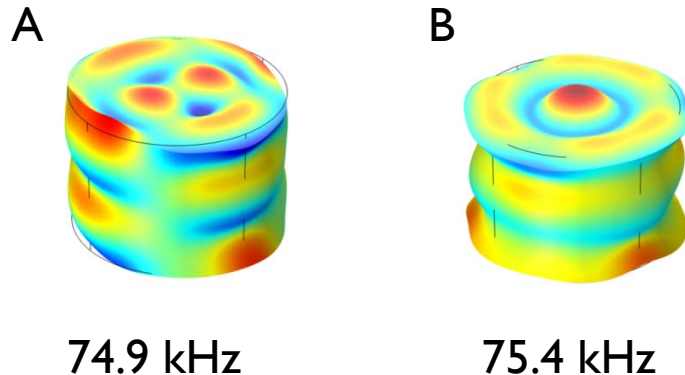
- ▶ The overlap is largest with the **TEM20** mode.
- ▶ We should care about only the elastic mode C.



Discussion (negative g)

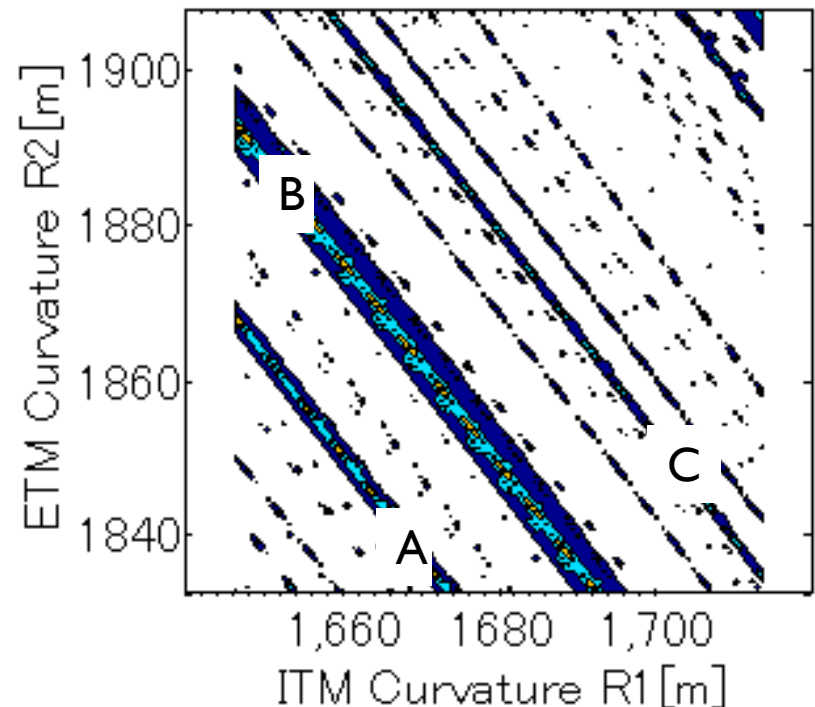
- ▶ $\max(\mathcal{R}) = 16000$
(4 times larger than the positive g)

- ▶ In addition to C, the below 2 modes are also unstable :



- ▶ For both A and B, their overlaps are largest with **TEM02**.

Max. of the parametric gain $\log_{10}(\mathcal{R})$
(around $R1=1680$ m, $R2=1780$ m)



Conclusion

- ▶ In the previous figs., we see more colored region in the positive case.
- ▶ The negative g case has more elastic mode to care than the positive case.
(Since when curvature R changes with 2%, $g = 1 - L/R$ varies more drastic for the negative g than for the positive g .)
- ▶ **But the PI does not seem to be a serious problem in both cases(Not Always $\mathcal{R} > 1$).**
- ▶ With the ASC report by Michimura,
the negative g is finally supported.
(@ the MIF meeting 1/19)

Fin.



Appendix A: Calculation error

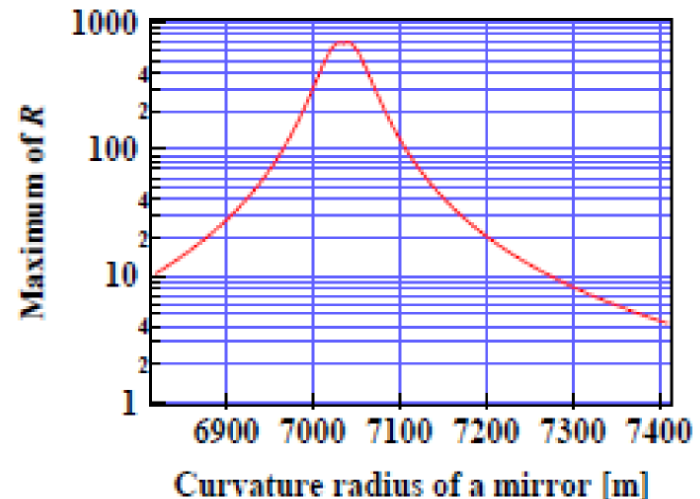
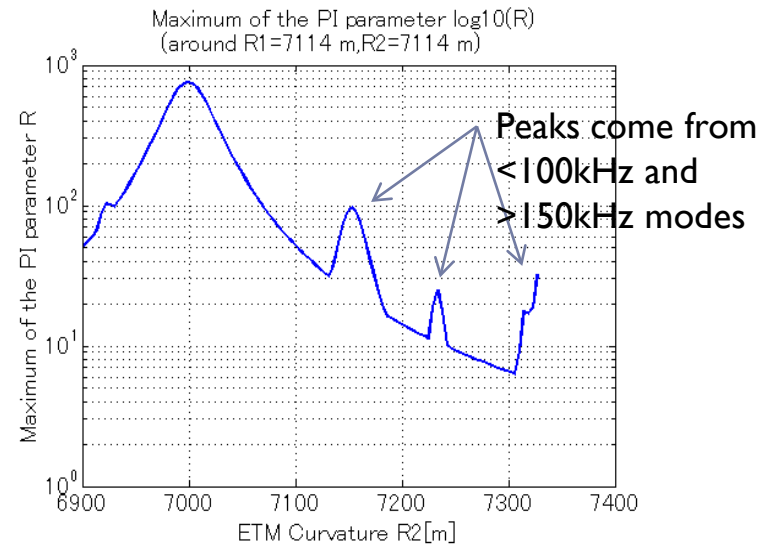
- ▶ **Q.** By setting the curvature requirement like $+1\%/-0\%$ (not $\pm 0.5\%$ as usual), can we get rid of the PI risk in advance?
- ▶ **A.** Since the error of the FEM is $1\sim 5\%$, the error of $\max(\mathcal{R})$ should be $> 2\%$. So we cannot get rid of the risk completely.

(To get rid of the PI in advance, the error of the FEM should be $\ll 0.5\%$. And there is no such a method.)

Appendix B: Check with the old result

- ▶ Often reported that the FEM calculations return with $> 1\%$ errors.
- ▶ Check with the Yamamoto's result(old parameters) [2]
→ My result agree with it by $< 2\%$ errors for the peak position and value.
→ OK

[2]H.Yamamoto et al., *Amaldi proceedings*(2009)

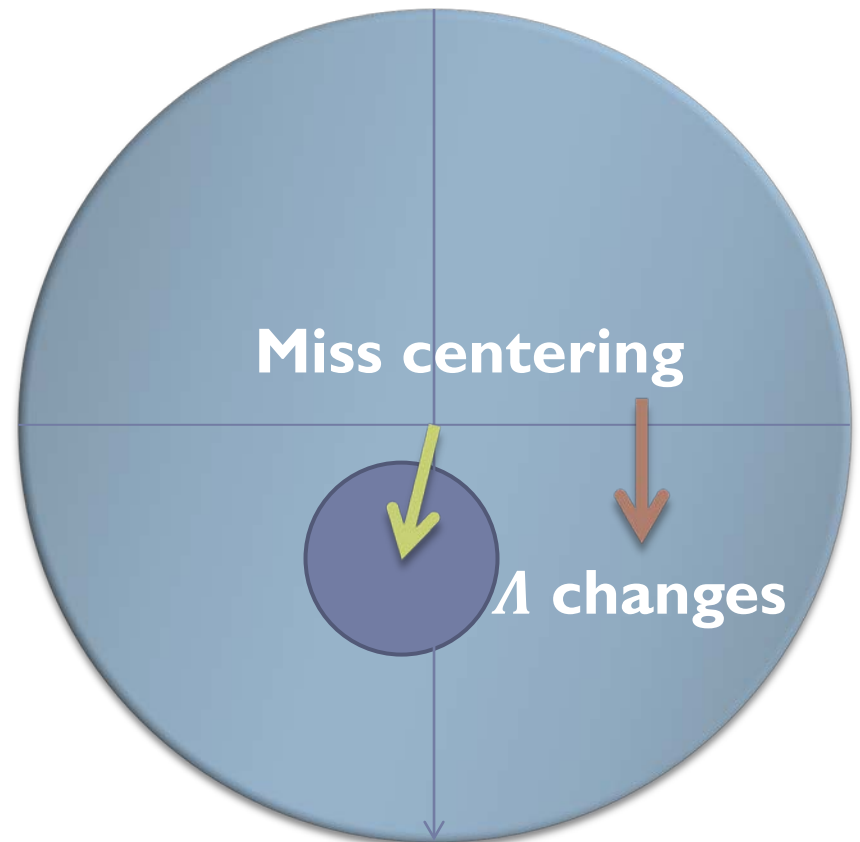


Appendix C: Loss of higher TEM modes

- ▶ For higher TEM modes, the power loss at mirrors is large.
- ▶ So, the Finesse becomes smaller than $\mathcal{F} = 1550$.
- ▶ **Q.** How is that effect?
- ▶ **A.** The highest peak comes from TEM₂₀ and TEM₀₂, and their power loss are about 3 ppm. So that effect doesn't make role.

Appendix D: Beam miss-centering

- ▶ A beam miss-centering might be occurs with 1mm error.
- ▶ **Q.** By this error, the PI occurs?
- ▶ **A.** No. (By miss-centering, \mathcal{R} changes only 1%.)



Appendix E: When we face with the PI

- ▶ Because of the error of the FEM by COMSOL, the risk of the PI is not excluded completely.
- ▶ We should prepare for the PI by some method.
(e.g. Suppress the quality value of the elastic mode by some actuators [3].)

[3] Zhang et al., 2011

