Gravitational waves from simulations of binary neutron stars

Interaction between analytical and numerical techniques:

Numerical relativity and the **Effective-One-Body** approach

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Motivation

I am excited at the prospect of measuring GWs and in particular at the realization of LCGT!

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Plan of the talk

- Introduction
    - The Whisky code and binary neutron stars
- Pre-merger phase
    - Interaction between analytical and numerical techniques: Numerical relativity and the Effective-One-Body approach

The fundamental equations

We solve the following equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} \qquad (\text{field eqs}: \ 6+6+3+1)$$

$$\nabla_\mu T^{\mu\nu} = 0 \ , \qquad (\text{cons. en./mom.}: \ 3+1)$$

$$\nabla_\mu (\rho u^\mu) = 0 \ , \qquad (\text{cons. of baryon no}: \ 1)$$

$$p = p(\rho, \epsilon, \ldots) \ . \qquad (\text{EoS}: \ 1+\ldots)$$

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The complete set of equations is solved with the codes:

`Cactus/Carpet/Whisky`.

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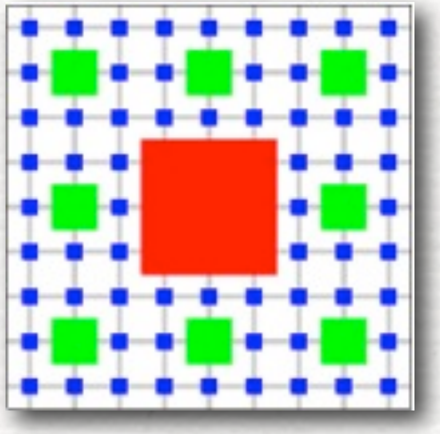

`Carpet` (www.carpetcode.org) provides box-in-box adaptive mesh refinement with vertex-centred grids.

Some gravitational-wave sources simulated with Whisky

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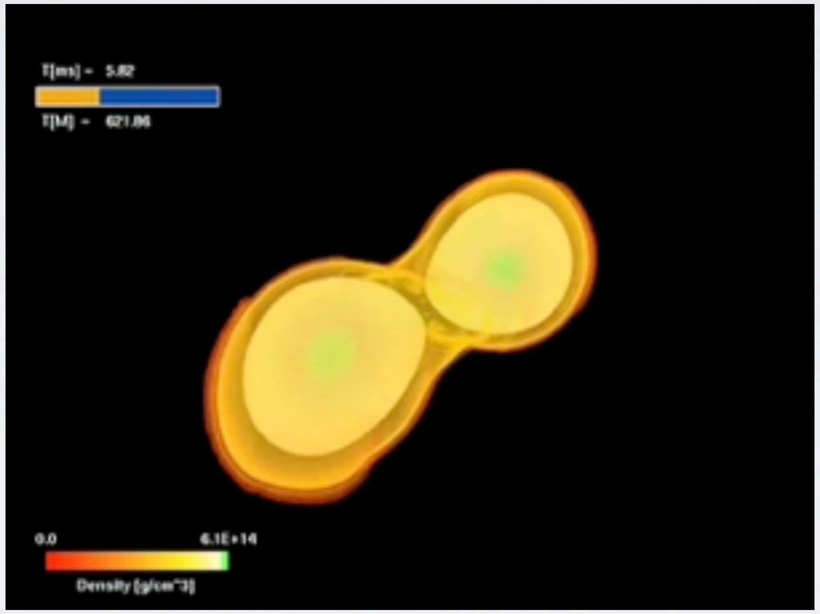
- Binary neutron stars



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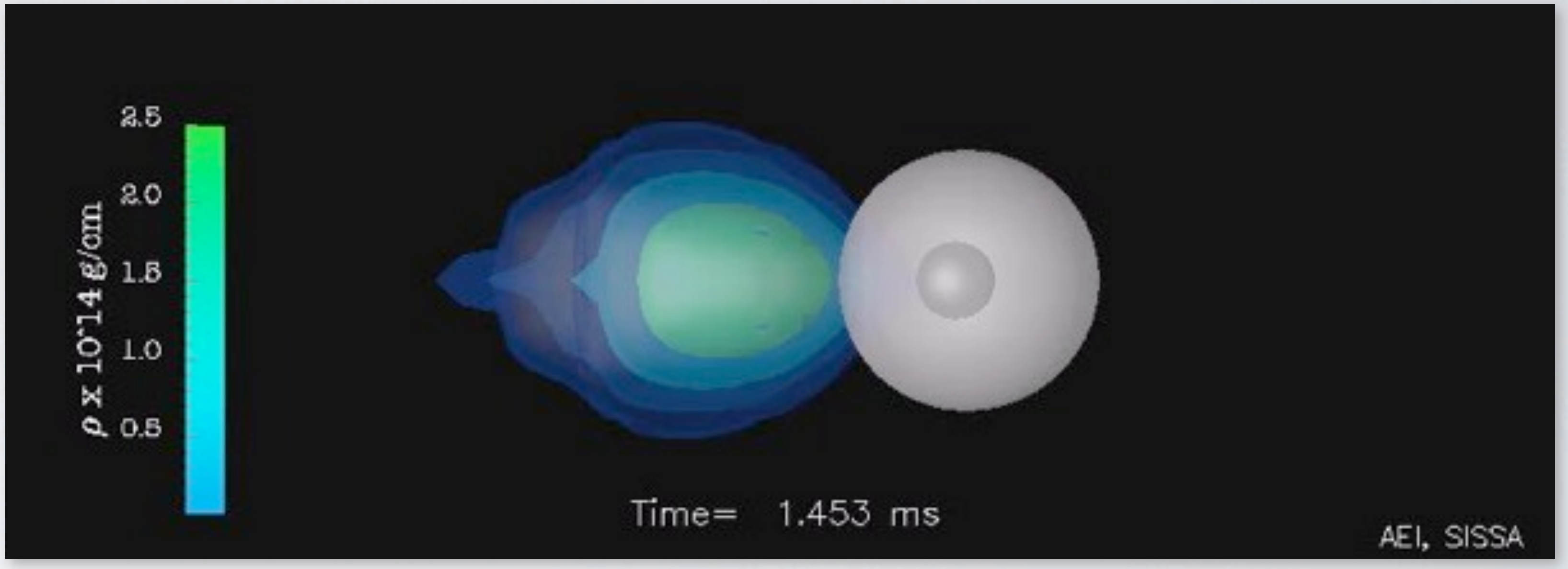

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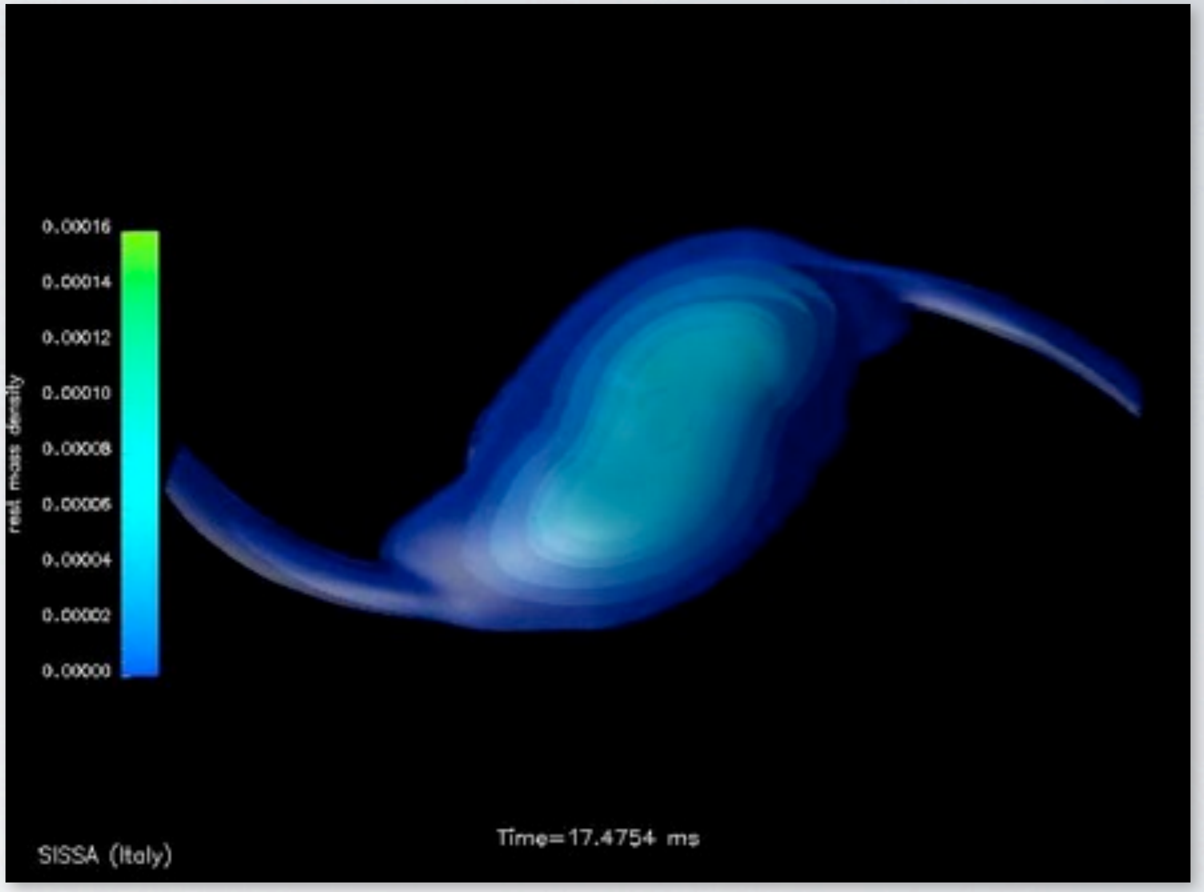

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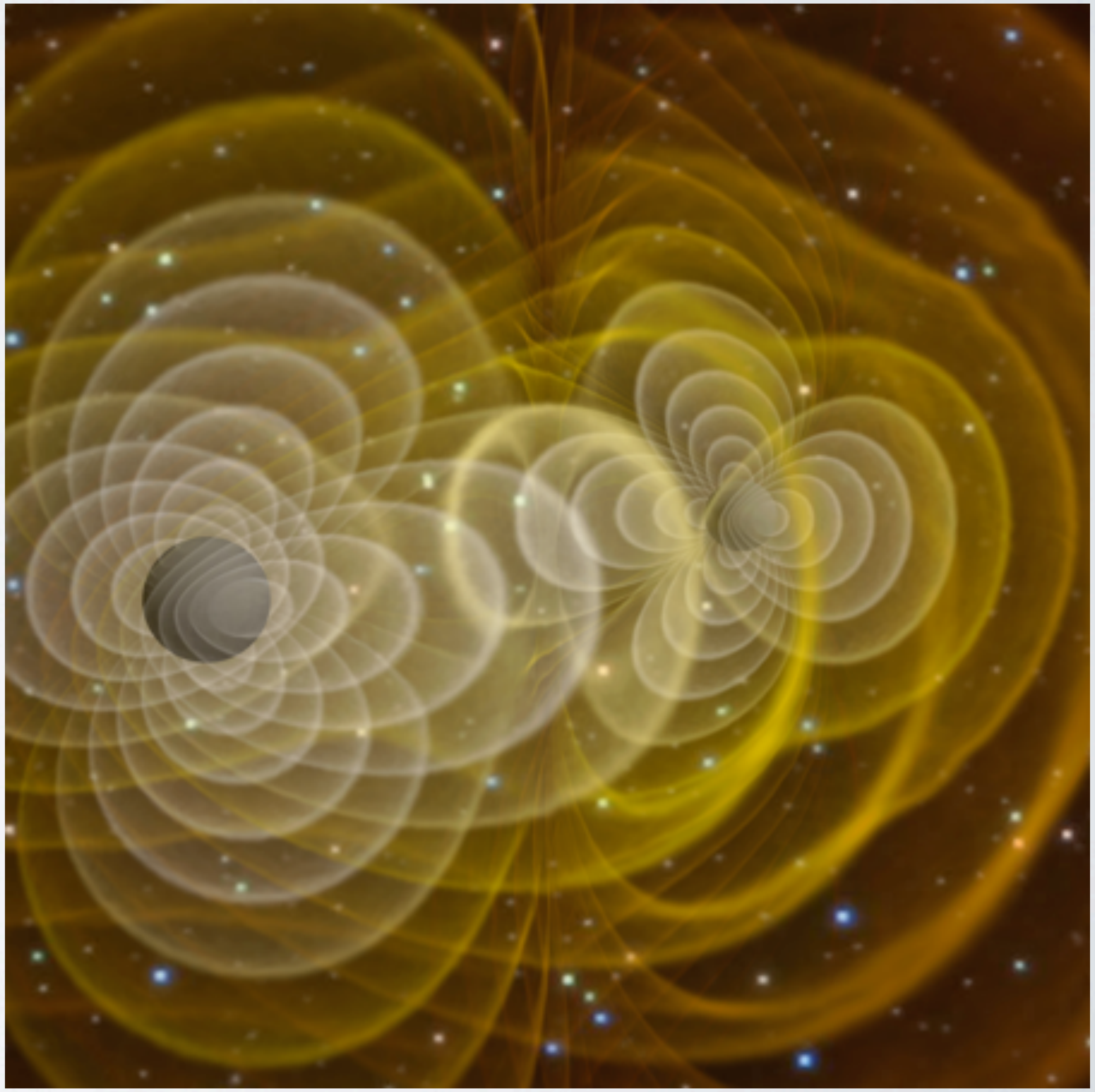

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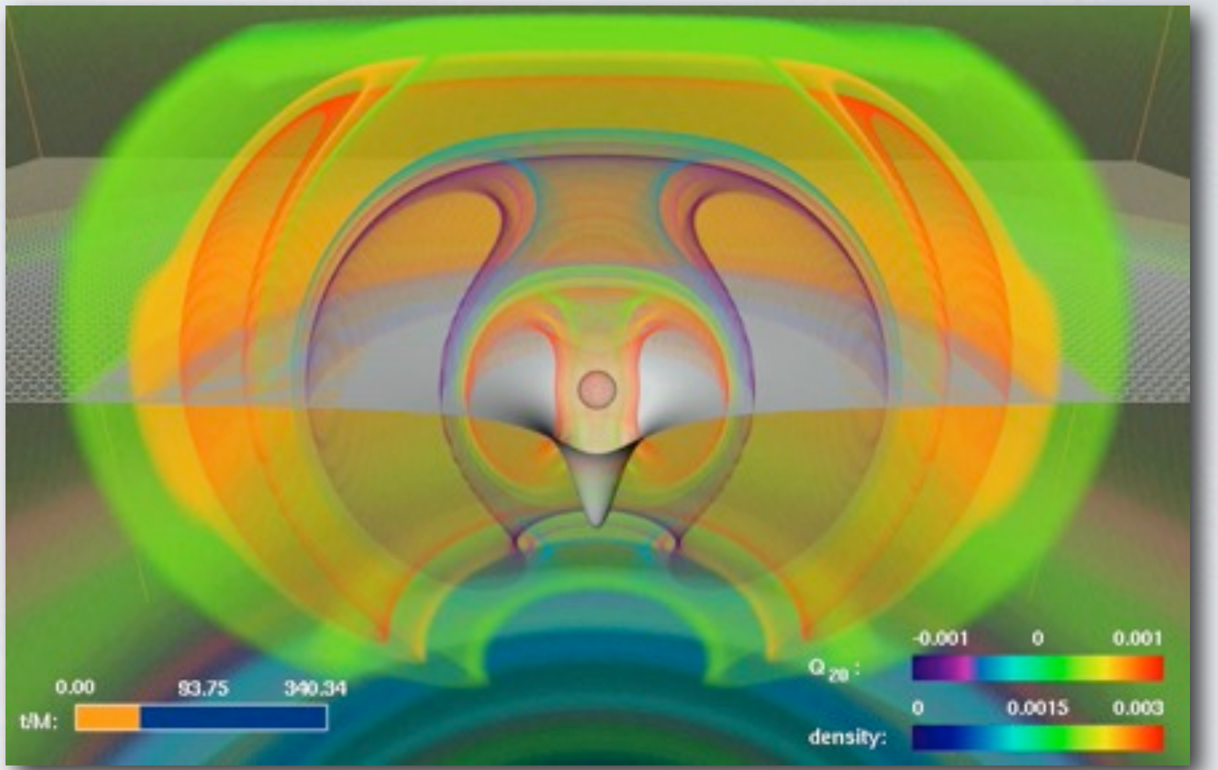

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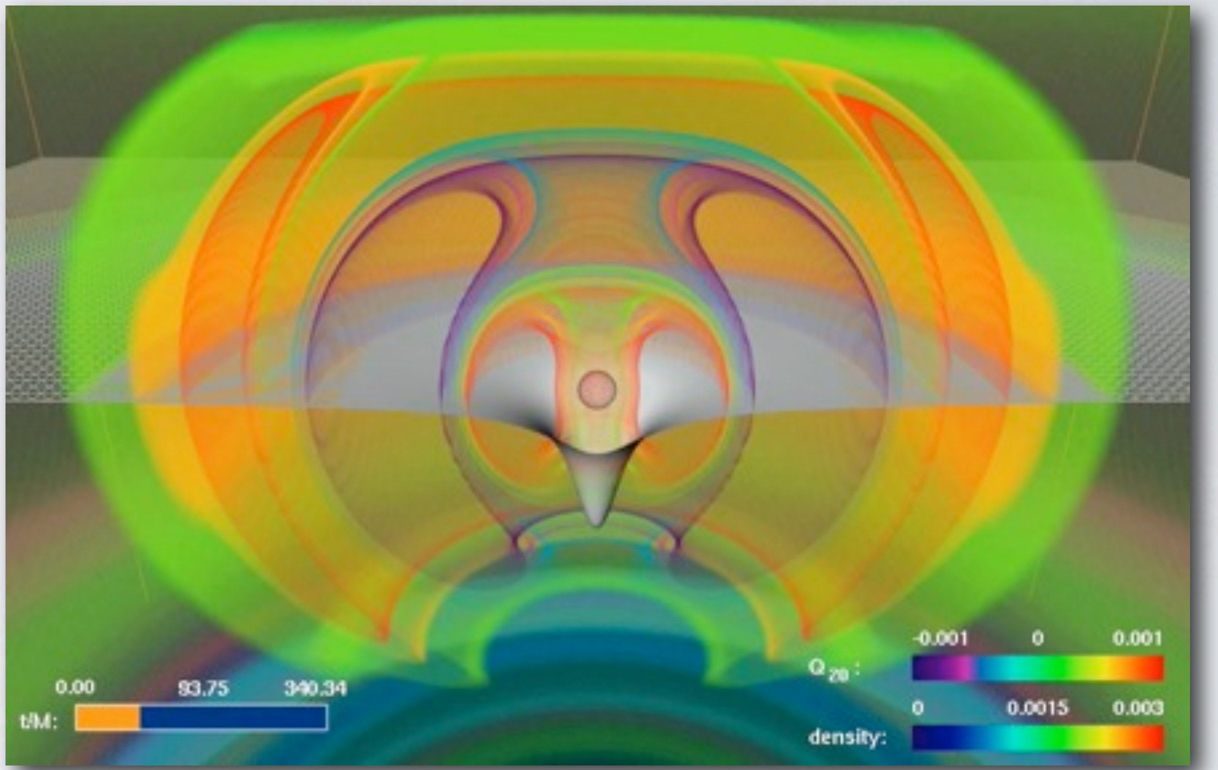
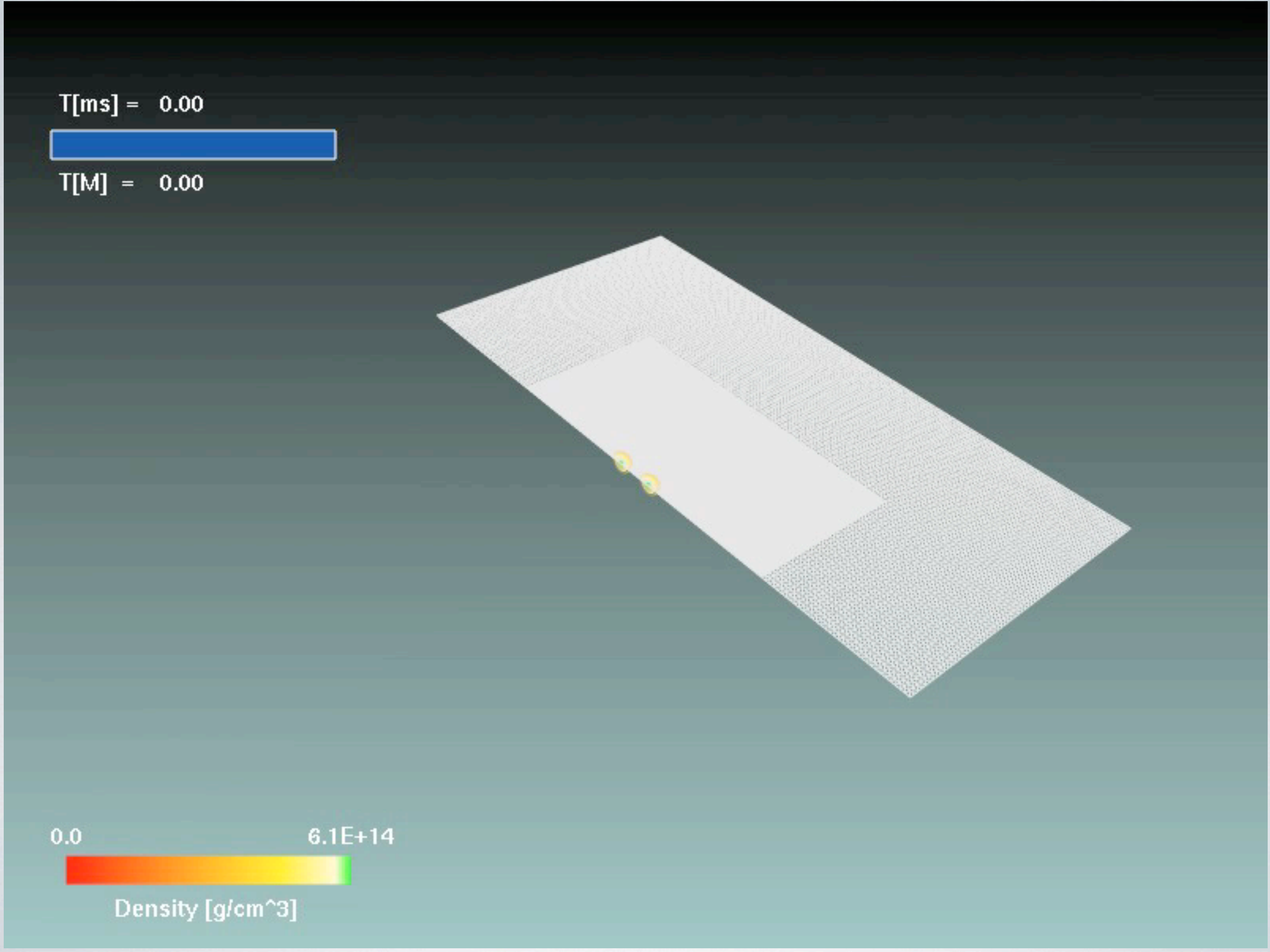
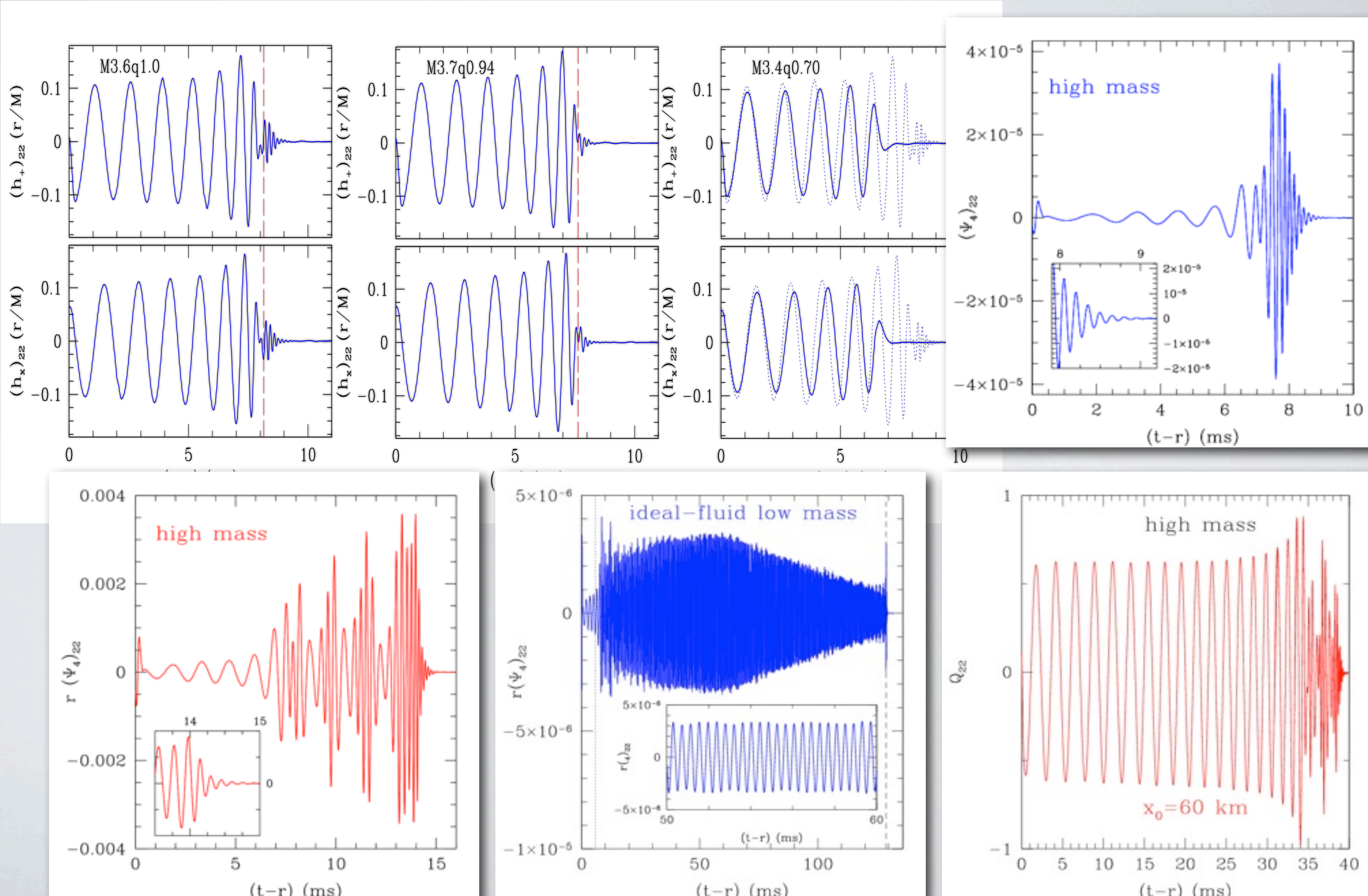


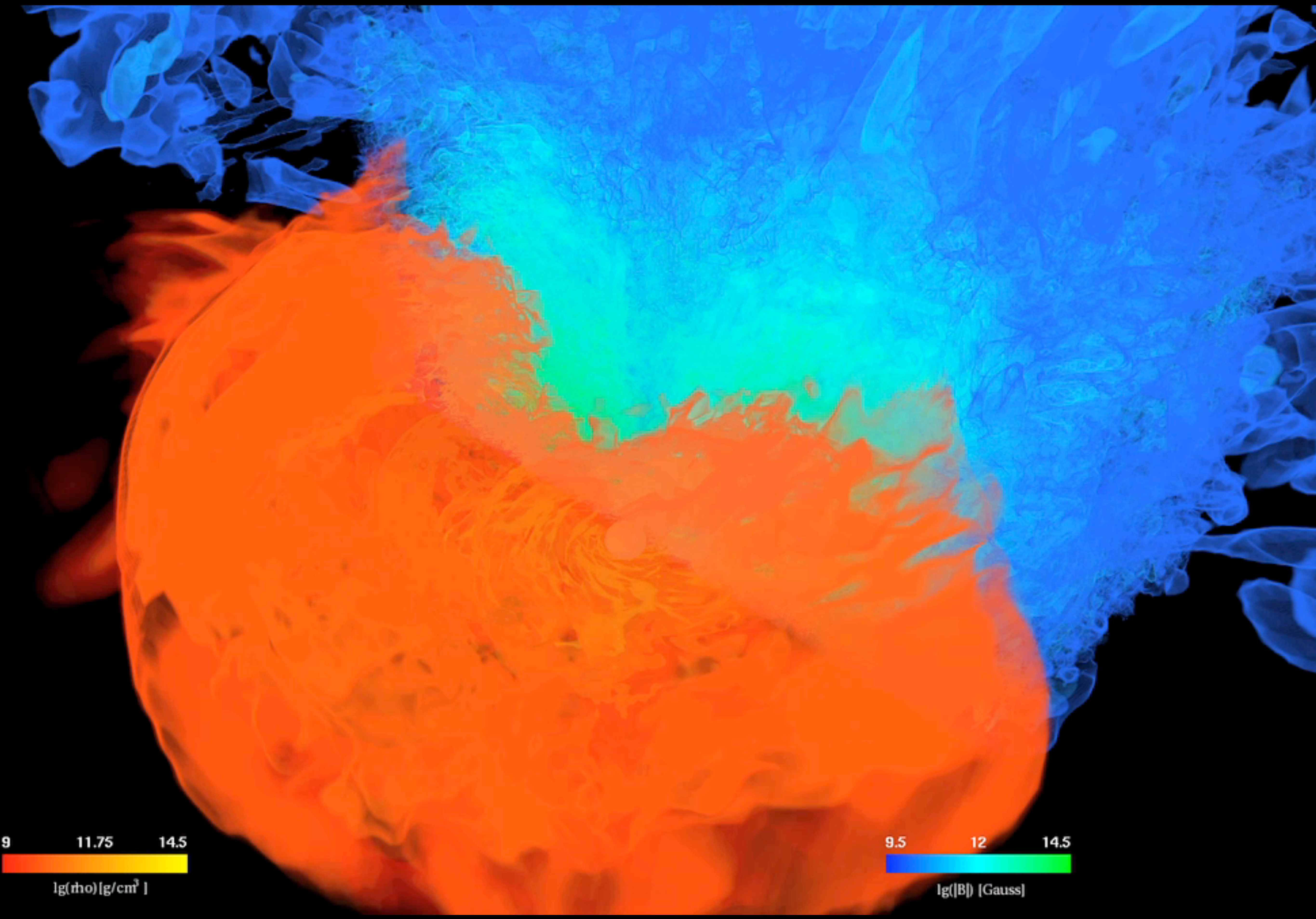

T[ms] = 0.00
T[M] = 0.00
0.0
6.1E+14
Density [g/cm^3]

A variety of waveforms

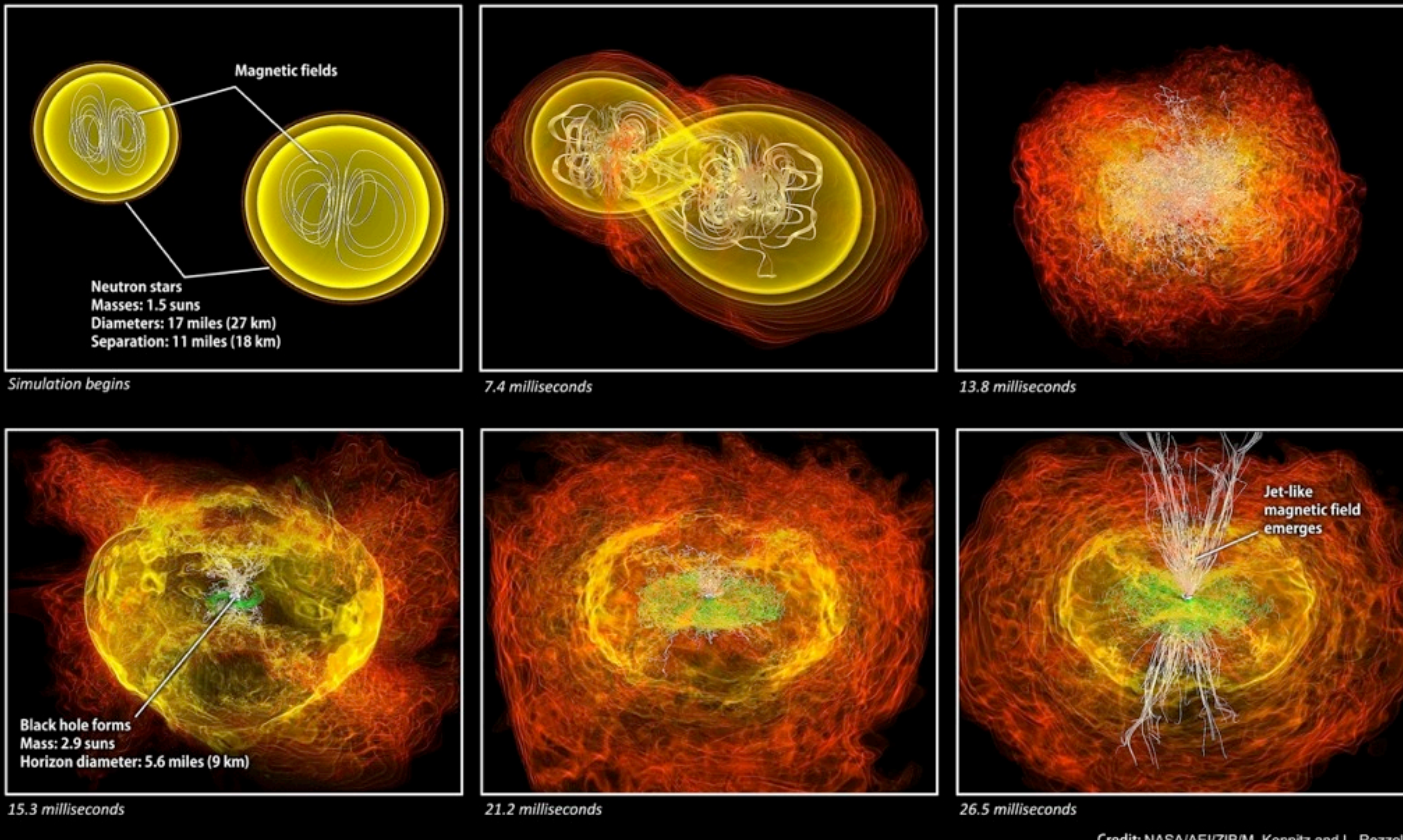


Animations:, Rezzolla, Koppitz

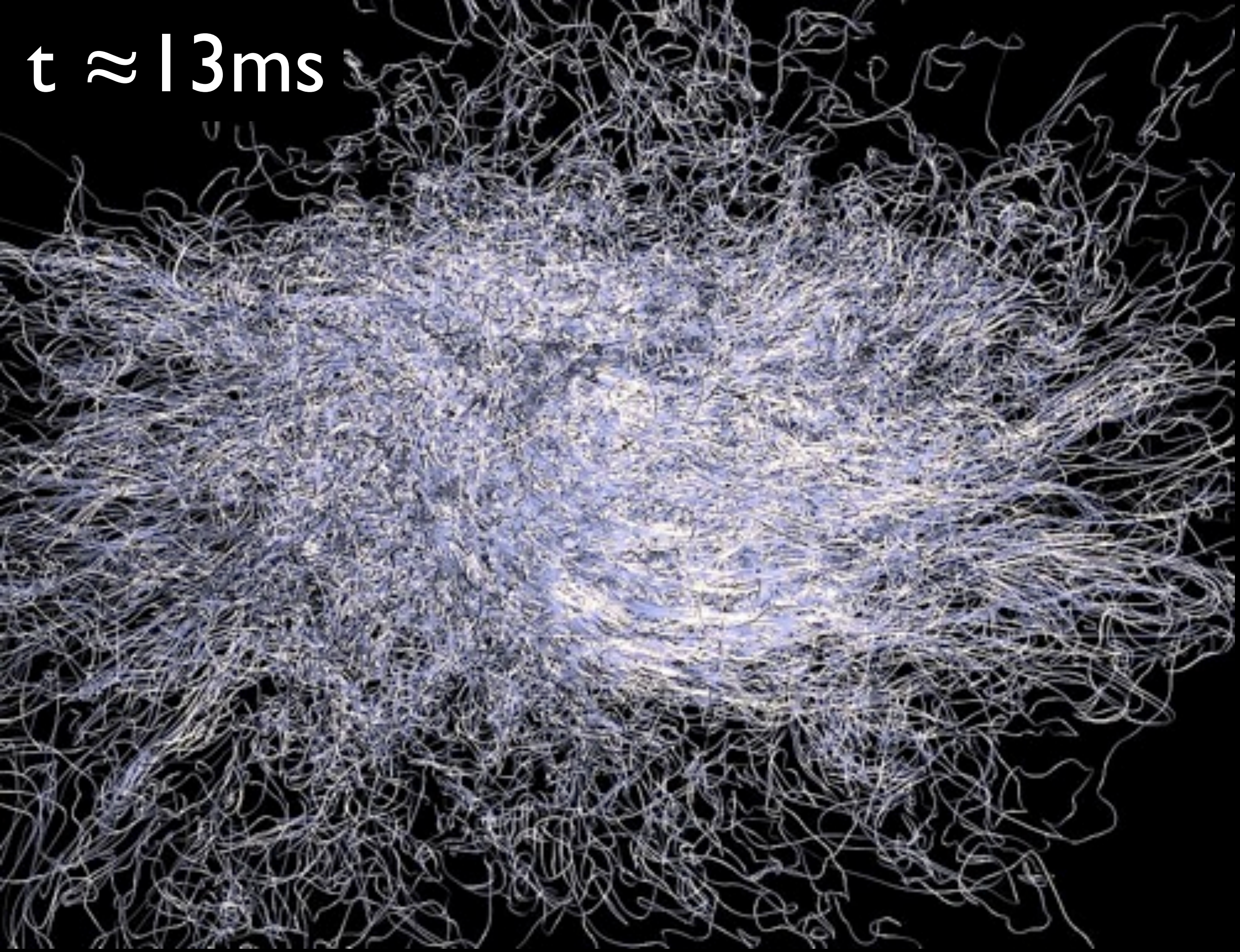
t ~15ms

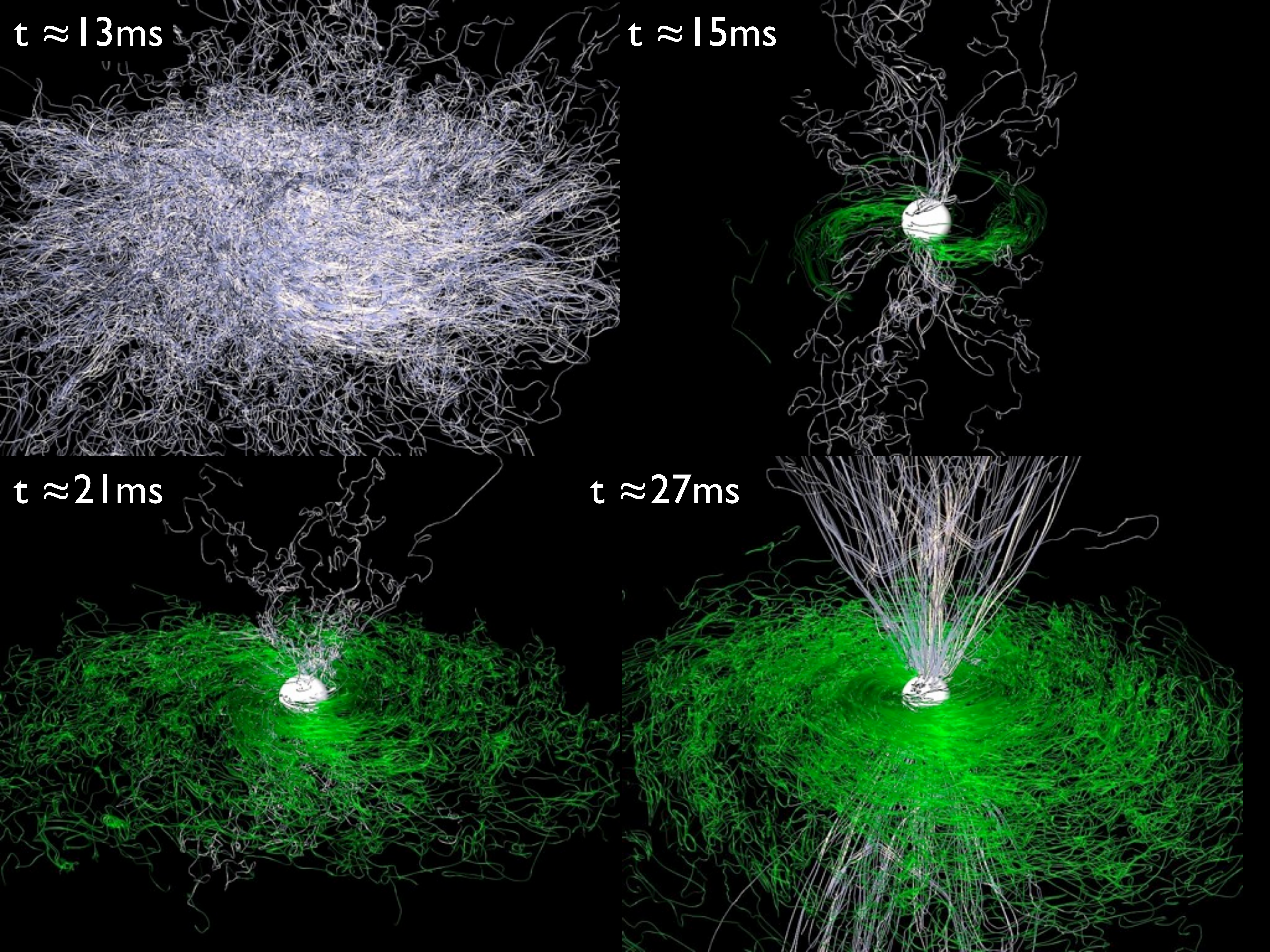

9
11.75
14.5
lg(rho) [g/cm³]
9.5
12
14.5
lg(|B|) [Gauss]

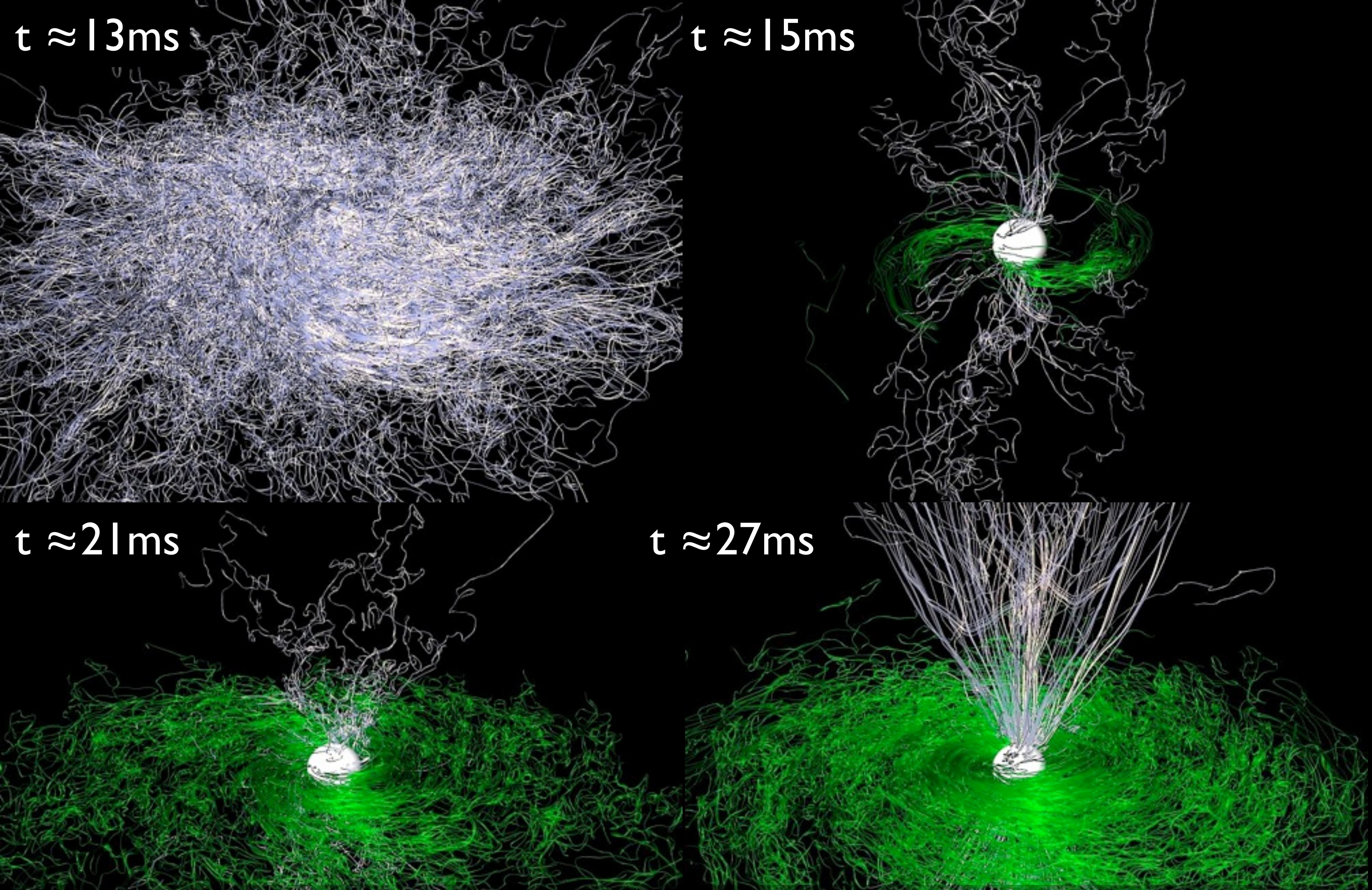
Crashing neutron stars can make gamma-ray burst jets



$J/M^2 = 0.83$ $\mathrm{M_{tor}} = 0.063 M_\odot$ $\mathrm{t_{accr}} \simeq M_{\mathrm{tor}}/\dot{M} \simeq 0.3\ \mathrm{s}$

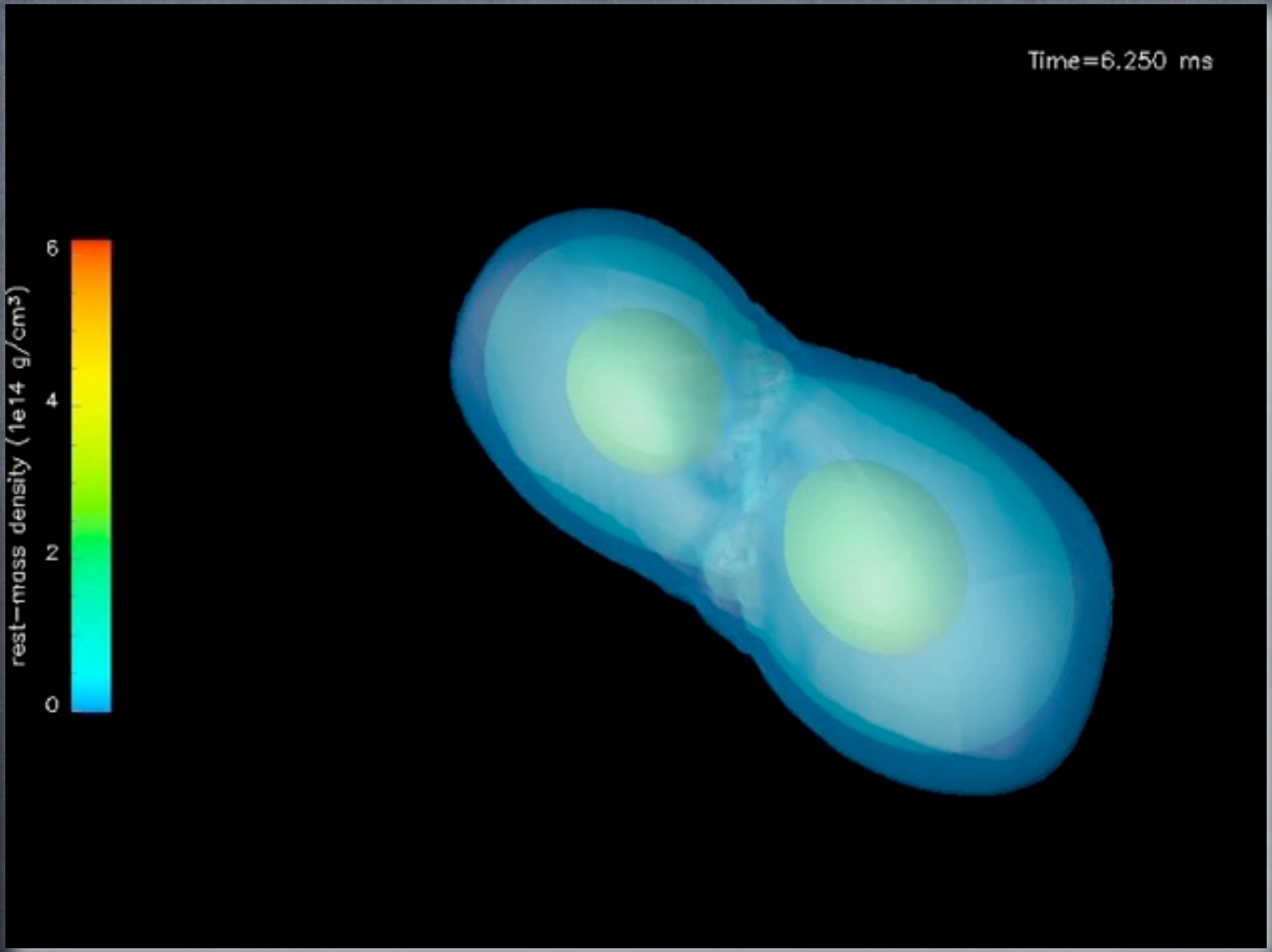

t ≈13ms


t ≈13ms
t ≈15ms
t ≈21ms
t ≈27ms



A *magnetic jet* is produced from *ab-initio* calculations.
Opening half-angle is ≈ 30° (observational importance).

Pre-merger phase: analytical and numerical techniques



Motivation for the synergy between numerical and analytical methods

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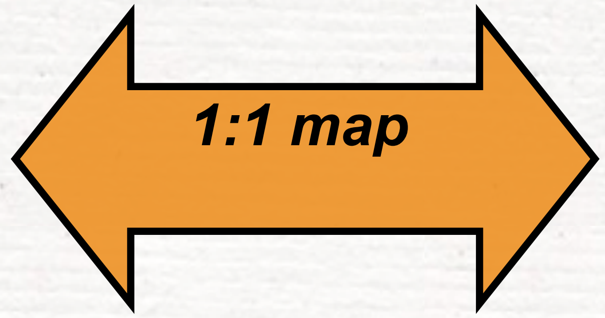
- **GW data analysis** requires **templates**
- **Templates** for the merger and immediate post-merger phases require **numerical simulations**
- But the parameter space (masses, spins, EOS, ...) is very large
- Cannot rely only on numerical simulations to densely cover the parameter space
- The **PostNewtonian formalism** is successfully used for this also in case of the GWs produced by binary systems; but it has limitations
- The **Effective-One-Body approach** is another approximation method that improves the description of the late inspiral and merger phases
  - It combines information from the analytic PN method and from numerical simulations. It extends the domain of validity of perturbation theory so as to approximately cover some non-perturbative features
  - References: Buonanno Damour, PRD59, 084006 (1999); PRD 62, 064015 (2000); for introductory lectures see e.g. Damour Nagar, arXiv:0906.1769

Effective-One-Body approach: short overview

The fundamental idea of the EOB: The <u>two body dynamics</u> is represented by that of a <u>single effective particle in an effective potential</u>.

Real 2-body system *(m_1, m_2)* (in the c.o.m. frame)



an effective particle of mass *$\mu = m_1 m_2/(m_1+m_2)$* in some effective metric *$g_{\mu\nu}^{eff}(M)$*

The effective metric:

$$g_{\mu\nu}^{eff} dx^\mu dx^\nu = -A(r)c^2 dT^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

is only a mathematical object, whose coefficients are a priori unknown:

$$A(r) = 1 + a_1 \frac{GM}{c^2 r} + a_2 \left(\frac{GM}{c^2 r}\right)^2 + a_3 \left(\frac{GM}{c^2 r}\right)^3 + \ldots$$

$$B(r) = 1 + b_1 \frac{GM}{c^2 r} + b_2 \left(\frac{GM}{c^2 r}\right)^2 + \ldots$$

and are determined by imposing:

1) The <u>corrispondence with the 3PN two-body Hamiltonian</u>;
2) The compatibility with Schwarzschild in the single-body limit;
3) The coincidence of the effective mass with the reduced mass of the two-body system.

Effective-One-Body approach: short overview

Result:

$$H_{\rm EOB} = M\sqrt{1 + 2\nu(\hat{H}_{\rm eff} - 1)}$$

$M_A,\ M_B$

masses of the stars

$$M \equiv M_A + M_B$$

$$\nu \equiv \frac{M_A M_B}{M^2}$$

$$z_3 \equiv 2\nu(4 - 3\nu)$$

$$p_{r_*} \equiv \sqrt{\frac{A}{B}}\, p_r$$

Simple effective Hamiltonian

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A\left(1 + \frac{p_\varphi^2}{r^2} + z_3 \frac{p_{r_*}^4}{r^2}\right)}.$$

crucial EOB "radial potential" A(r)

$$A = 1 - \frac{2}{r} + \frac{2\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{\nu}{r^4}$$

at 3PN, and then one can extend:

$$A = 1 - \frac{2}{r} + \frac{2\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{\nu}{r^4} + \frac{a_5}{r^5} + \frac{a_6}{r^6}$$

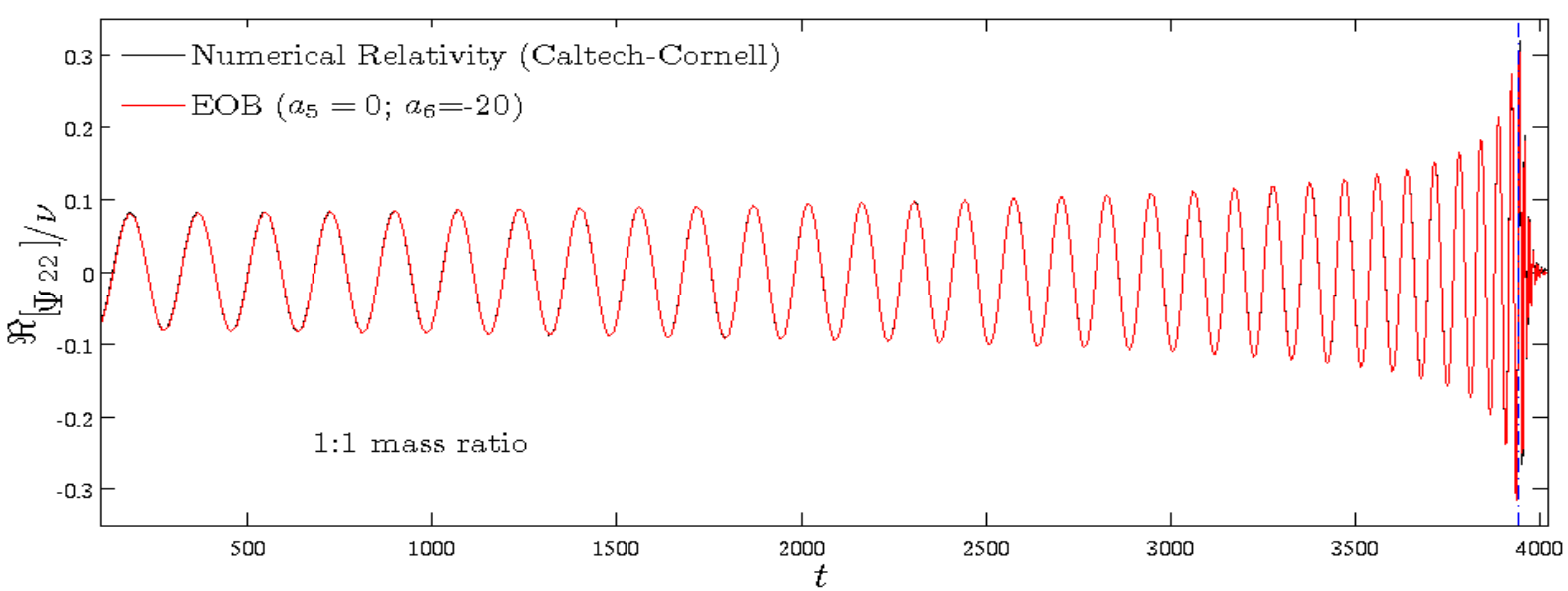
and find the unknown coefficient in other ways.

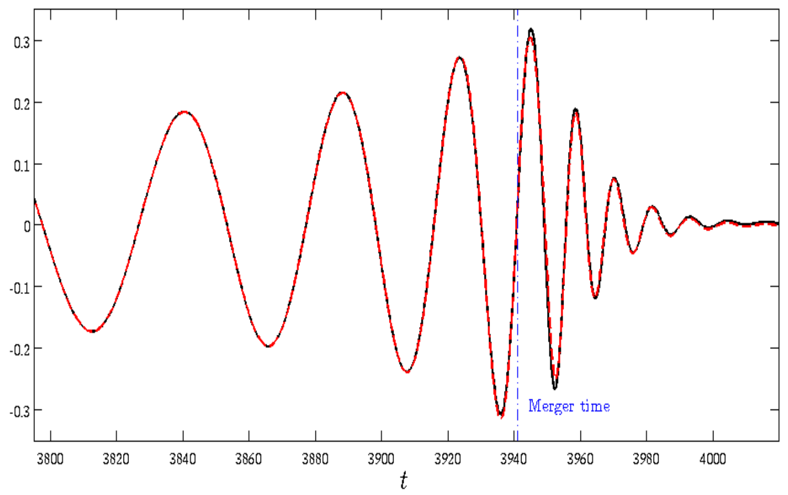
Effective-One-Body approach: success with BBH

The EOB formalism works very well for binary black-hole mergers.

And arguably better than other PN approximations (T4), especially when considering unequal mass ratios and spin effect.

(Damour et al. PRD 78; Hannam et al. PRD 78; Damour Nagar PRD79, Buonanno et al. PRD80)





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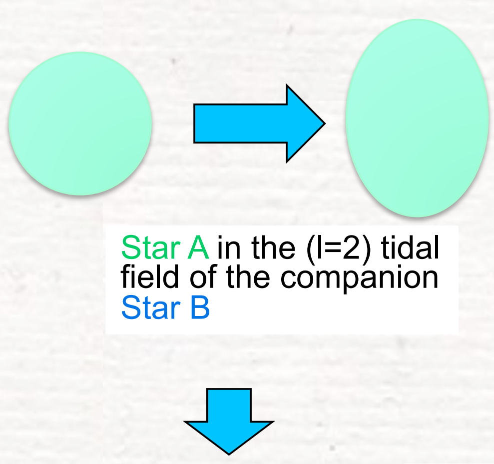

$$M_{ij}^A = \mu_2^A G_{ij}^A$$

External tidal field

$$2k_\ell \equiv (2\ell-1)!!\frac{G\mu_\ell}{R^{2\ell+1}}$$

(dimensionless)

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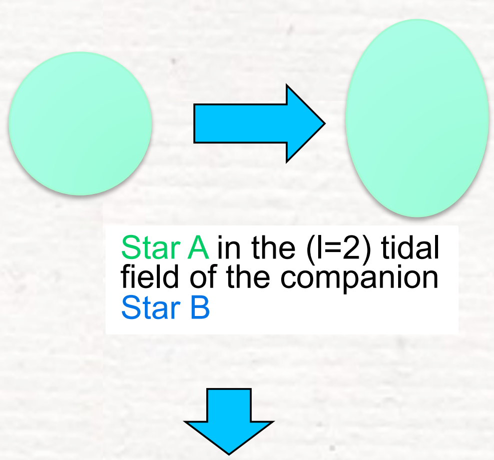

$$M^A_{ij} = \mu^A_2 G^A_{ij}$$ External tidal field

$$2k_\ell \equiv (2\ell - 1)!! \frac{G\mu_\ell}{R^{2\ell+1}}$$
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- Also thanks to the work of Uryu et al. [PRD80:124004 (2009)], Damour and Nagar were prompted to include tidal terms beyond the leading-order ones in the EOB approach: Damour & Nagar PRD81:084016 (2010).

Effective-One-Body approach: tidal corrections

Incorporating tidal effects in the EOB Hamiltonian is in principle straightforward:

$$A(r) = A_0(r) + A^{tidal(N)}(r)$$

where $A^{tidal(N)}(r)$ is computed in a Newtonian-like form (but with relativistic Love numbers)

$$A^{tidal(N)}(r) = \sum_{\ell \geq 2} A_\ell^{tidal(N)} = -\sum_{\ell \geq 2} \kappa_\ell^T u^{2\ell+2} \qquad u \equiv \frac{1}{r}$$

$$\kappa_\ell^T = 2\frac{M_B M_A^{2\ell}}{M^{2\ell+1}} \frac{k_\ell^A}{c_A^{2\ell+1}} + 2\frac{M_A M_B^{2\ell}}{M^{2\ell+1}} \frac{k_\ell^B}{c_B^{2\ell+1}}$$

and then one can formally add higher PN corrections (next-to-leading-order tidal corrections):

$$A_\ell^{tidal} = A_\ell^{tidal(N)} \hat{A}_\ell^{tidal}$$

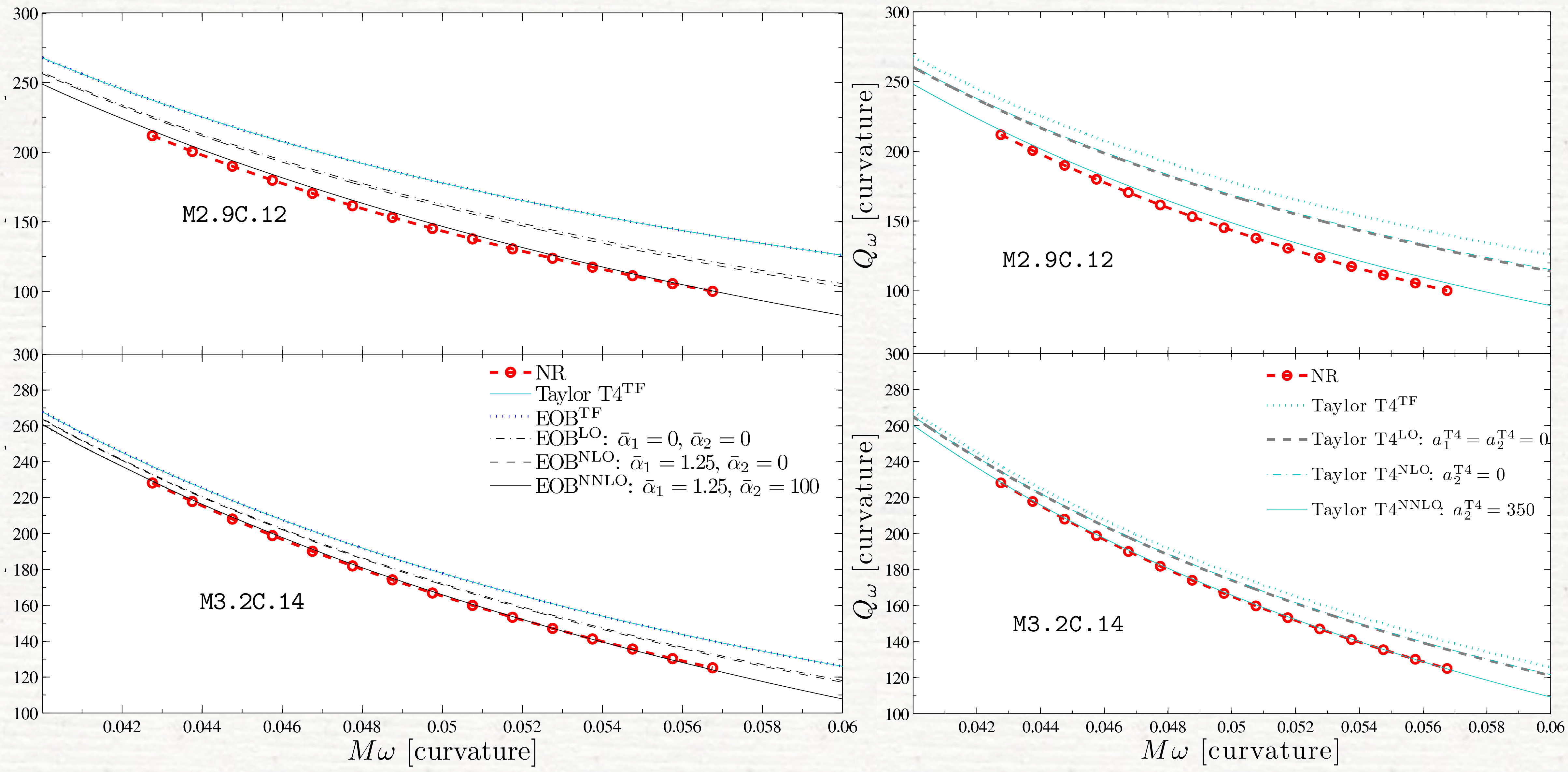
$$\hat{A}_\ell^{tidal} = (1 + \overline{\alpha}_{1PN} u + \overline{\alpha}_{2PN} u^2 \ldots.)$$

The $\overline{\alpha}_{nPN}$ can be computed analytically or estimated by comparing the results with numerical simulations.

$\bar{\alpha}_{1PN}^{\ell=2} = 1.25$ was analytically computed in Damour & Nagar PRD81 (2010).

EOB - T4 comparison in the frequency domain

The Q_ω diagnostics: $Q_\omega \equiv \omega^2/\dot{\omega}$



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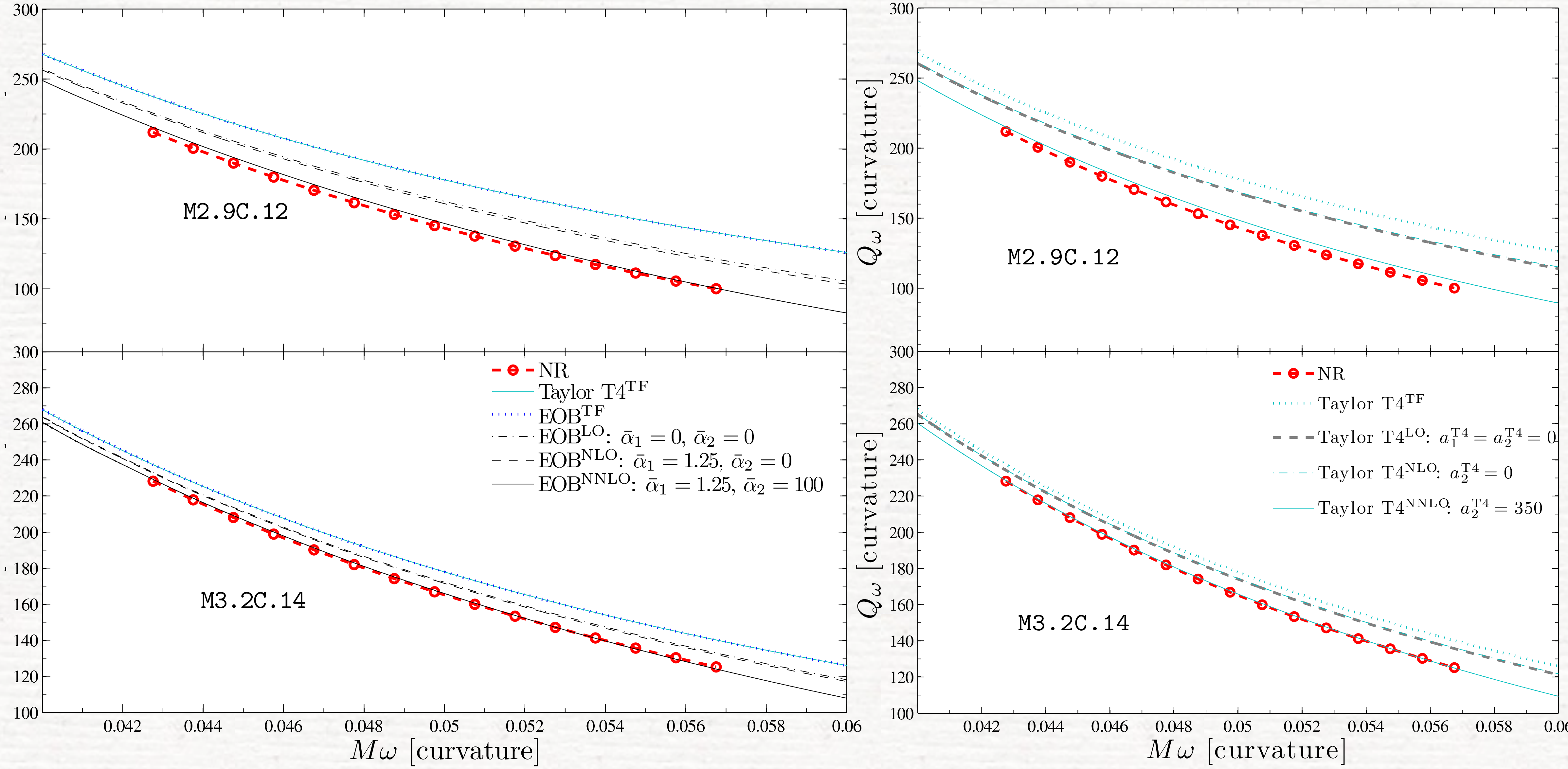

By comparing the same procedure for EOB and T4, it can be seen that the EOB framework gives smaller NLO and NNLO corrections than T4, so it seems more robust. Additionally, in T4 the 2PN term starts dominating already at large distances (r≈19M). Furthermore the NLO corrections for T4 are less consistent (with respect to the EOB ones) for different simulations.

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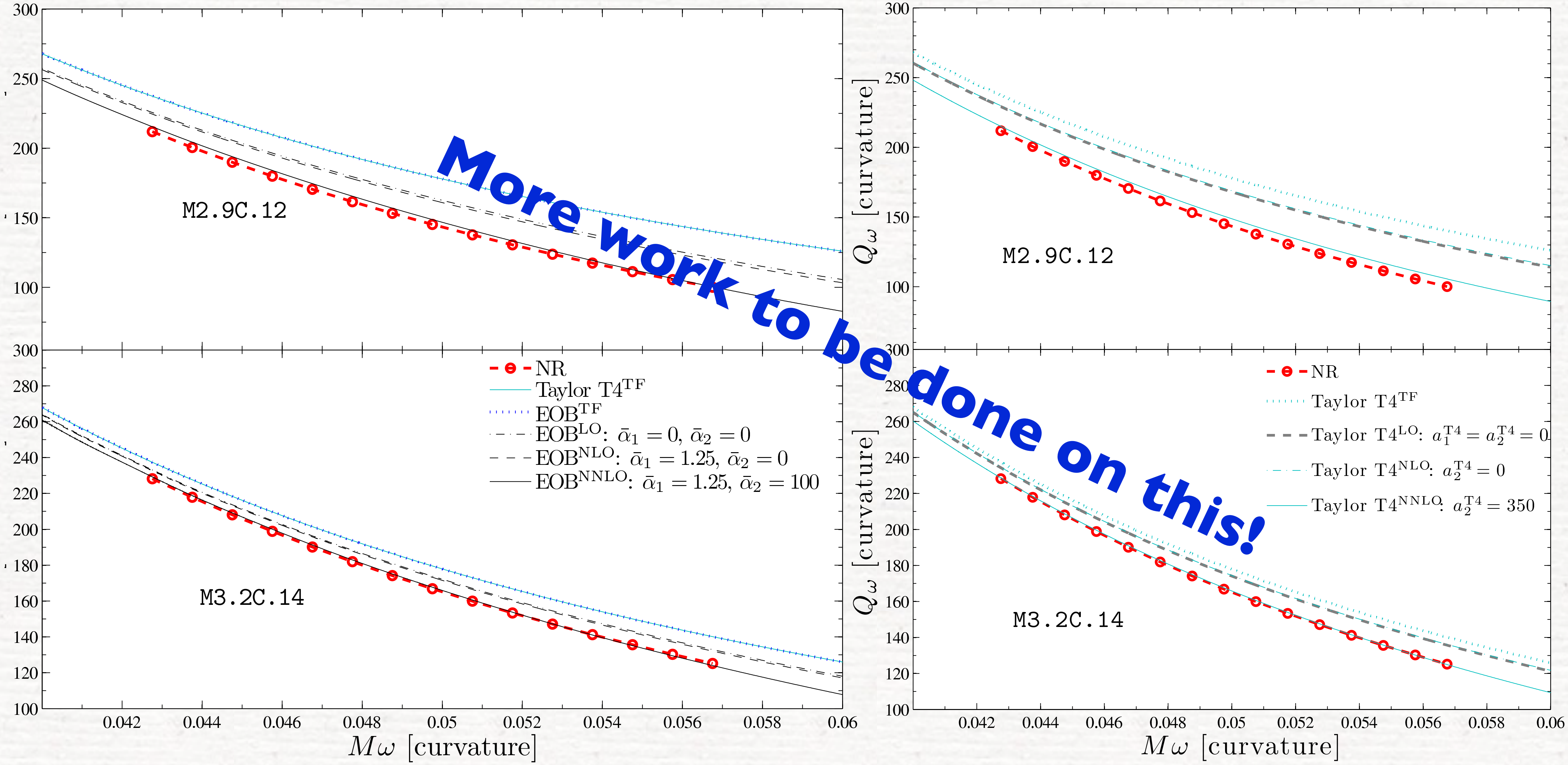

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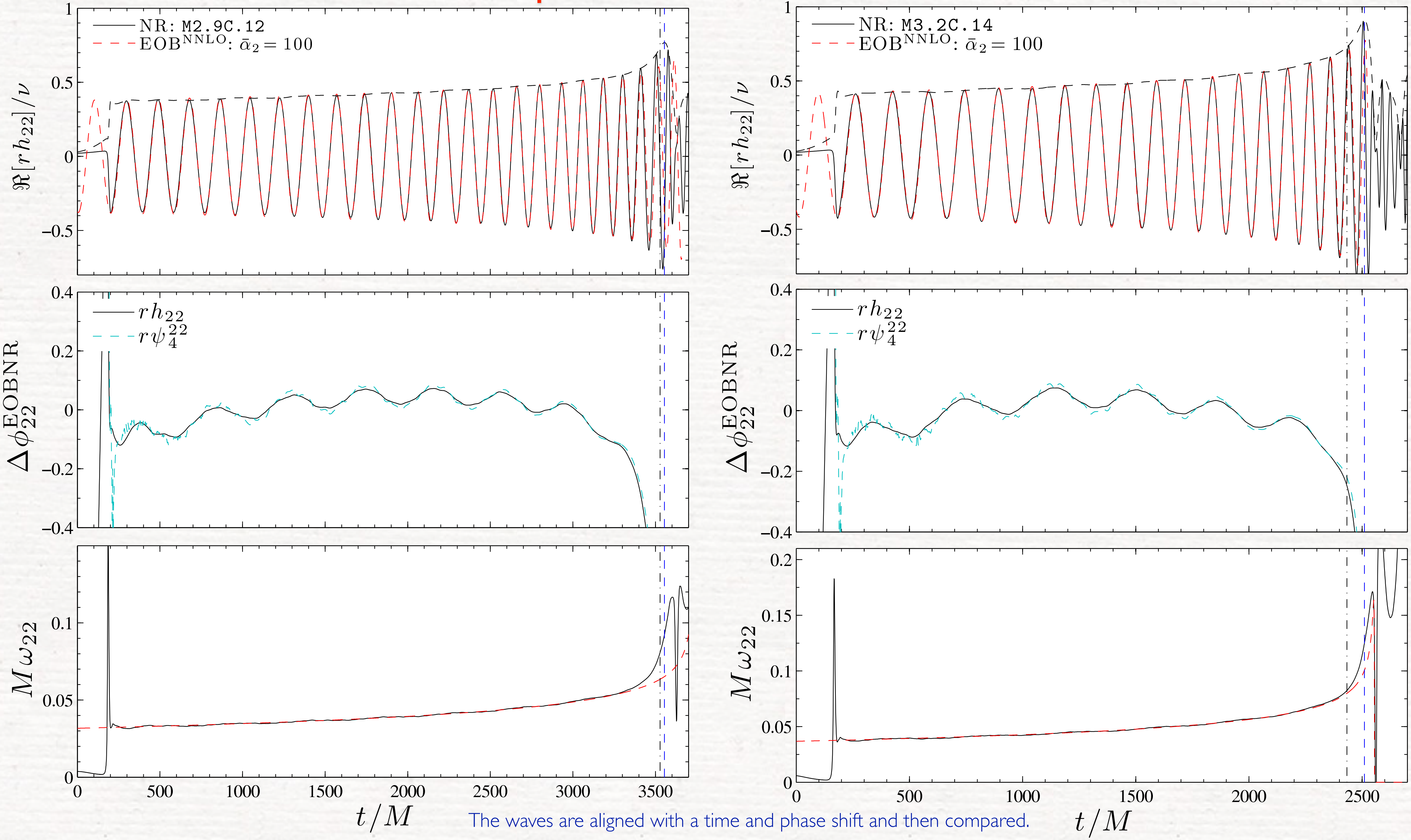
EOB - T4 comparison in the time domain



The waves are aligned with a time and phase shift and then compared.

- With the inclusion of NNLO (effective) tidal-effect correction the dephasing is <≈ 1 radian and ≈0.1 radians for most of the time (except the last 100M).
- With only leading-order tidal effects, the maximum dephasing would be 5 radians.

Estimation of numerical errors

Estimation of our error budget in the time domain:

- resolution: $\Delta\phi = 0.5$
- finite radius extraction: $\Delta\phi = 0.05$
- isentropic vs. non-isentropic EOS: $\Delta\phi = 0.15$
- quadrature sum: $\Delta\phi = 0.52$

Errors computed in the frequency-domain analysis (Q_ω) are similar or larger (because of the necessary cleaning procedure of the NR data; however the "cleaning error" is smaller than all other errors)

Conclusion

- We simulate compact systems with our state-of-the-art 3-dimensional general-relativistic code *Whisky* and compute gravitational waveforms.

- We use our waveforms to calibrate some parameters of the Effective One Body formalism.

- Improvements in the accuracy of numerical simulations and in the theoretical understanding of the analytical EOB approach give a viable analytical model with one parameter that reproduces numerical results of binary NS inspiral up to the merger (or the tidal disruption in case of BH-NS binaries).

- Further improvements are necessary, especially on the numerical side, in particular more accurate NR simulations, encompassing more compactnesses and different mass ratios will be needed to assess the relative merits of the EOB versus the Taylor-T4 description of tidally interacting BNS systems.

- However there seems to remain little conceptual problem in producing any number of GW templates with the EOB approach tuned with the results of numerical relativity, also for binary neutron-star mergers.