Adaptive Mirror Motion Estimation using Phase-Squeezed States

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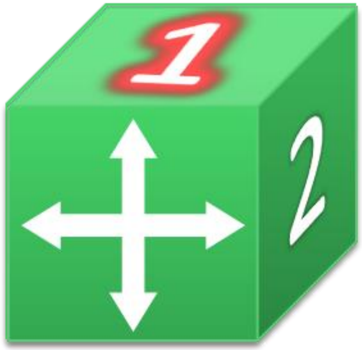
Outline

1. Introduction
2. Theory
3. Experiment
4. Summary



What do I do?

What? Objective

- Measure optical phase in high precision

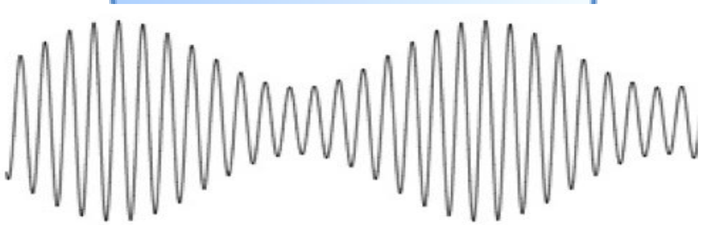
Gravitational Wave Detection?

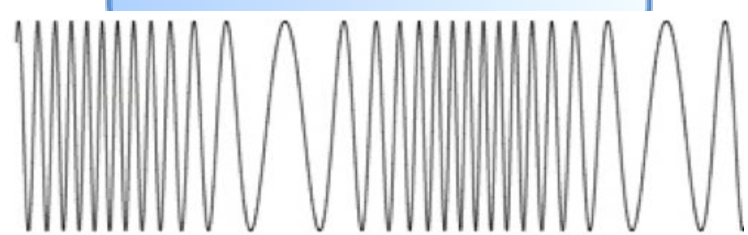
Why? Applications

- Precision measurement
- Coherent optical communication

AM (Amplitude Modulation)

FM (Frequency Modulation)

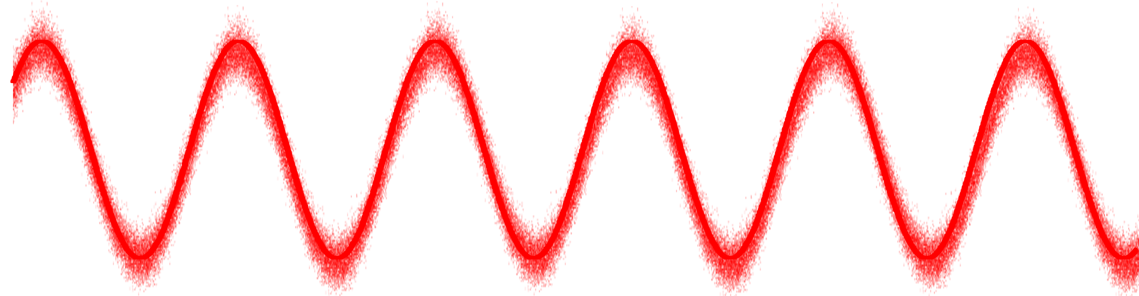




Introduction

◆Light

- ➢Amplitude $|\alpha|$
- ➢Phase φ



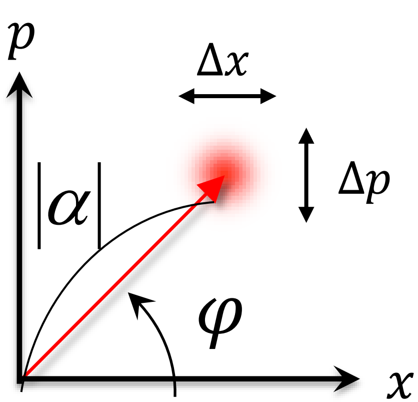
$$\mathbf{E} = \mathbf{E}_0 \alpha e^{-i\omega t}$$
$$\alpha = x + ip$$
$$\alpha = |\alpha| e^{i\varphi}$$

$$\hat{\mathbf{E}} = \mathbf{E}_0 \hat{a} e^{-i\omega t}$$
$$\hat{a} = \hat{x} + i\hat{p}$$

$\hat{x}, \hat{p}$: quadrature

$\hat{x}, \hat{p}$ are conjugates

$$[\hat{x}, \hat{p}] = \frac{i}{2} \quad \left(\hbar = \frac{1}{2}\right)$$

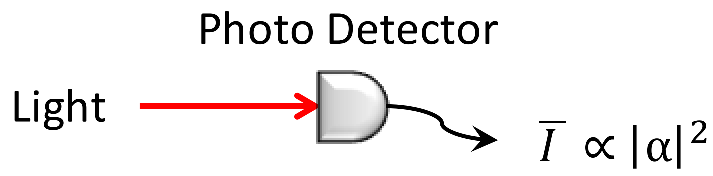


Heisenberg's uncertainty

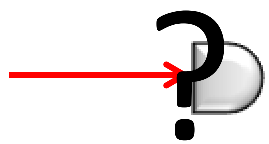
$$\Delta x \cdot \Delta p \geq \frac{1}{4}$$

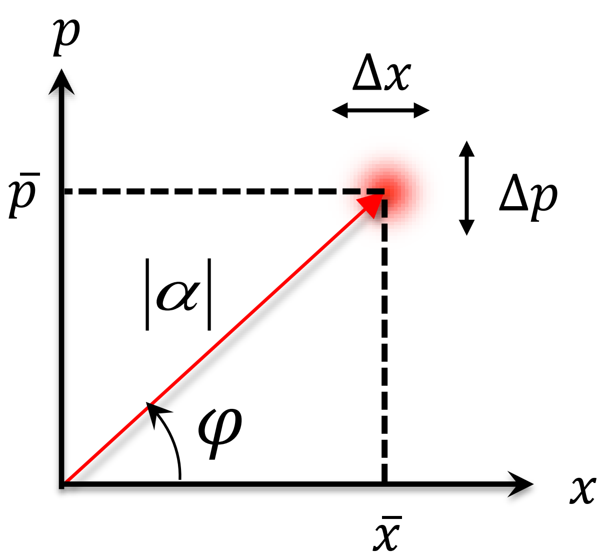
Optical Measurement

Measuring Amplitude



Measuring Phase

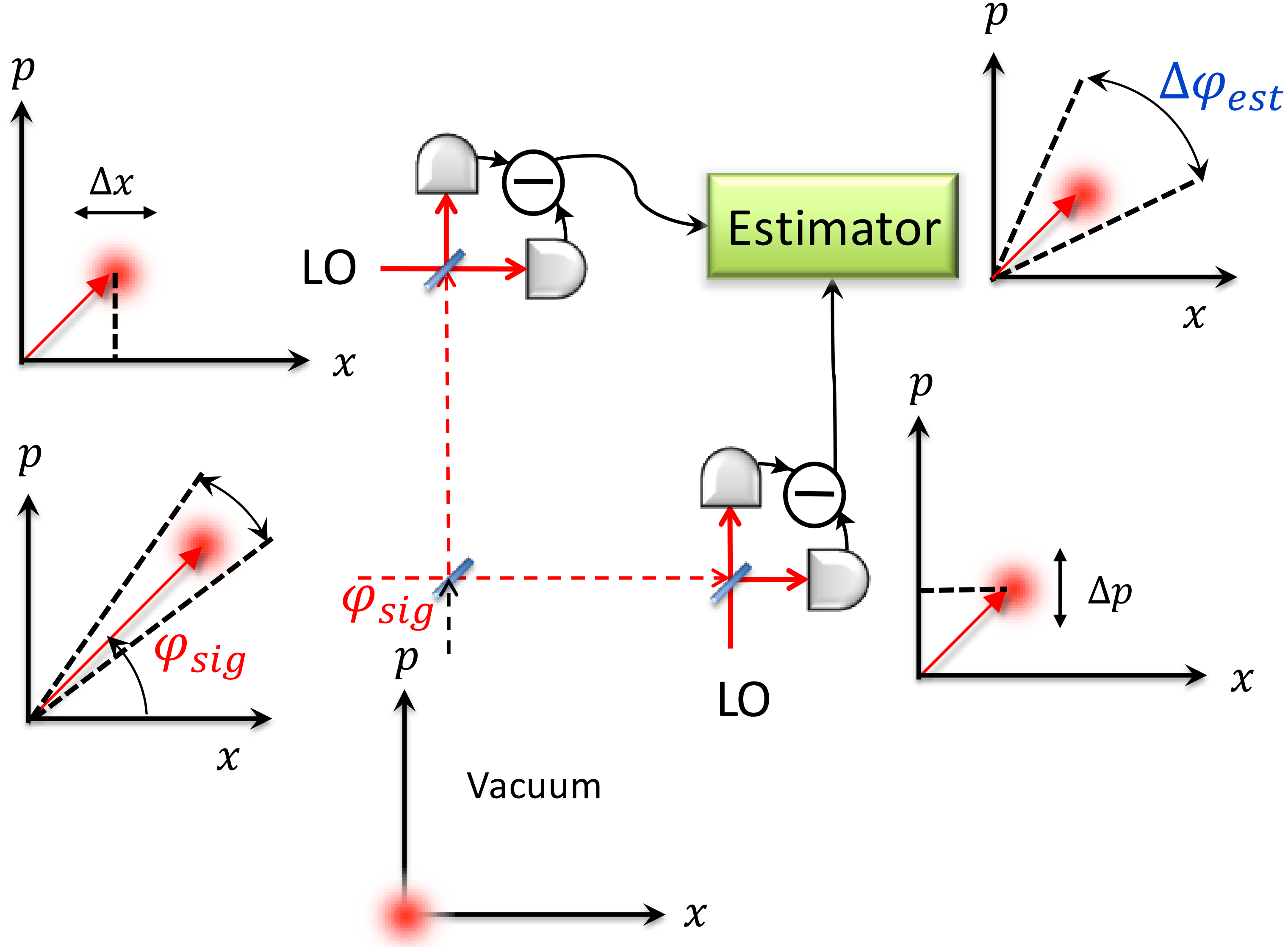




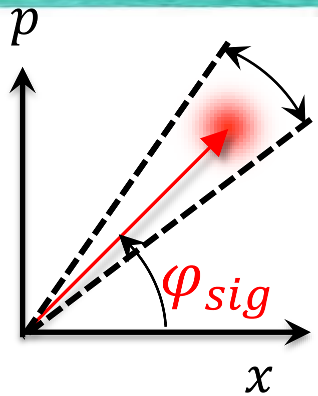
$\hat{x}, \hat{p}$ are conjugates

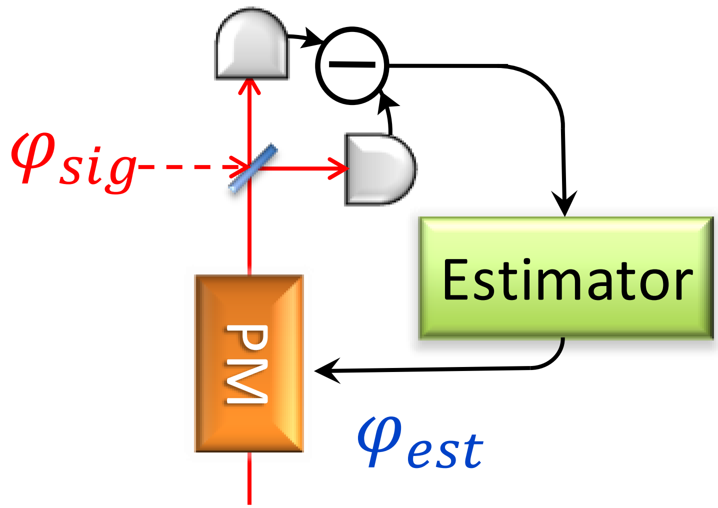
Impossible to measure
$\hat{x}, \hat{p}$ simultaneously

Optical Phase Measurement

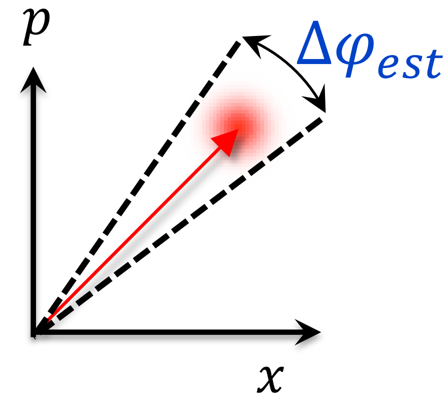


Adaptive Optical Phase Measurement

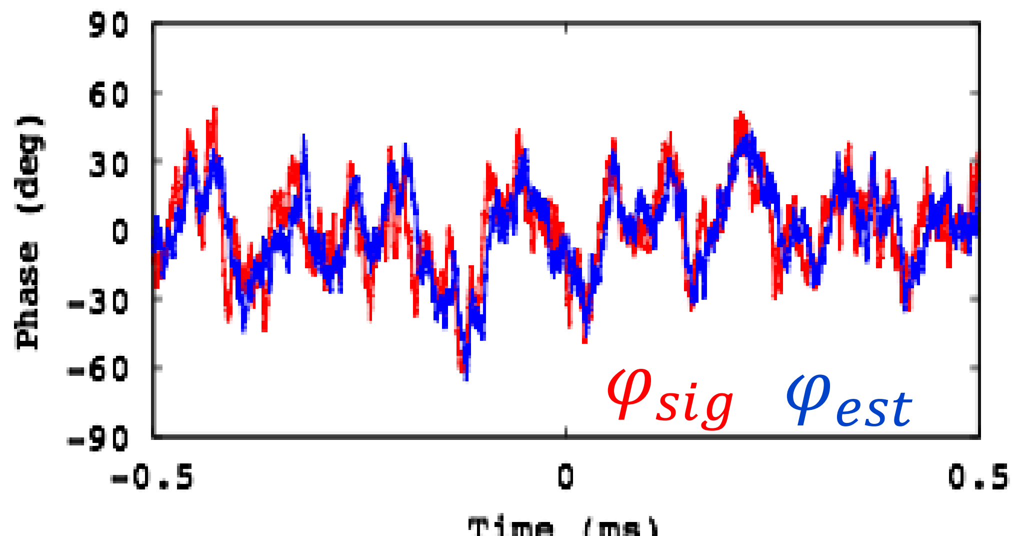




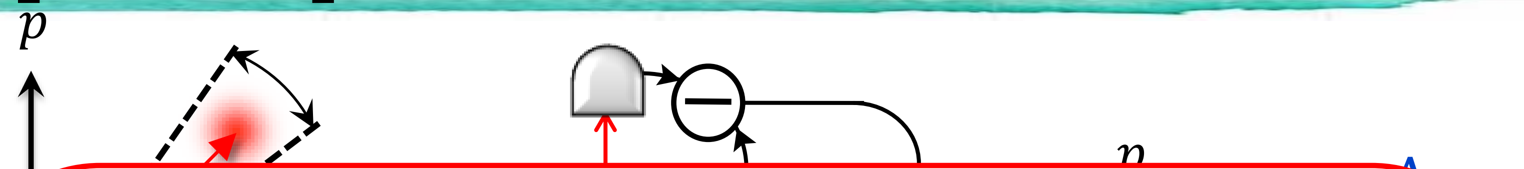
LO (Local Oscillator)



EOM
(Eletro-Optic Modulator)

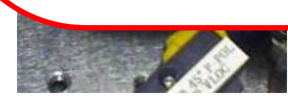


Adaptive Optical Phase Measurement

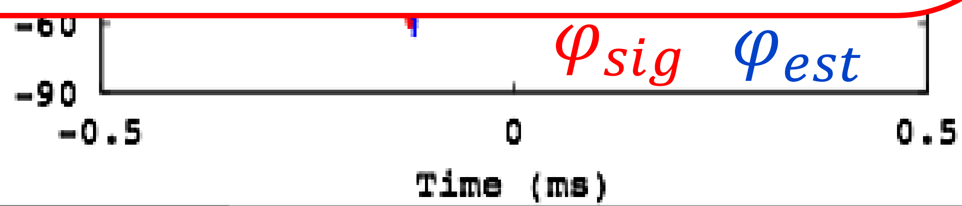


We have already demonstrated the superiority of adaptive optical phase measurement (for phase modulated by EOM)

Can we apply adaptive measurement to practical applications?

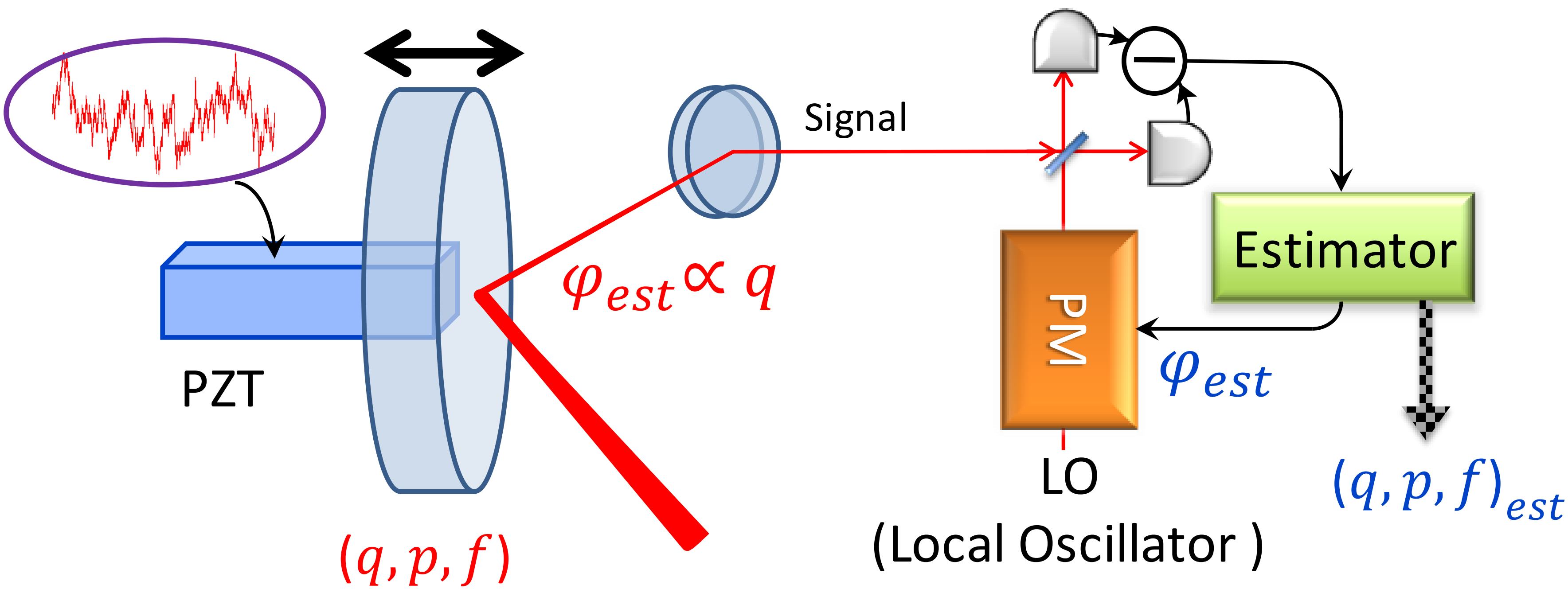


EOM
(Eletro-Optic Modulator)



Purpose

◆Estimate Position, Momentum, and applied Force of a Mirror



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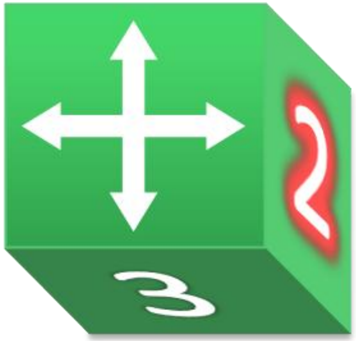
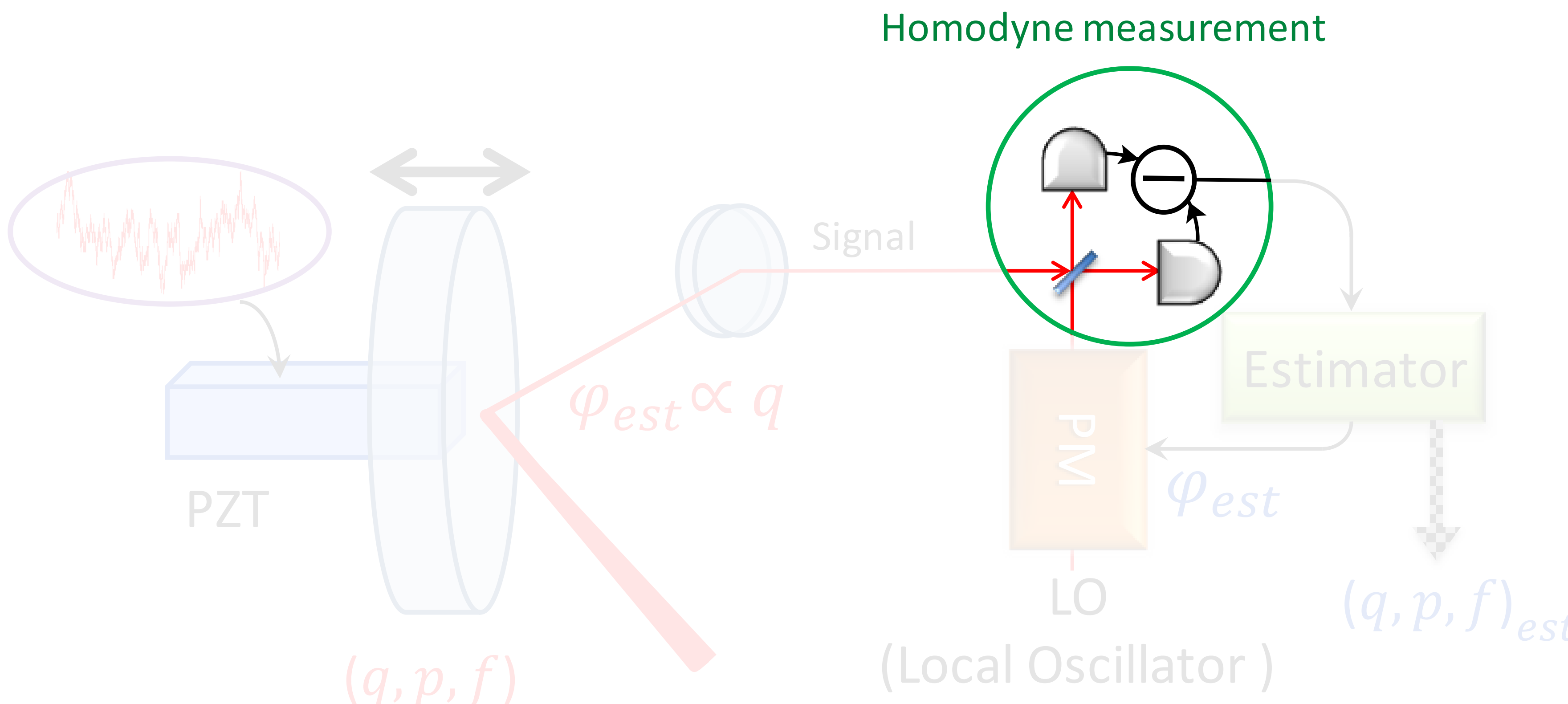
2. Theory
   - Adaptive optical phase measurement
   - Mirror motion model
   - Estimation method
3. Experiment
4. Summary

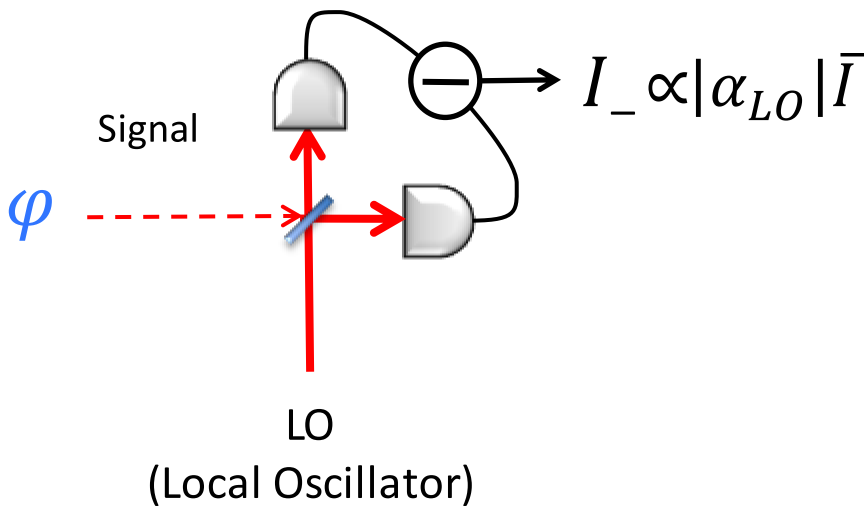


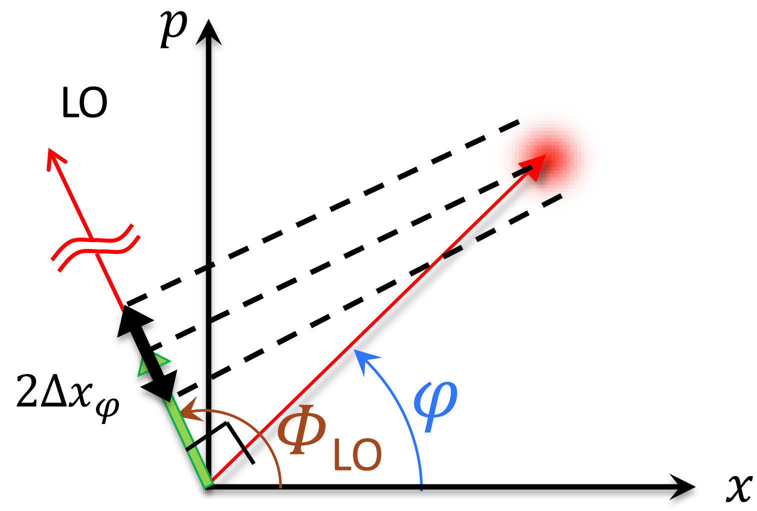
Purpose

- ◆Estimate Position, Momentum, and applied Force of a Mirror



Homodyne Measurement





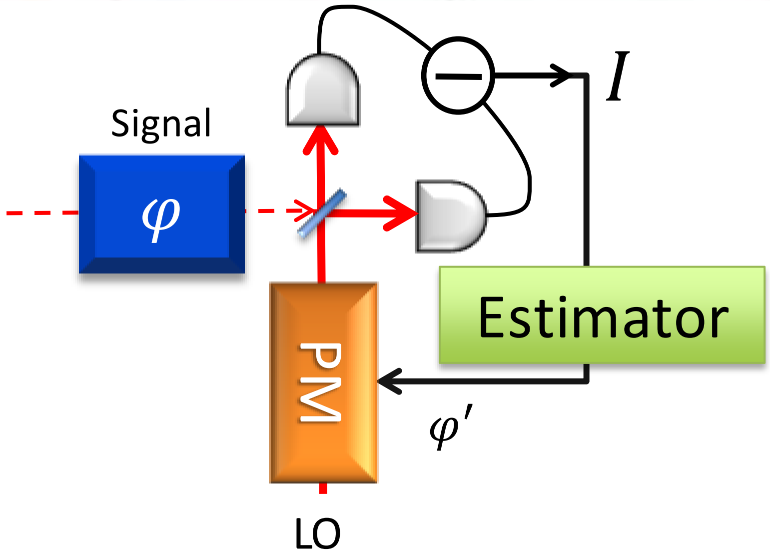
Most phase sensitive when

$$\varphi - \Phi_{LO} = \frac{\pi}{2}$$

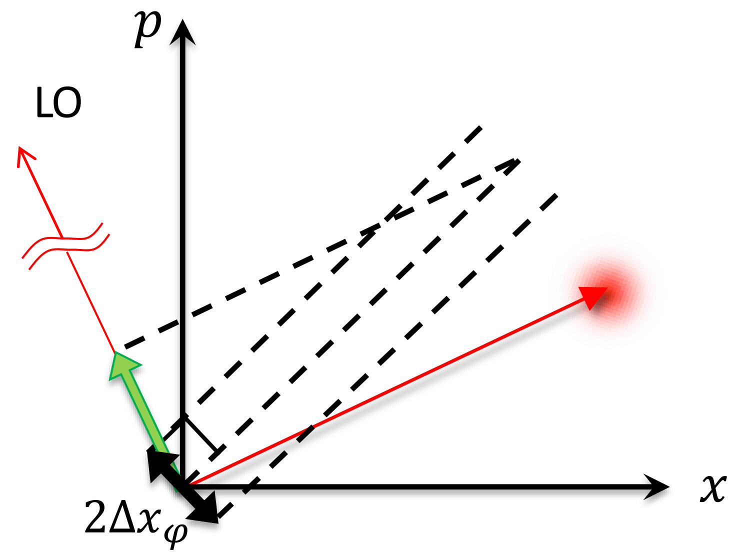
$$\bar{I} = 2|\alpha|\cos(\varphi - \Phi_{LO})$$

$$\Delta I^2 = \Delta x_{\varphi}{}^2$$

Adaptive Homodyne Measurement



PM: Phase Modulator
LO: Local Oscillator

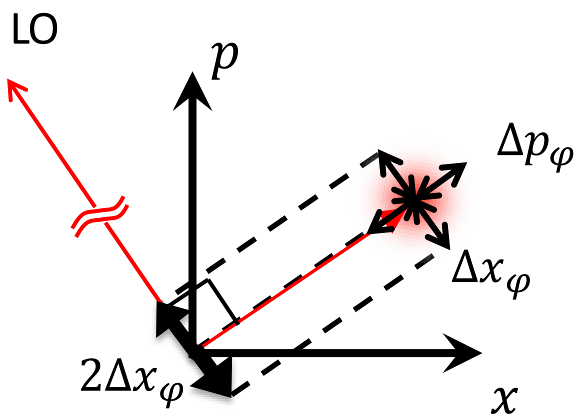


$$\Phi_{LO} = \varphi' + \frac{\pi}{2}$$

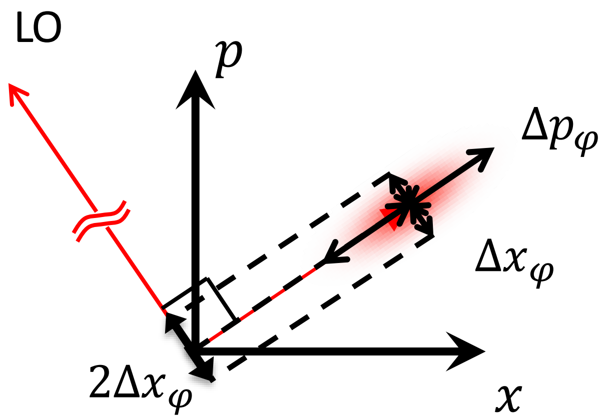
$$\bar{I} = 2|\alpha|\cos(\varphi - \Phi_{LO})$$
$$\Delta I^2 = \Delta x_\varphi{}^2$$

$$\bar{I} = 2|\alpha|\sin(\varphi - \varphi') \approx 2|\alpha|(\varphi - \varphi')$$
$$\Delta I^2 = \Delta x_\varphi{}^2$$

Phase-Squeezed State



Coherent state



Phase-squeezed state

$$\Delta x_\varphi \cdot \Delta p_\varphi = \frac{1}{4}$$

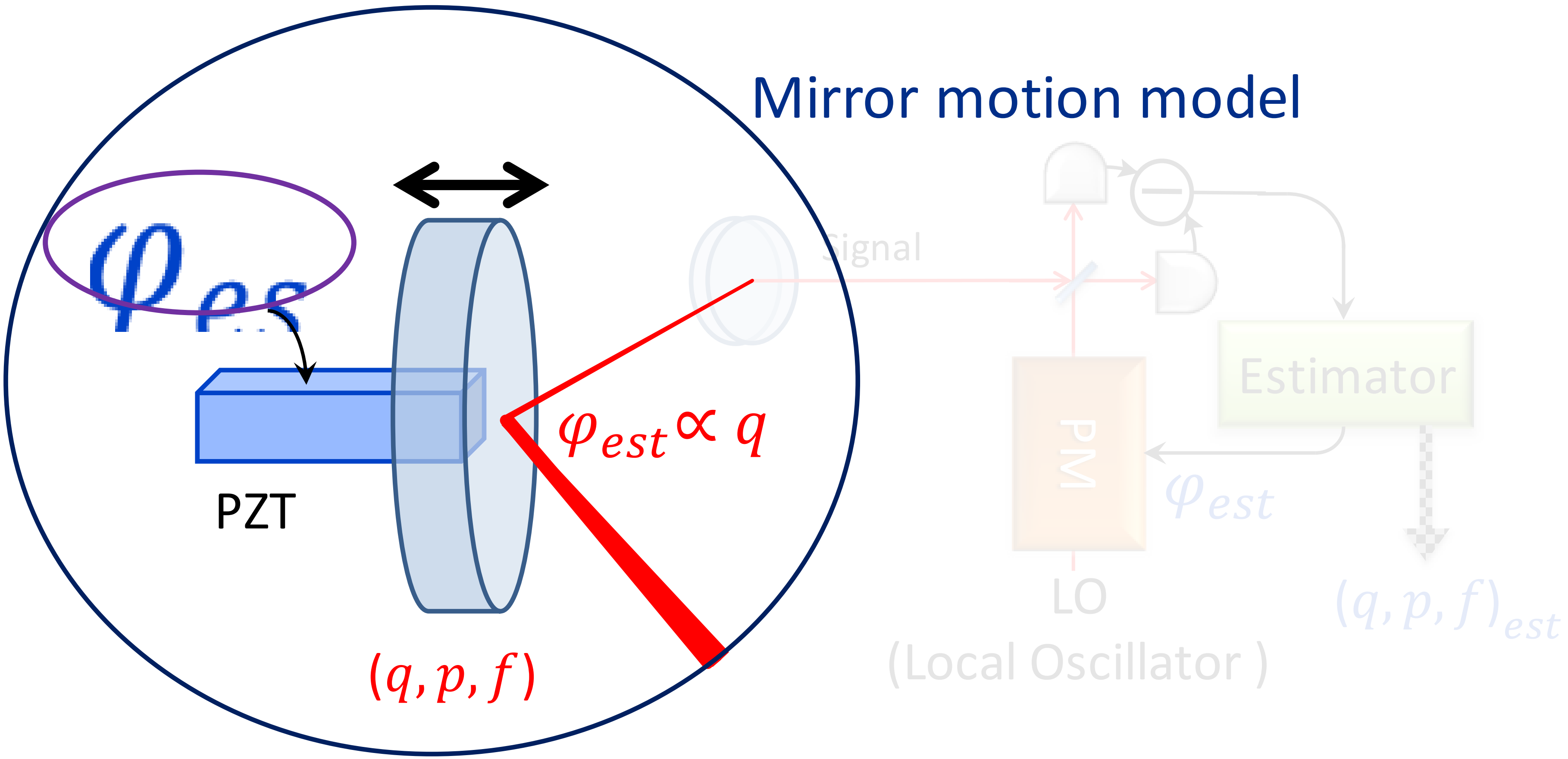
$$\Delta x_\varphi{}^2 = \Delta p_\varphi{}^2 = \frac{1}{4}$$

$$\Delta x_\varphi \cdot \Delta p_\varphi = \frac{1}{4}$$

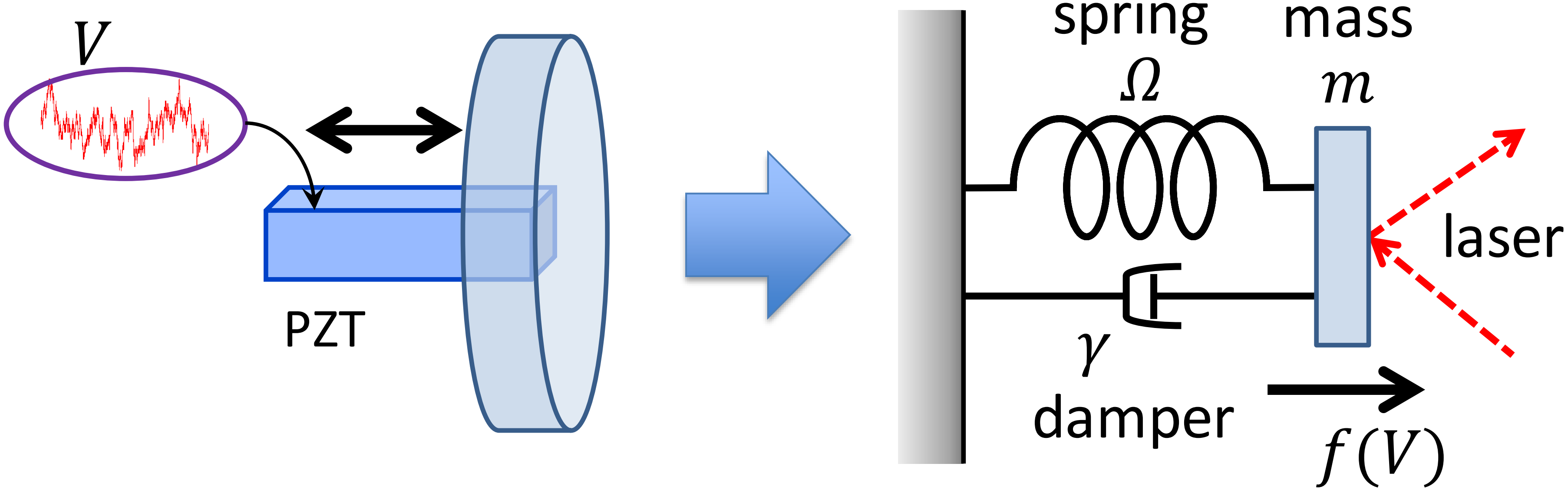
$$\Delta x_\varphi{}^2 = \frac{1}{4} e^{-2r}, \quad \Delta p_\varphi{}^2 = \frac{1}{4} e^{+2r}$$

Purpose

◆Estimate Position, Momentum, and applied Force of a Mirror



Mirror Motion Model

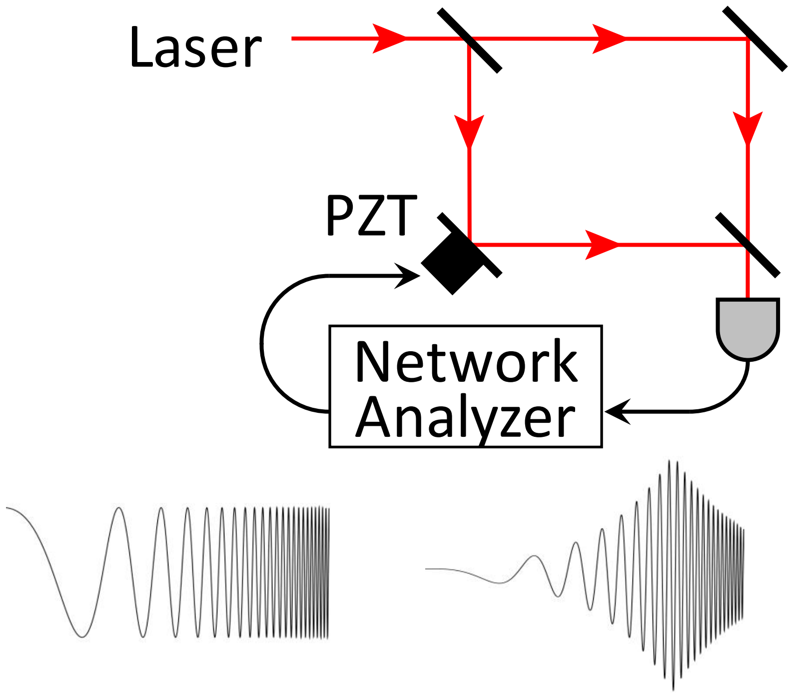


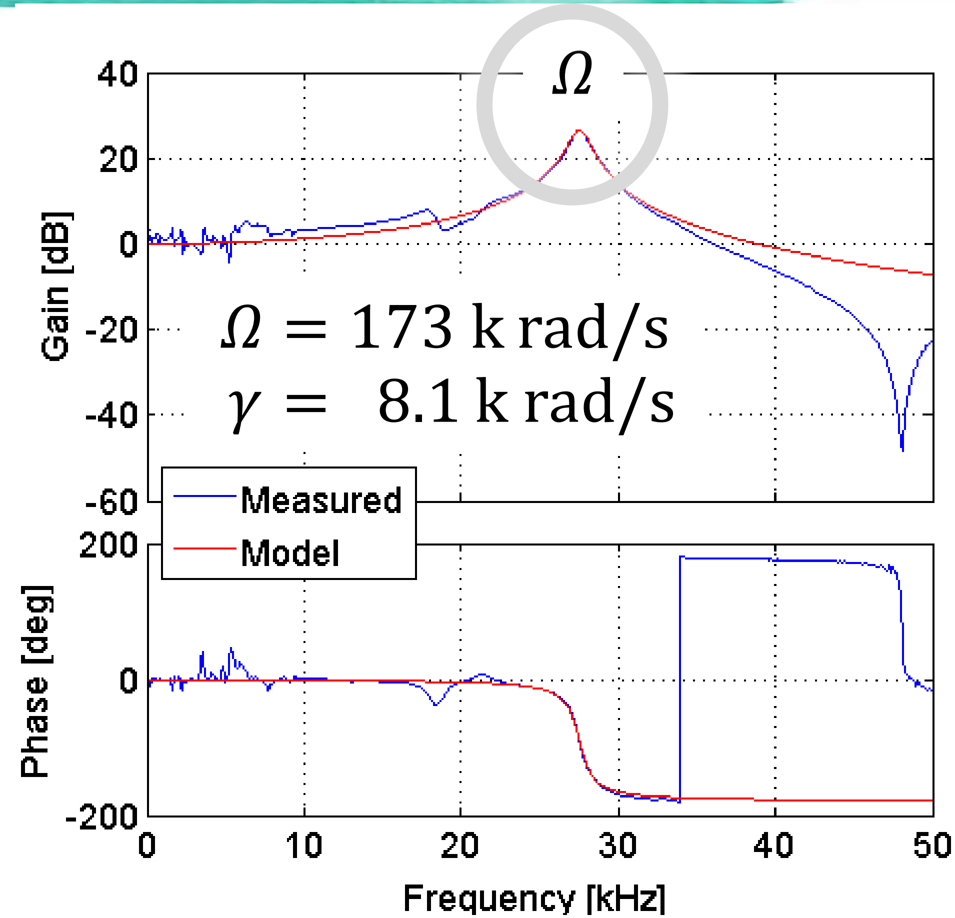
Spring-mass-damper model
⇒Motion Equation:

$$
\begin{cases}
\dfrac{dp}{dt} = -m\Omega^2 q - \gamma p + f \\
f = \beta V
\end{cases}
$$

$$m = 5.88 \times 10^{-4}\ [\mathrm{kg}]$$
$$\beta = 1.99 \times 10^{-1}\ [\mathrm{N/V}]$$

Measurement of Model Parameter





Use the Measured Transfer function and Voltage applied to PZT to compare estimated (q, p, f) to the true values

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Purpose

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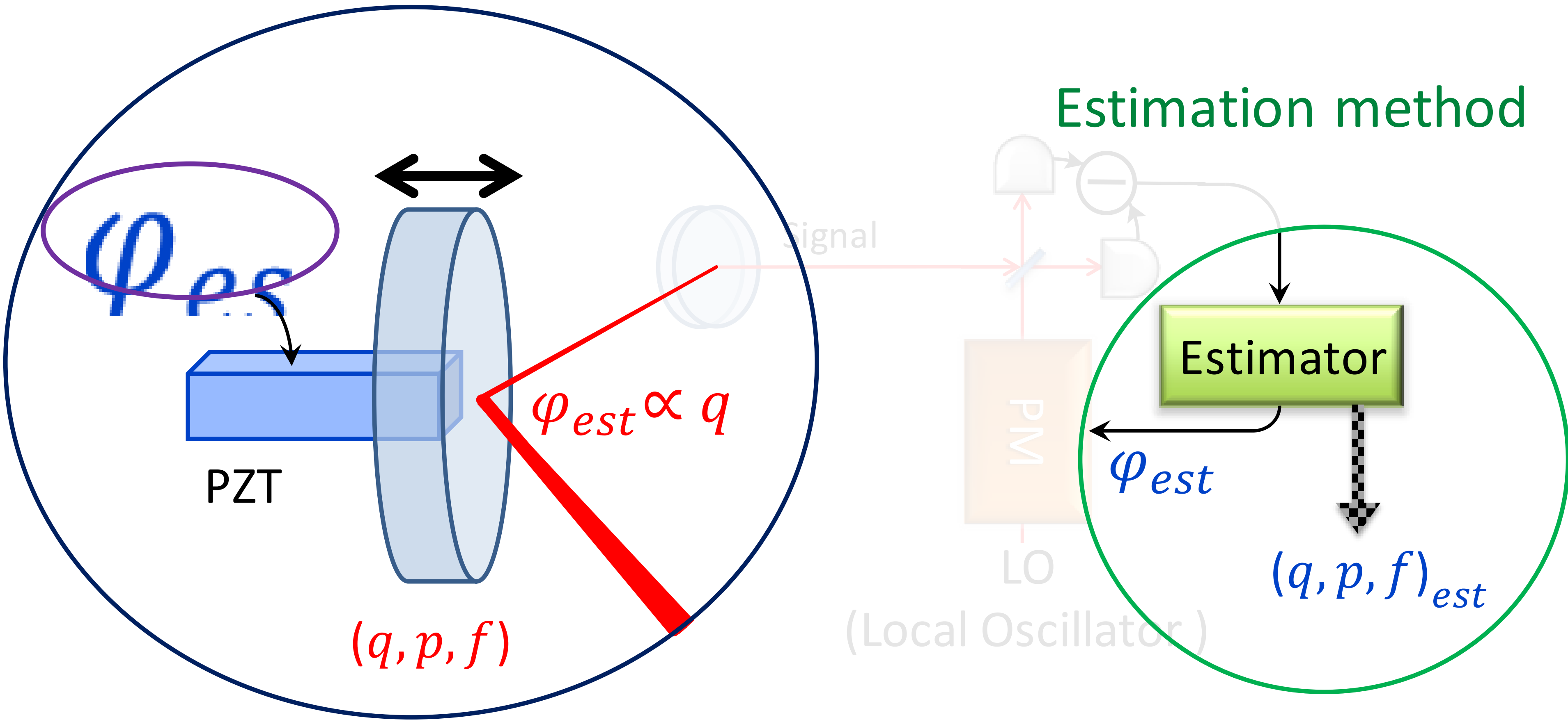
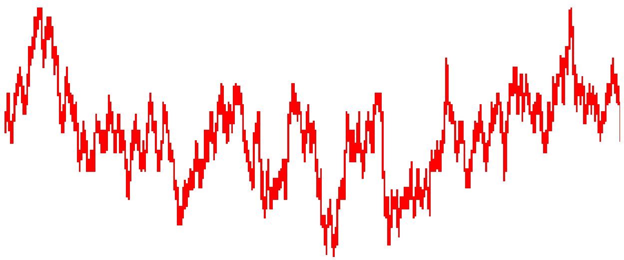


Estimation Method

◆Applied force: random walk bound in zero



$$\frac{df}{dt} = -\lambda f + \sqrt{\kappa}\underline{w(t)}$$

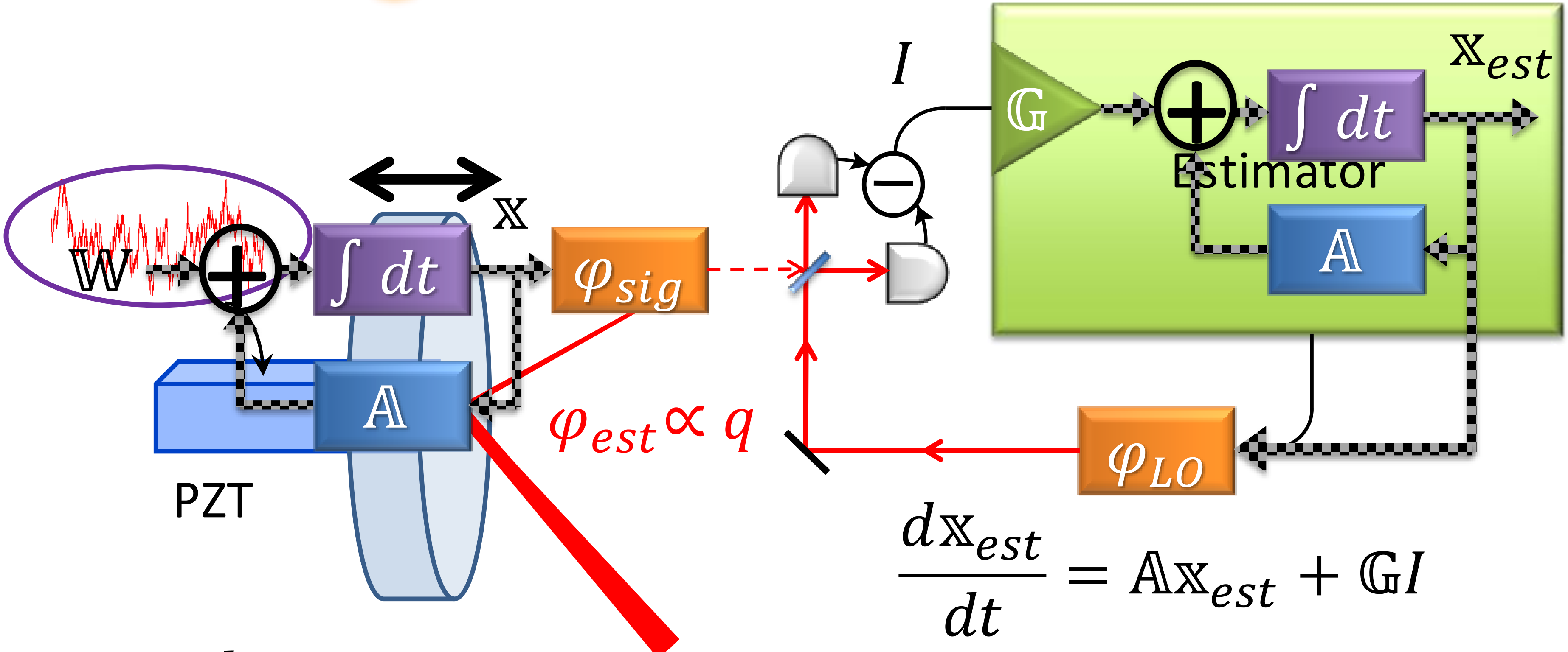
White Gaussian Noise

◆Time evolution of the system

$$\underbrace{\frac{d}{dt}\begin{pmatrix} q \\ p \\ f \end{pmatrix}}_{\mathbb{x}} = \underbrace{\begin{pmatrix} 0 & 1/m & 0 \\ -m\Omega^2 & -\gamma & 1 \\ 0 & 0 & -\lambda \end{pmatrix}}_{\mathbb{A}} \begin{pmatrix} q \\ p \\ f \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \sqrt{\kappa}w(t) \end{pmatrix}}_{\mathbb{W}}$$

Optimal Feedback Filter(1)

------- Signal ------- -- Measure&Estimate --



$$\frac{d\mathbb{x}_{est}}{dt} = \mathbb{A}\mathbb{x}_{est} + \mathbb{G}I$$

$$\frac{d\mathbb{x}}{dt} = \mathbb{A}\mathbb{x} + \mathbb{W}$$

Optimal Feedback Filter(2)

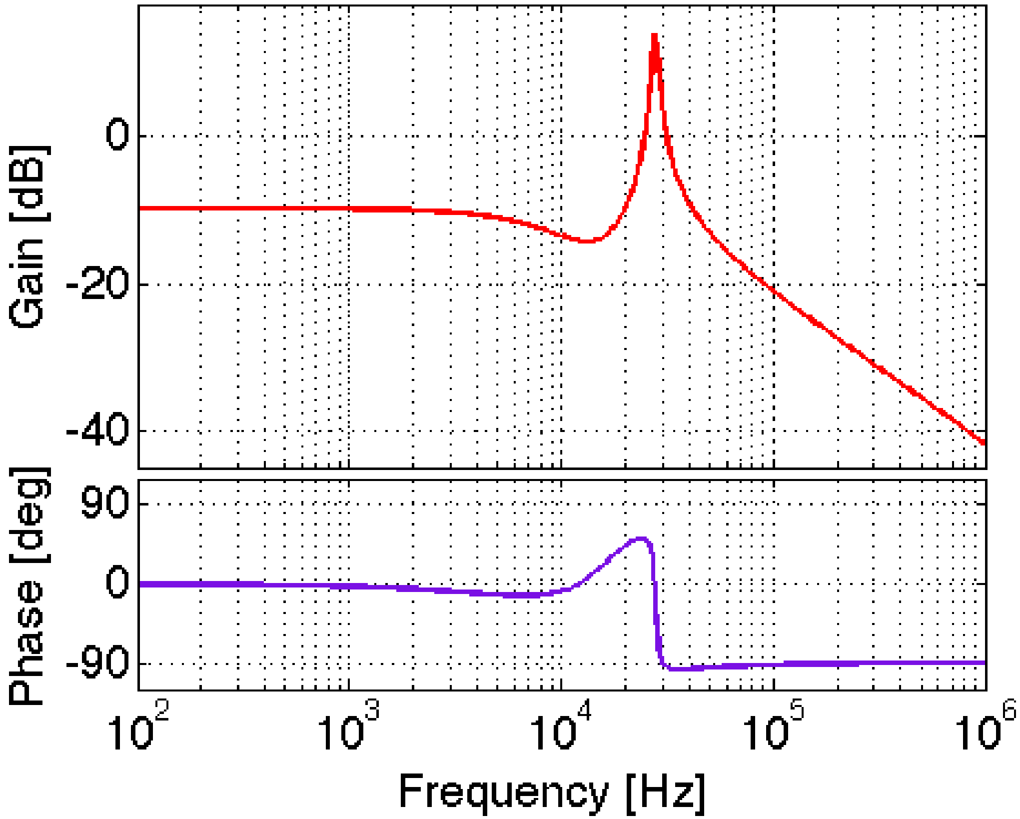
$$\frac{d\mathbb{x}_{est}}{dt} = \mathbb{A}\mathbb{x}_{est} + \mathbb{G}I$$

Fourier Transform

$$\mathbb{x}_{est} = (j\omega\mathbb{I} - \mathbb{A})^{-1}\mathbb{G}I$$

- Position
- Momentum
- Force

Estimator of Position

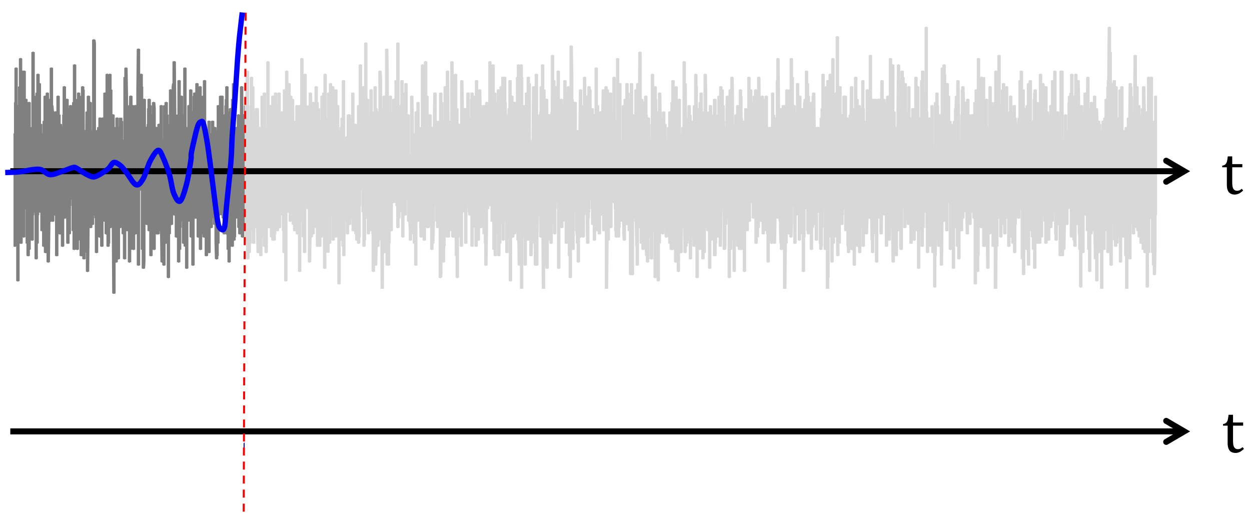


Optimal Feedback Filter(3)

$$\mathbb{x}_{est} = (j\omega\mathbb{I} - \mathbb{A})^{-1}\mathbb{G}I$$

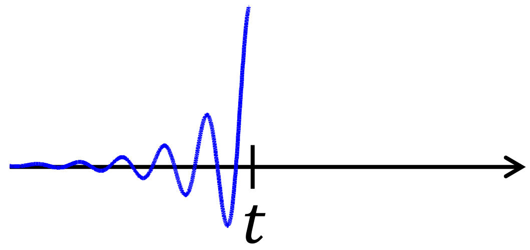
Inverse Fourier Transform

$$\mathbb{x}_{est}(t) = \int_{t_0}^{t} \underline{\mathbb{H}(t)}\, \underline{I(t-\tau)}$$

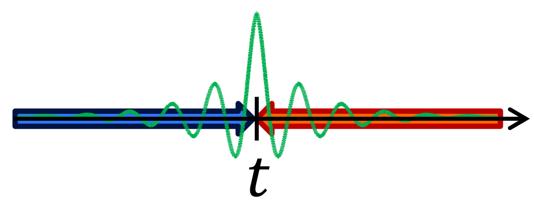


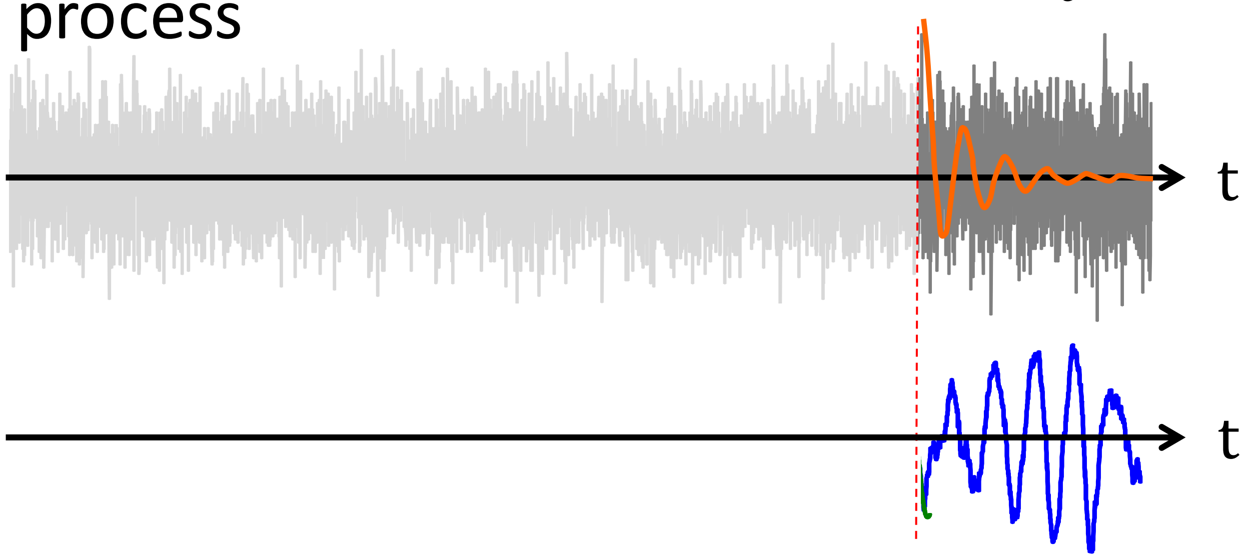
Estimation Method

- ◆ Filtering (only past)
  - ○ real time
  - × less precise



- ◆ Smoothing (past and future)
  - ○ more precise
  - × post process





Outline

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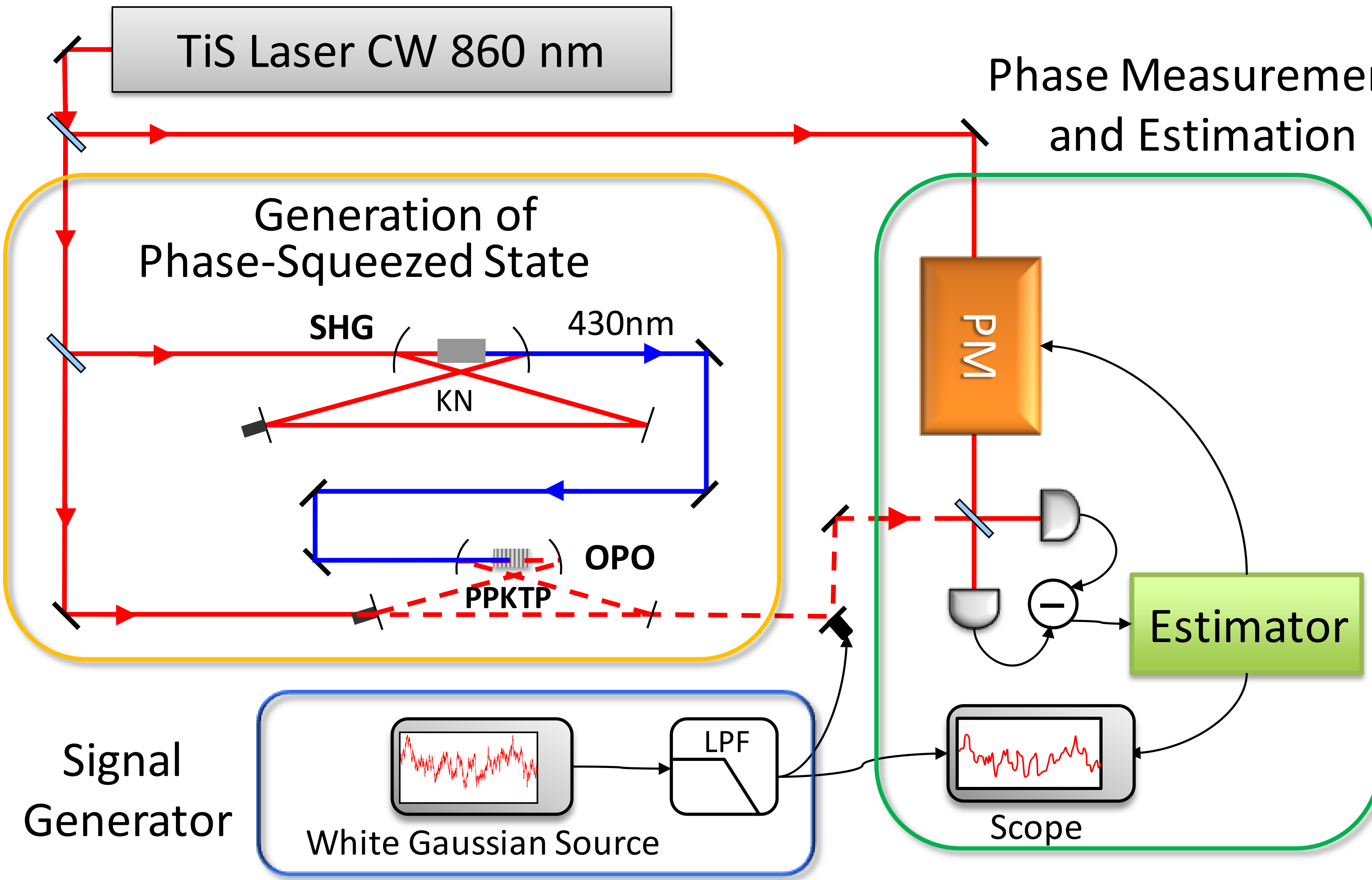
2. Theory

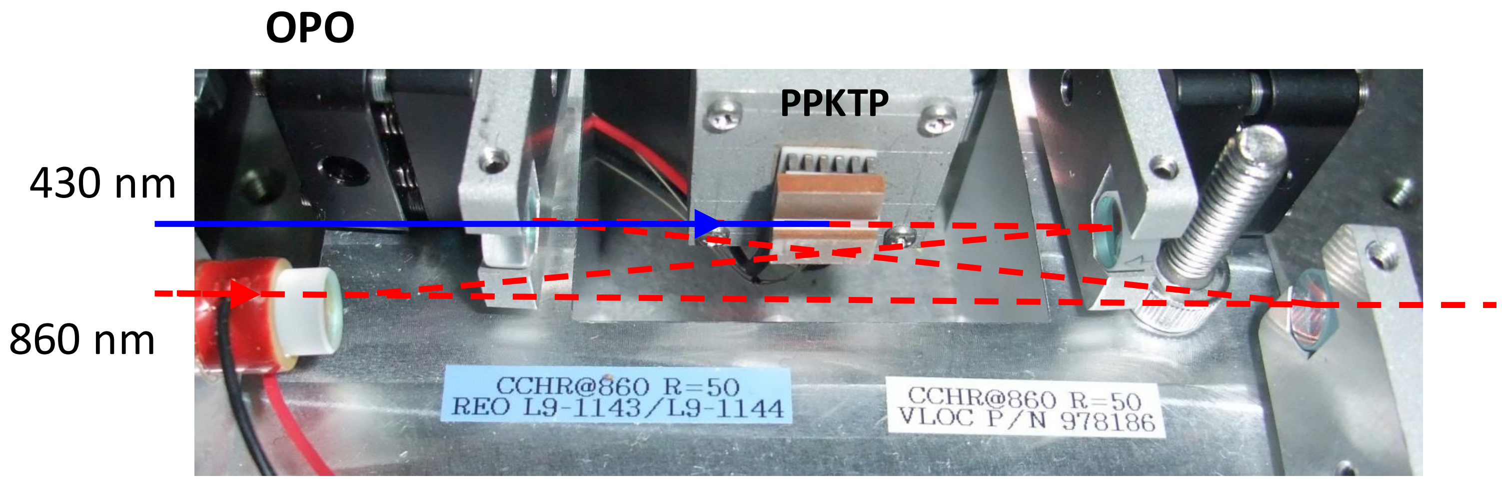
3. Experiment
   - κ dependence
   - $|\alpha|^2$ dependece
   - Phase-Squeezed State

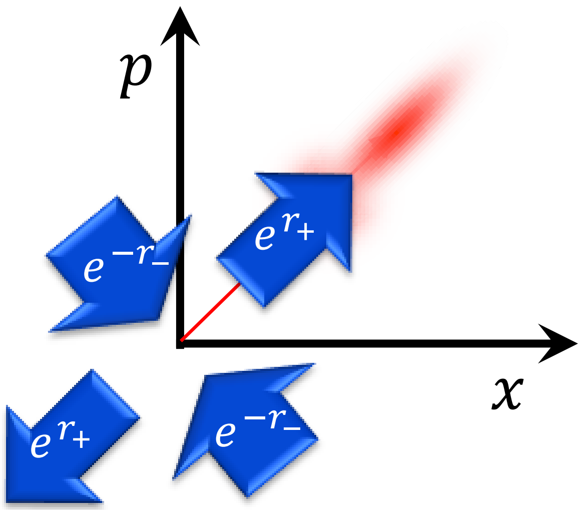
4. Summary

Experiment Setup



Generate Phase-Squeezed States

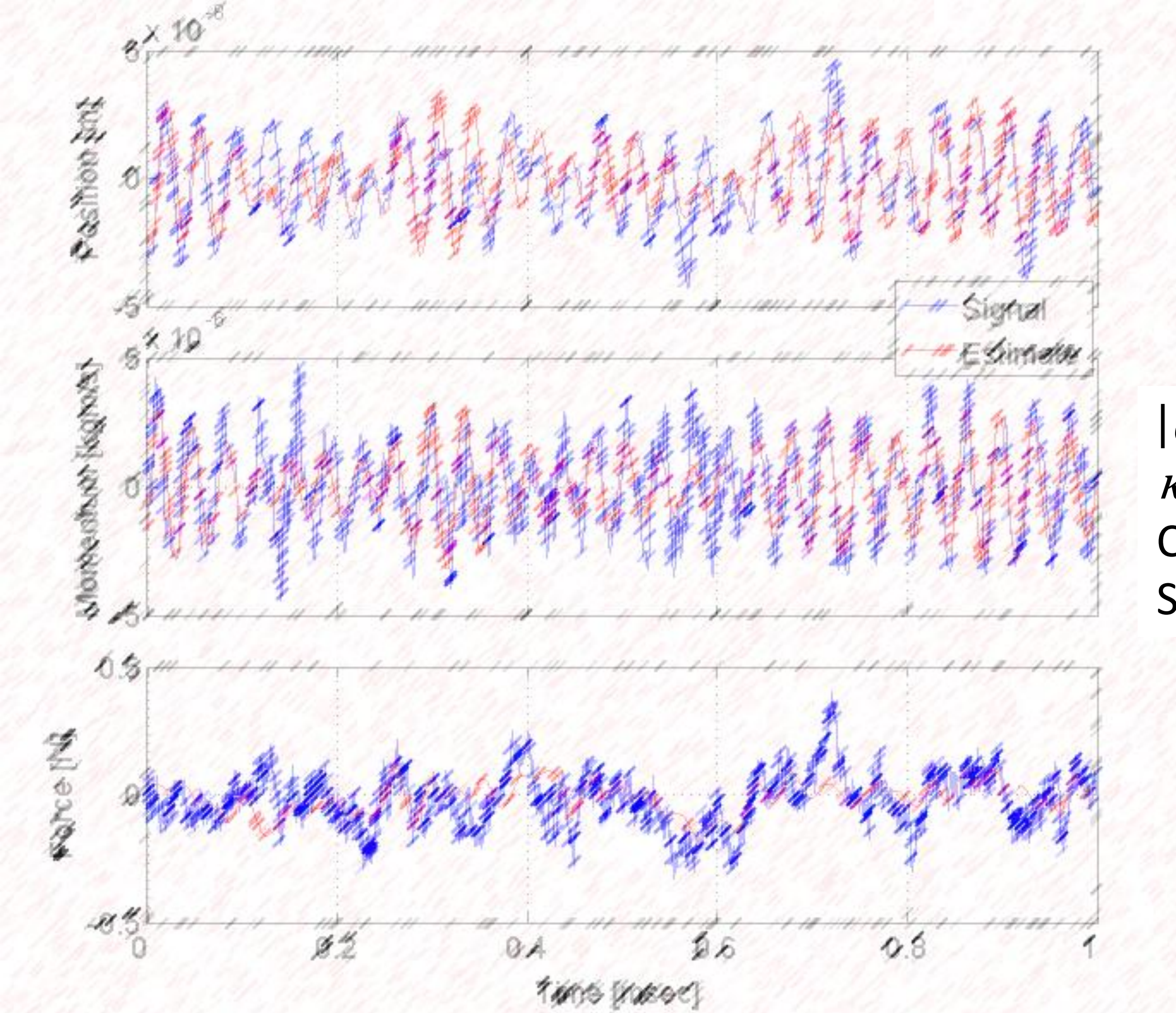




Squeeze Level
$e^{-2r} = 0.47$(-3.3 dB)

OPO: Optical Parametric Oscillator
PPKTP:Periodically Polled KTP

Time Domain Results

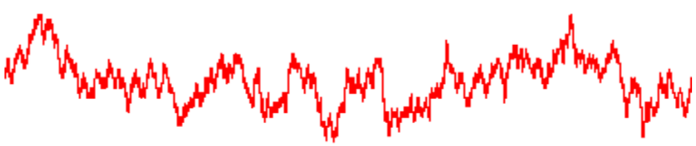


$|\alpha|^2 = 1 \times 10^6\ \mathrm{sec}^{-1}$
$\kappa \quad = 3 \times 10^9\ \mathrm{N}^2\mathrm{sec}^{-1}$
Coherent state
Smoothing method

Experiment Setup

$|\alpha|^2 = 1 \times 10^6\ \mathrm{sec}^{-1}$
Coherent state

$$\frac{df}{dt} = -\lambda f + \sqrt{\kappa} w(t)$$



$\lambda = 58$ k rad/s

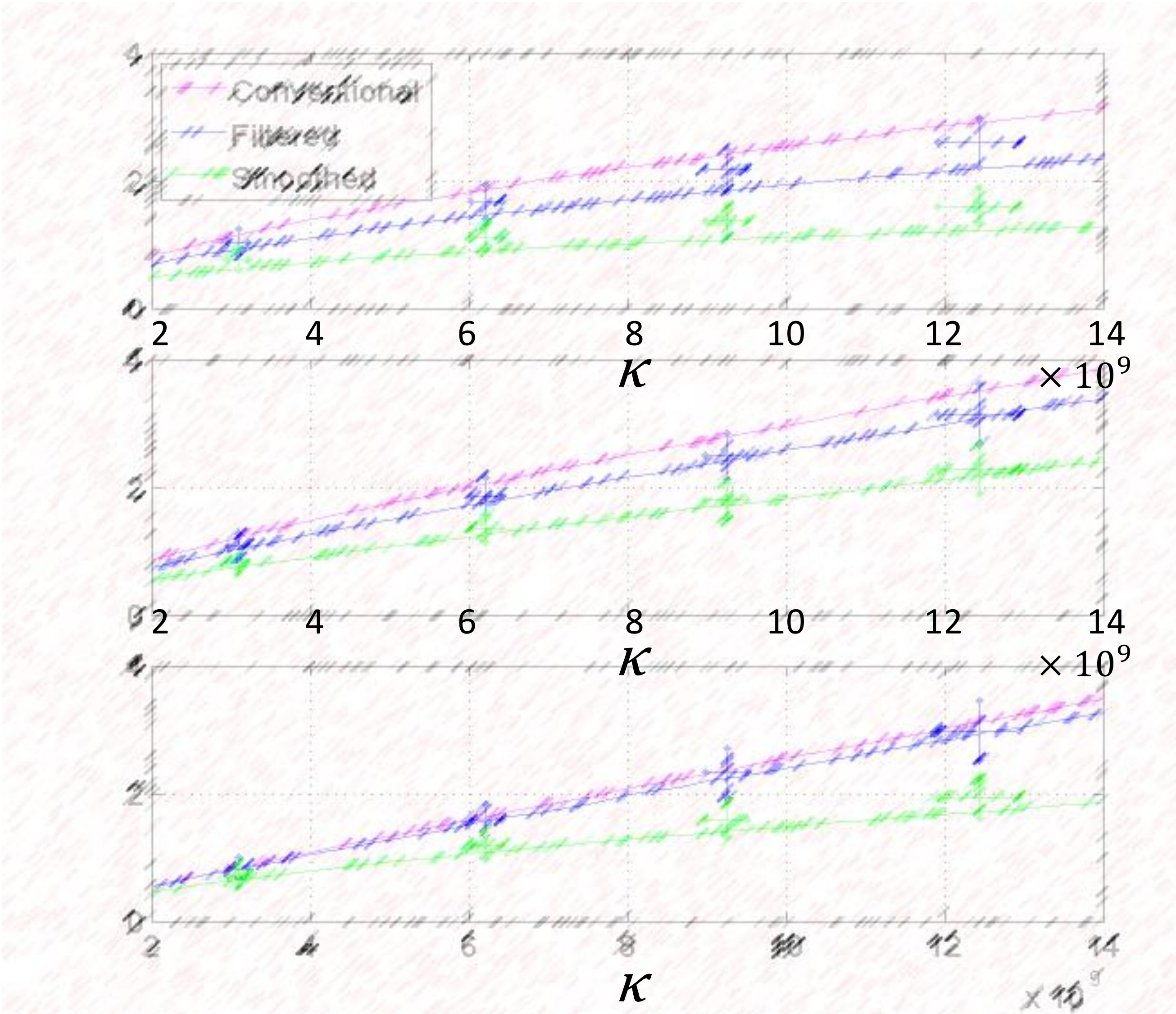
Signal Generator

Signal-Amplitude (κ) Dependence

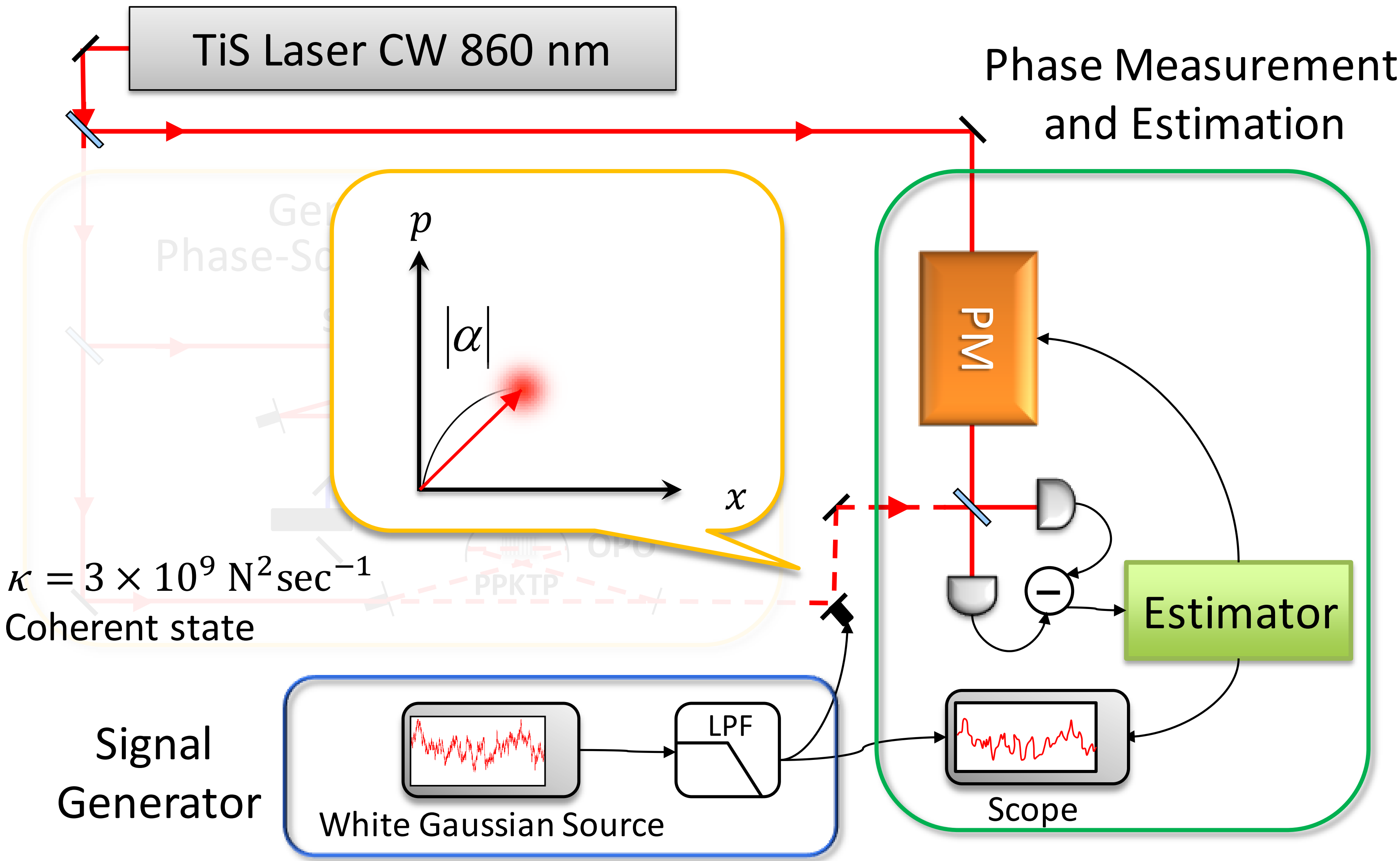
Position Variance $[\times 10^{-16}\ \mathrm{m}^2]$

Momentum Variance $[\times 10^{-12}\ \mathrm{kg}^2\mathrm{m}^2/\mathrm{s}^2]$

Applied Force Variance $[\times 10^{-2}\ \mathrm{N}^2]$



Experiment Setup

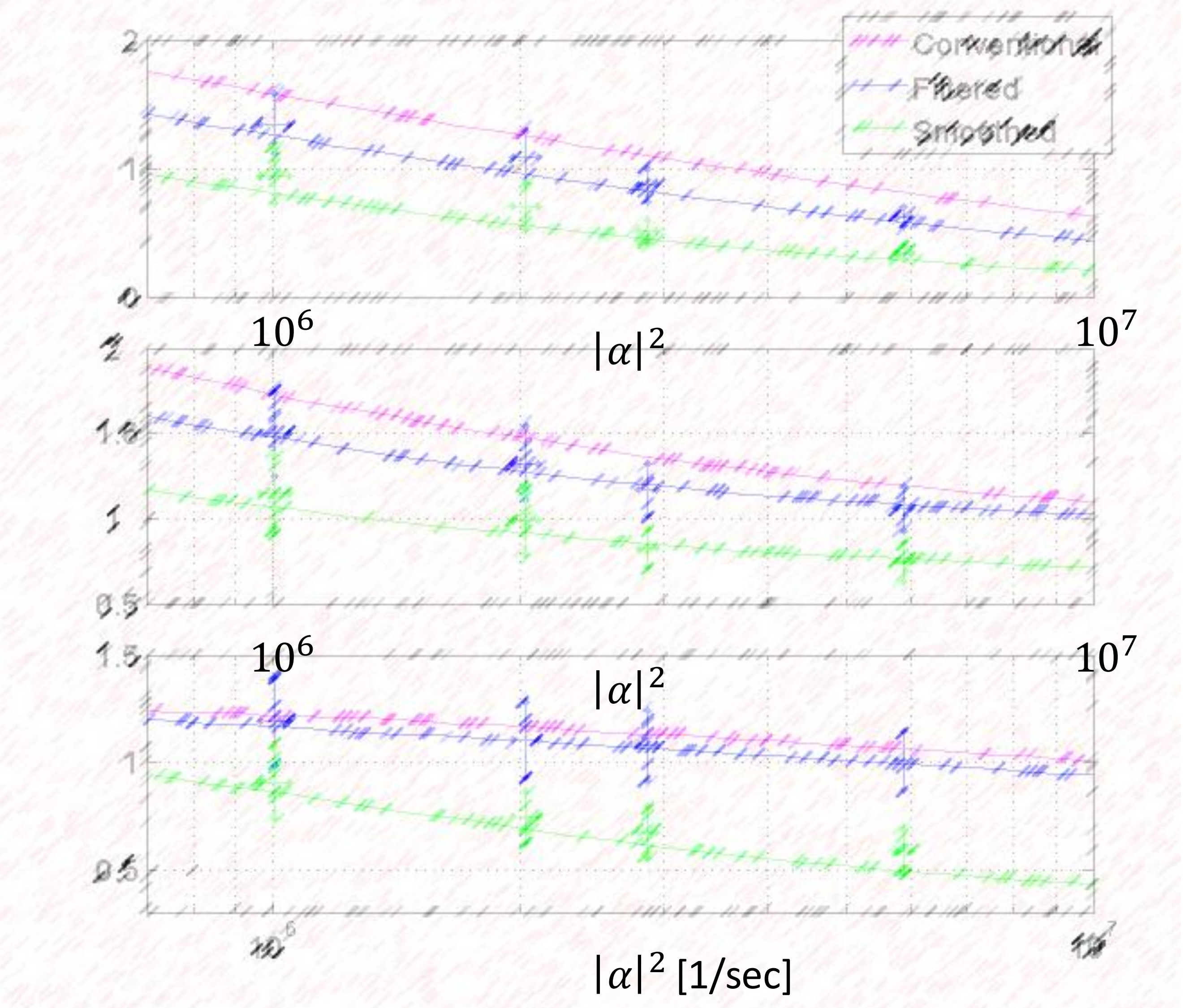


$|\alpha|^2$ Dependence

Position Variance $[\mathrm{x}10^{-16}\ \mathrm{m}^2]$

Momentum Variance $[\mathrm{x}10^{-12}\ \mathrm{kg}^2\mathrm{m}^2/\mathrm{s}^2]$

Applied Force Variance $[\mathrm{x}10^{-2}\ \mathrm{N}^2]$

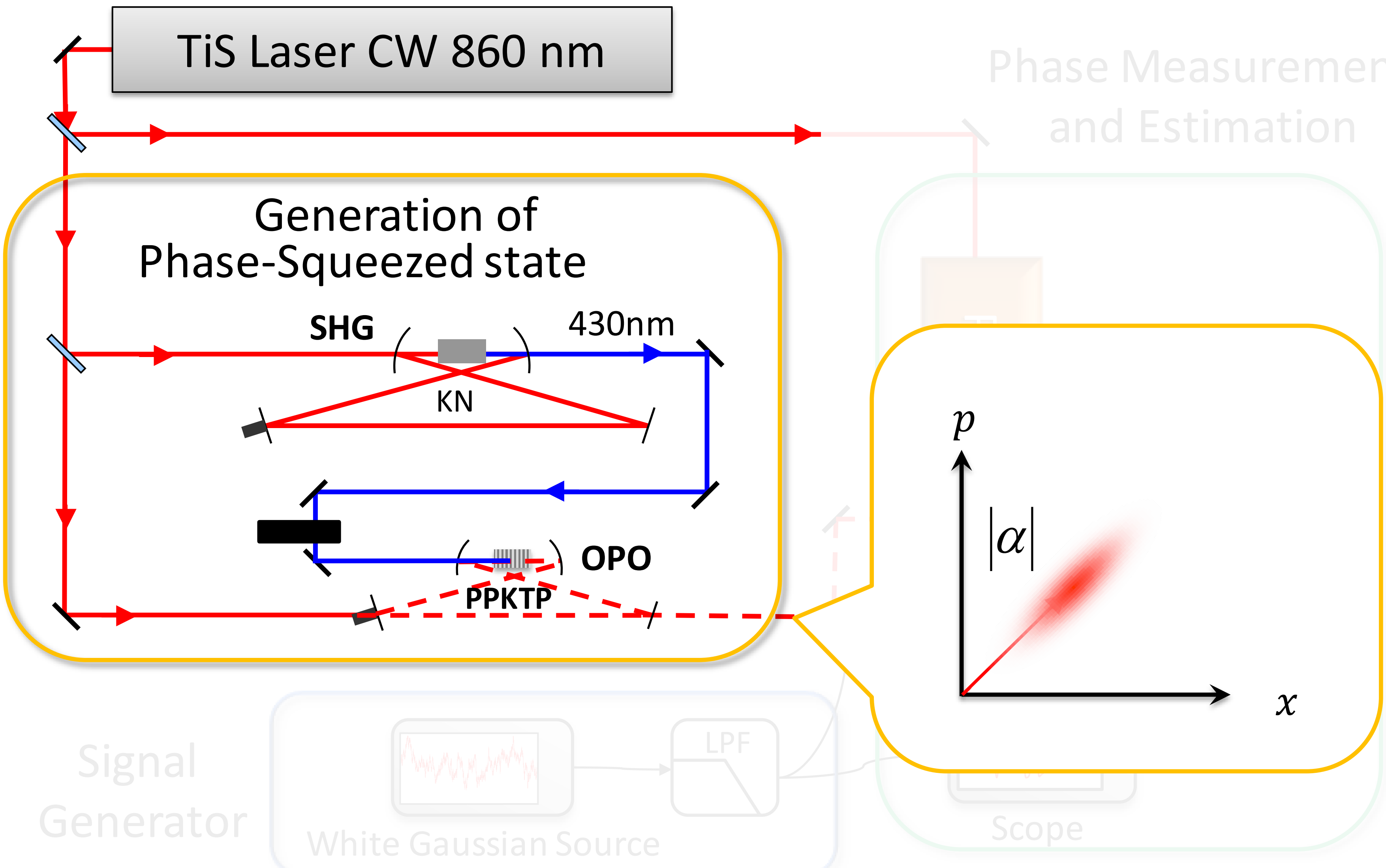


Experiment Setup

$\kappa = 3 \times 10^{9}\ \mathrm{N}^2\mathrm{sec}^{-1}$

Phase-Squeezed state

Pump beam power=80mW

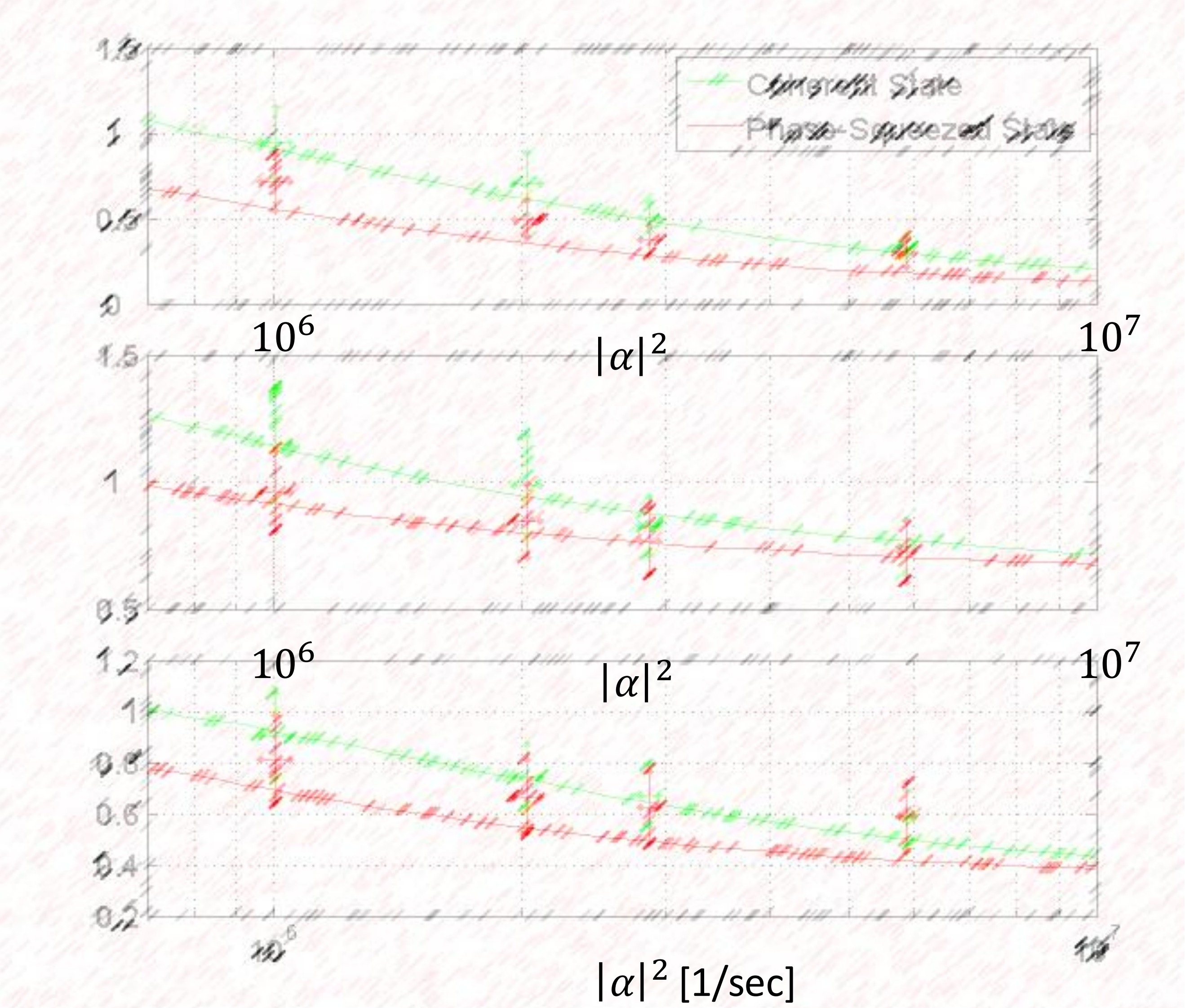


Results of Phase-Squeezed State

Position Variance [x10^{-16} m^2]

Momentum Variance [x10^{-12} kg^2m^2/s^2]

Applied Force Variance [x10^{-2} N^2]



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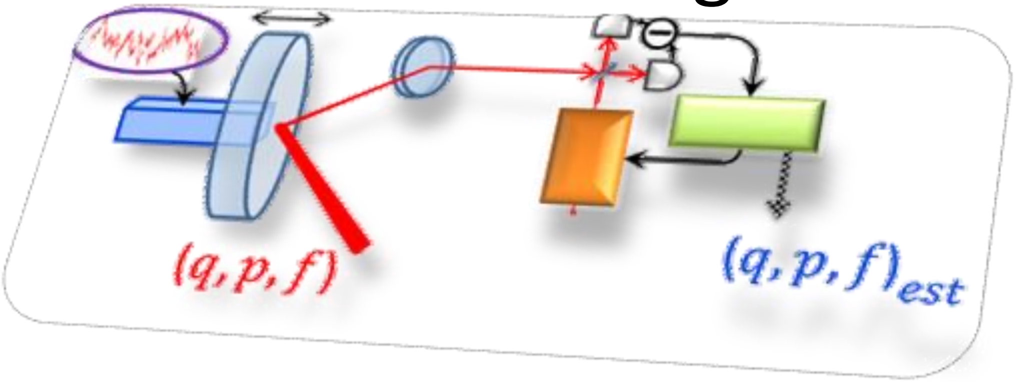
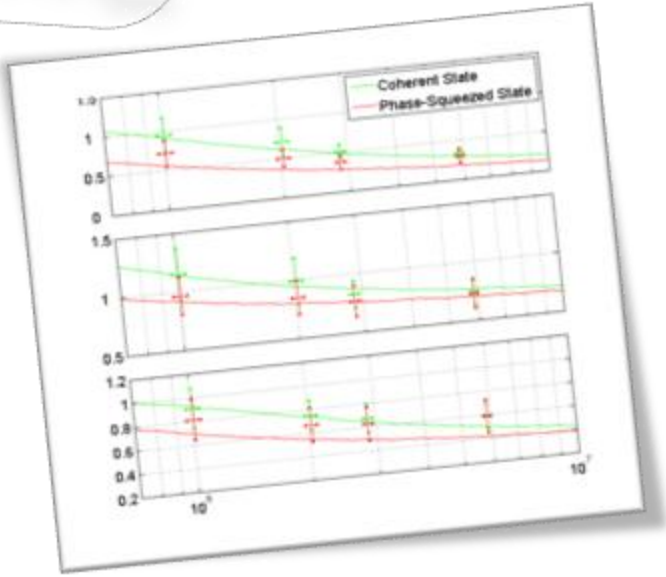
Summary

◆We demonstrated adaptive phase measurement for opto-mechanical sensing



◆We achieved better $(q,\, p,\, f\,)$ estimation with smoothing and by using phase-squeezed states



Thank you for your attention