Adaptive Mirror Motion Estimation using Phase-Squeezed States

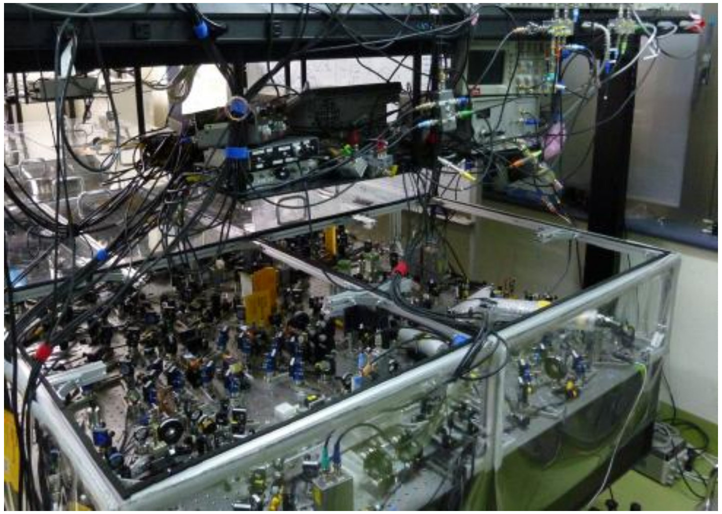
M2 Kohjiro Iwasawa

Furusawa/Yonezawa Lab.
Applied Phys. Engineering.
The Univ. of Tokyo

Who am I?

◆Former classmate of Takanori Sekiguchi

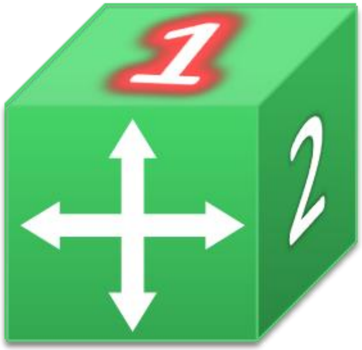
◆Former roomate of Keigo Takayama



◆I measure optical phase

Outline

1. Introduction
2. Theory
3. Experiment
4. Summary



What do I do?

What? Objective

- Measure optical phase in high precision

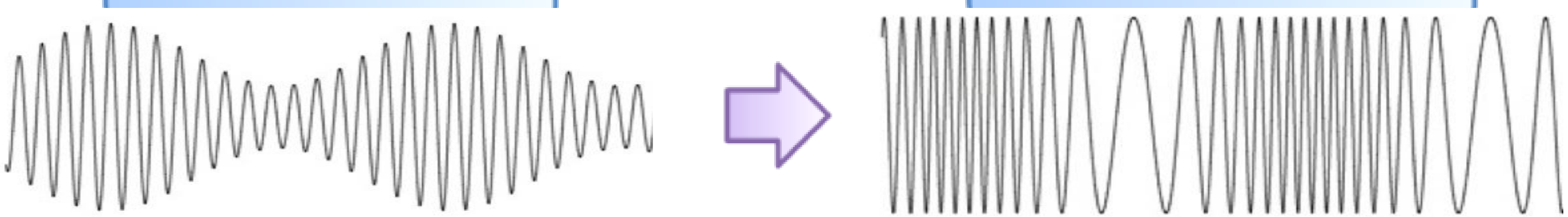
Gravitational Wave Detection?

Why? Applications

- Precision measurement
- Coherent optical communication

AM (Amplitude Modulation)

FM (Frequency Modulation)

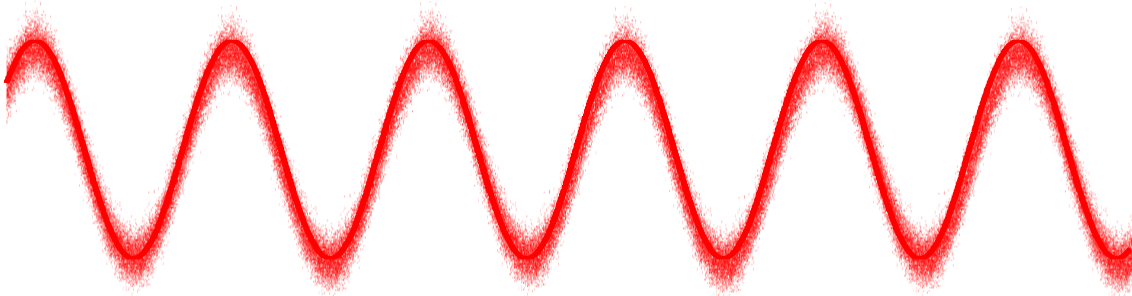


Introduction

◆Light

➢Amplitude $|\alpha|$

➢Phase φ



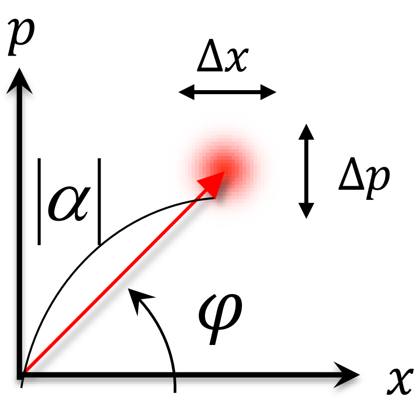
$$\mathbf{E} = \mathbf{E}_0 \alpha e^{-i\omega t}$$
$$\alpha = x + ip$$
$$\alpha = |\alpha| e^{i\varphi}$$

$$\hat{\mathbf{E}} = \mathbf{E}_0 \hat{a} e^{-i\omega t}$$
$$\hat{a} = \hat{x} + i\hat{p}$$

$\hat{x}, \hat{p}$: quadrature

$\hat{x}, \hat{p}$ are conjugates

$$[\hat{x}, \hat{p}] = \frac{i}{2} \quad \left(\hbar = \frac{1}{2}\right)$$

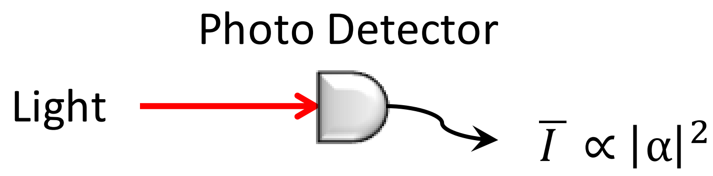


Heisenberg's uncertainty

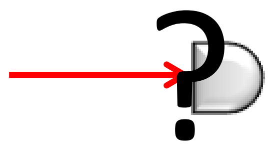
$$\Delta x \cdot \Delta p \geq \frac{1}{4}$$

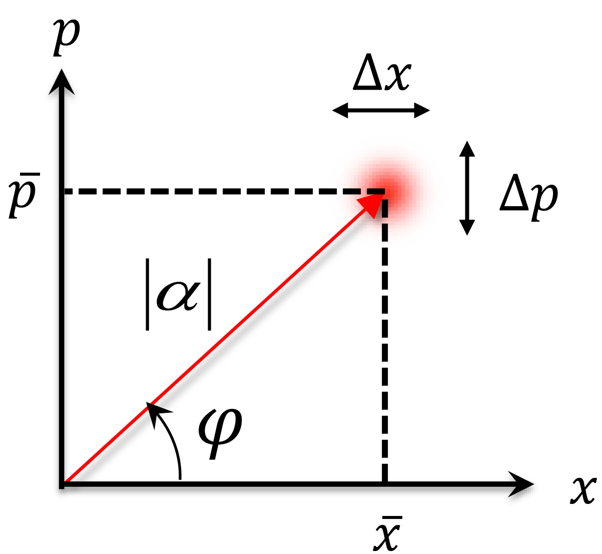
Optical Measurement

Measuring Amplitude



Measuring Phase

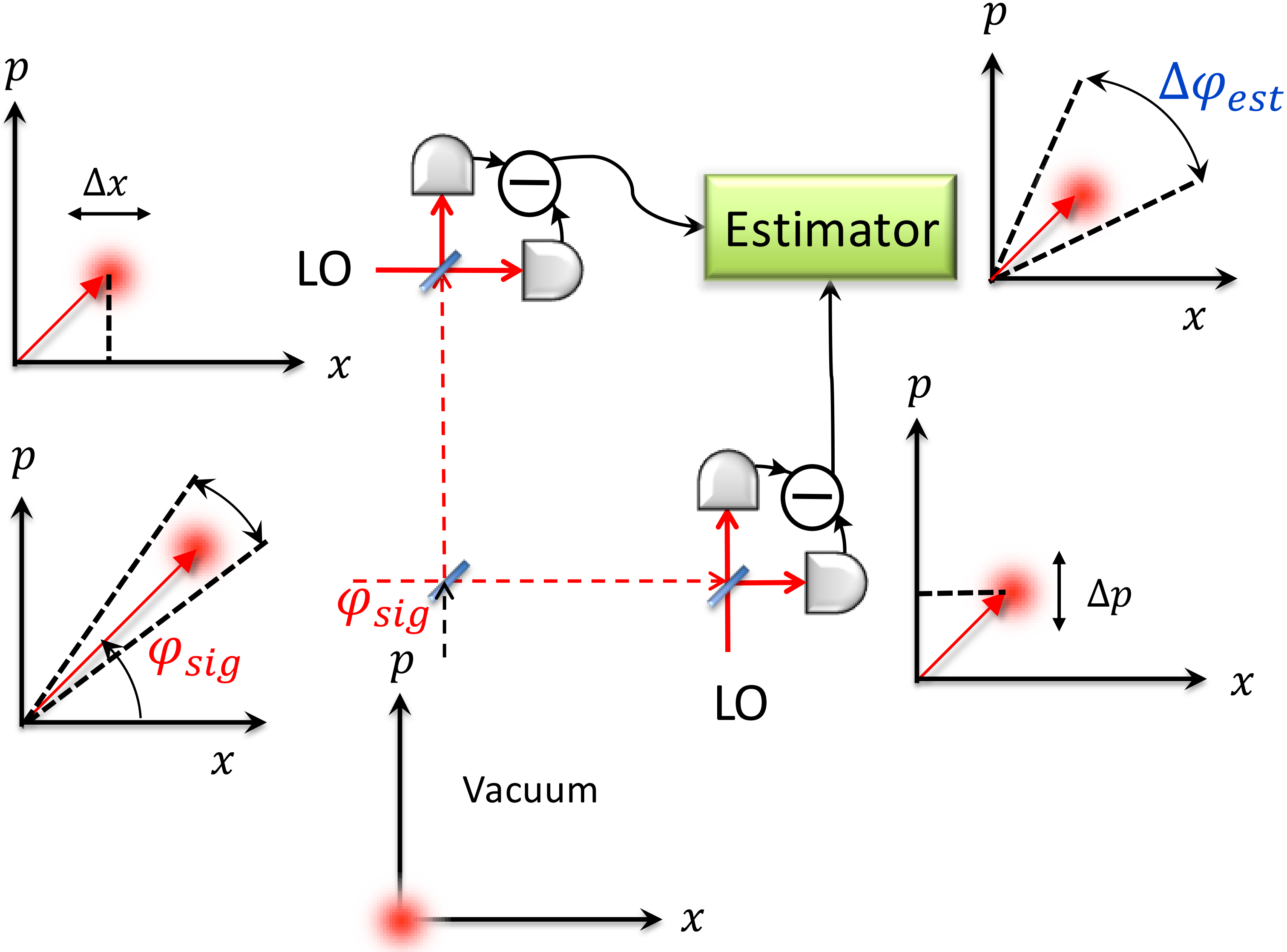




$\hat{x}, \hat{p}$ are conjugates

Impossible to measure $\hat{x}, \hat{p}$ simultaneously

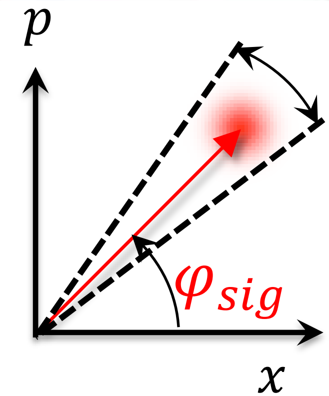
Optical Phase Measurement

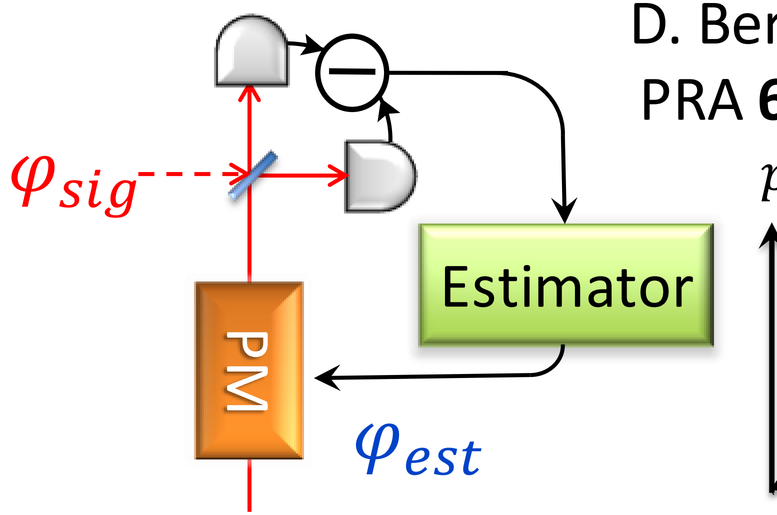


Kohjiro Iwasawa / Furusawa Lab. / Univ. of Tokyo

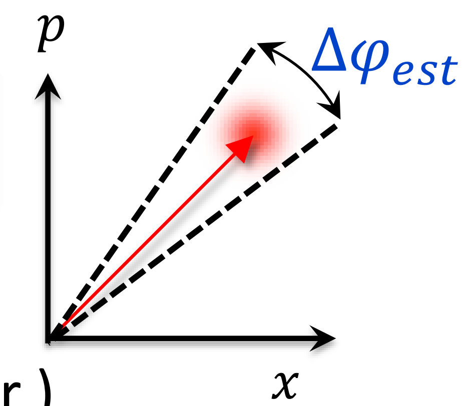
Adaptive Optical Phase Measurement

D. Berry & H. Wiseman, PRA **65** 043803 (2002).

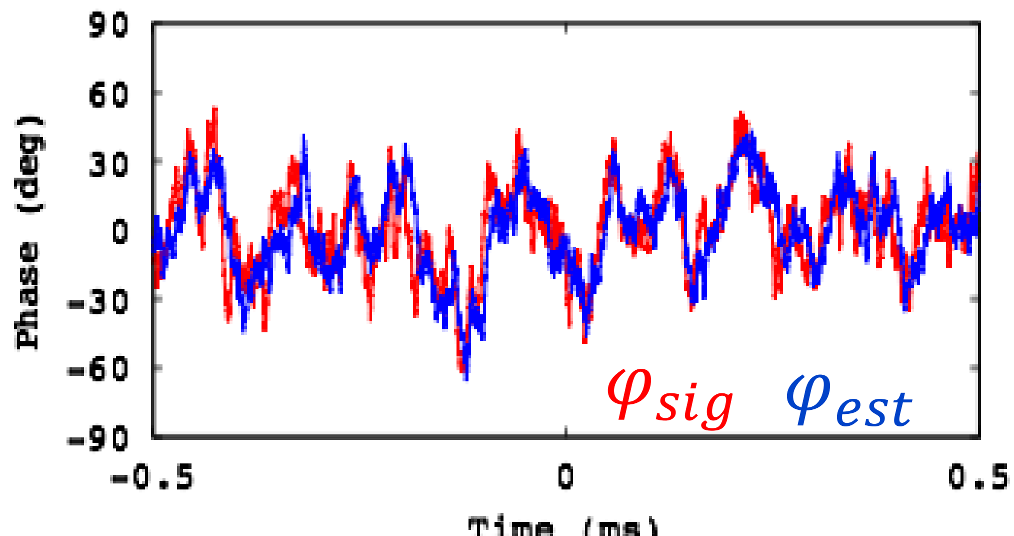




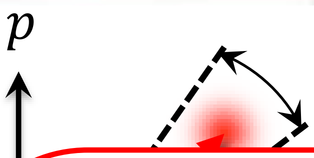
LO (Local Oscillator)

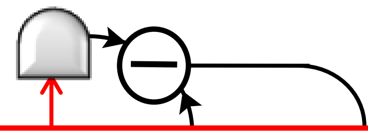


EOM
(Eletro-Optic Modulator)



Adaptive Optical Phase Measurement



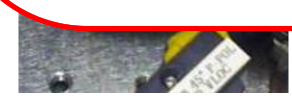


D. Berry & H. Wiseman,
PRA **65** 043803 (2002).

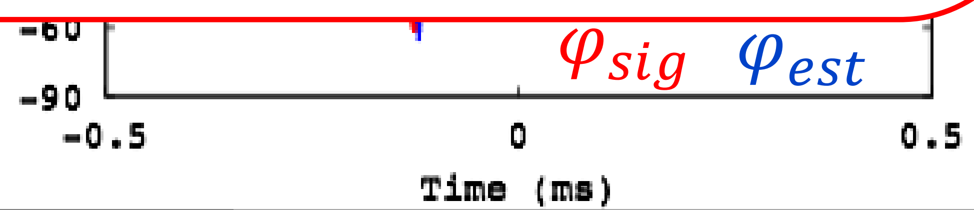
We have already demonstrated the superiority of adaptive optical phase measurement (for phase modulated by EOM)

PRL **104**, 093601 (2010).

Can we apply adaptive measurement to practical applications?



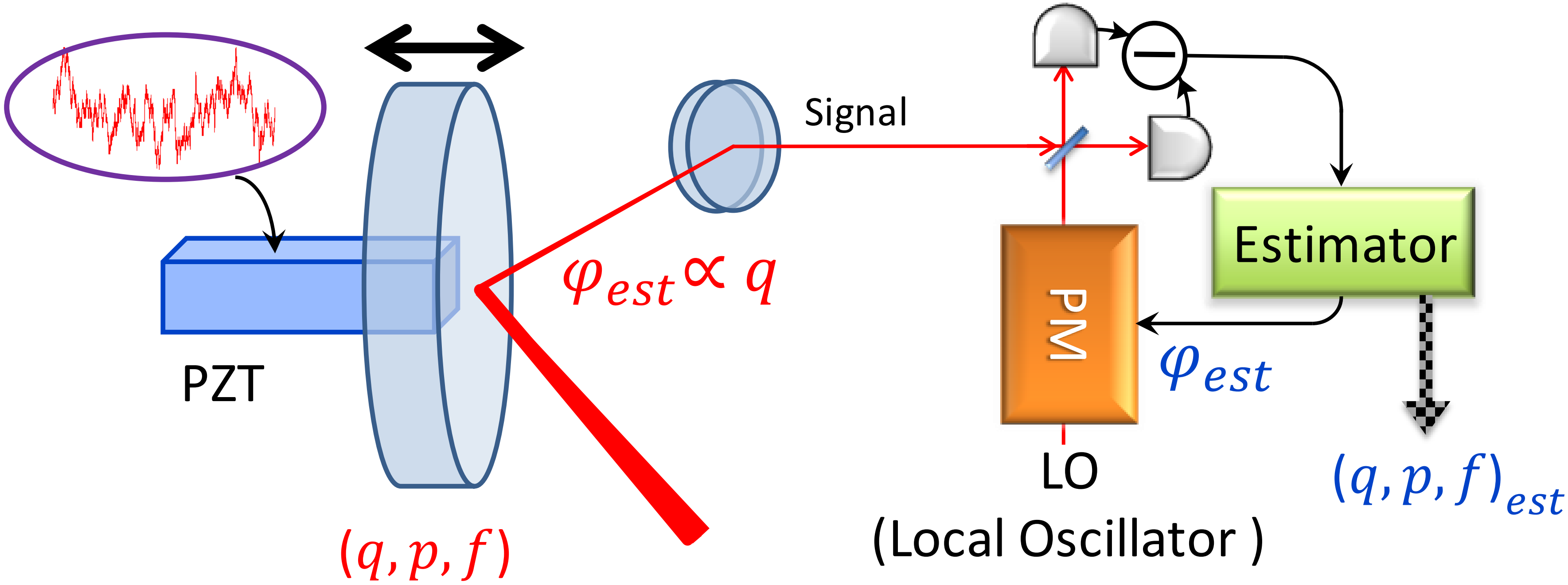
EOM
(Eletro-Optic Modulator)



Purpose

- Estimate Position, Momentum, and applied Force of a Mirror

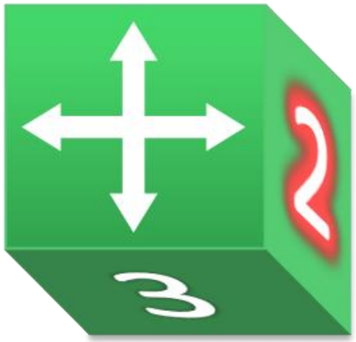
M. Tsang *et al*. PRA **79**, 053843 (2009).



Outline

1. Introduction

2. Theory
   - Adaptive optical phase measurement
   - Mirror motion model
   - Estimation method

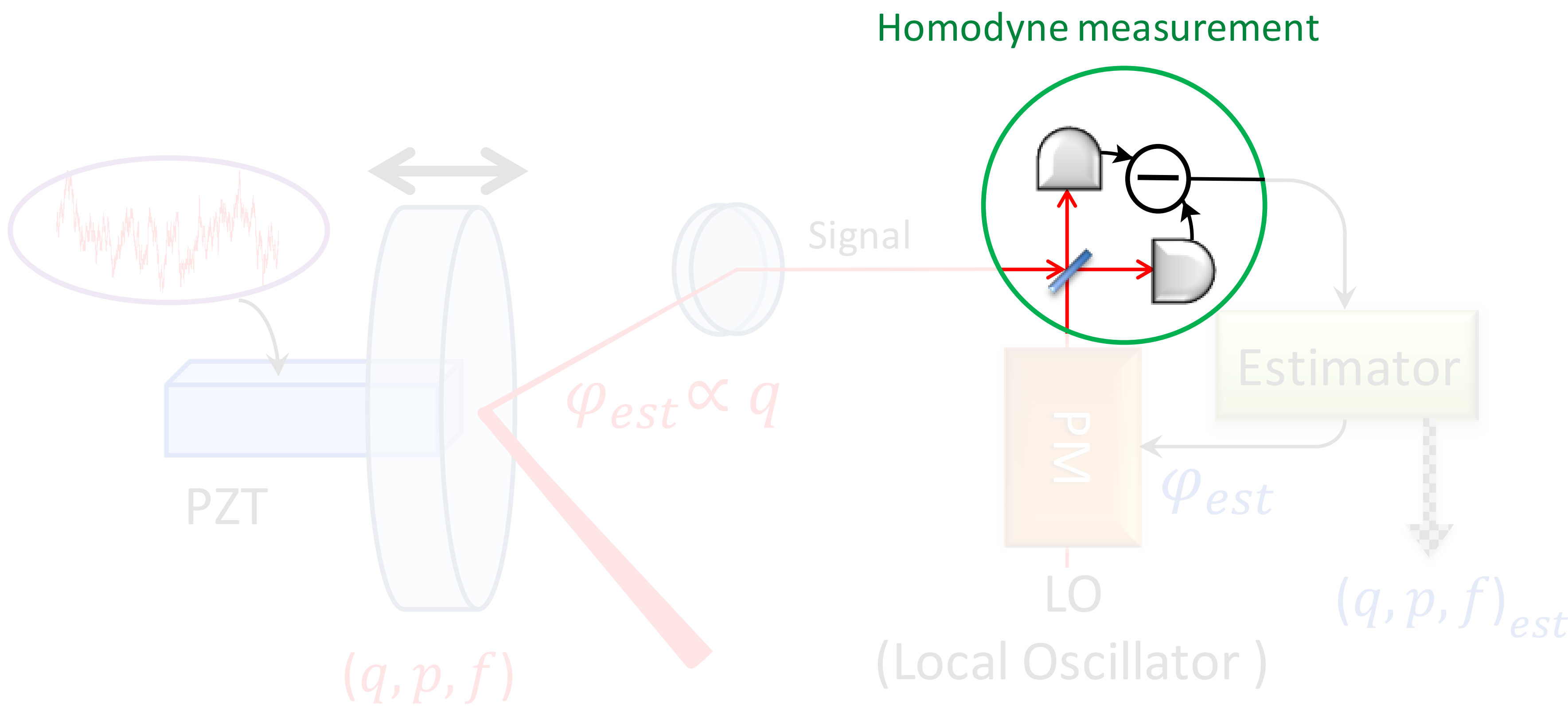


3. Experiment

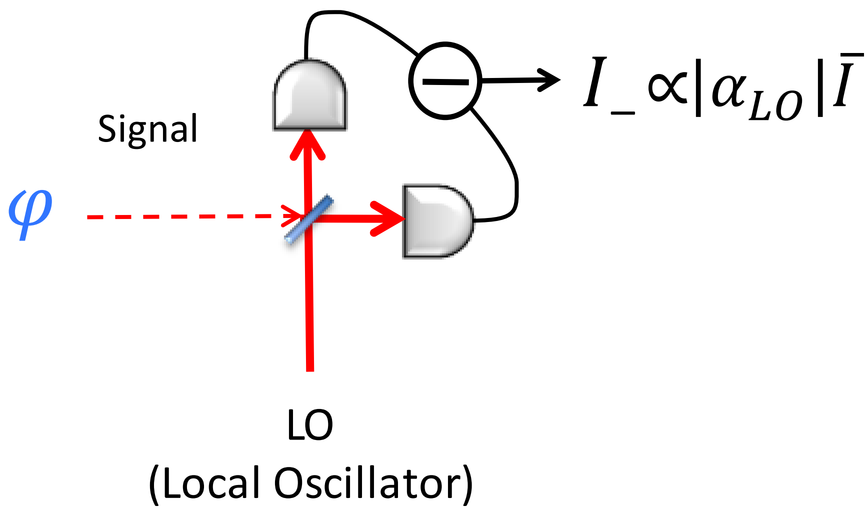
4. Summary

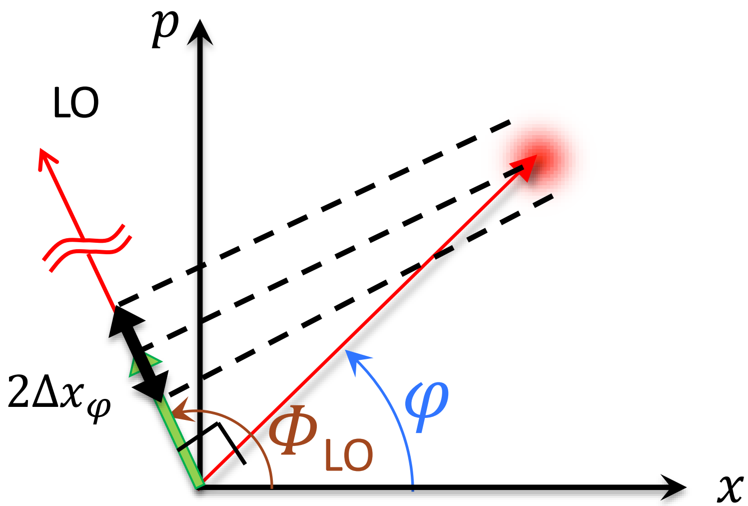
Purpose

◆Estimate Position, Momentum, and applied Force of a Mirror



Homodyne Measurement





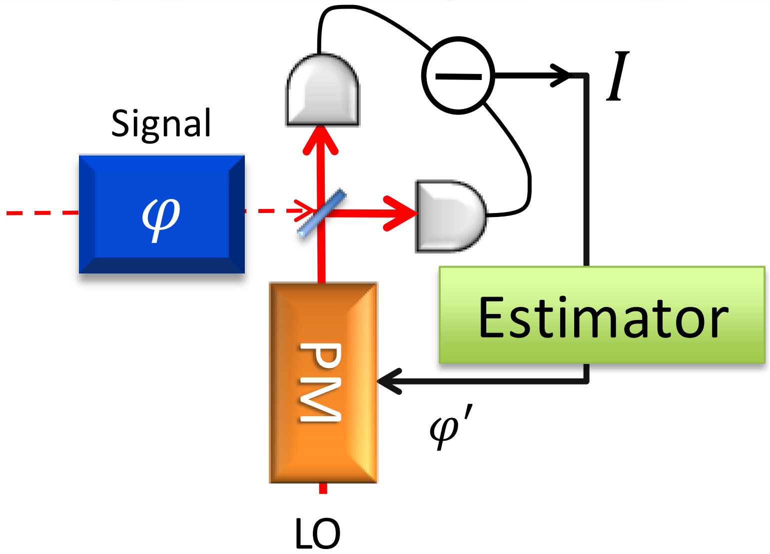
Most phase sensitive when

$$\varphi - \Phi_{LO} = \frac{\pi}{2}$$

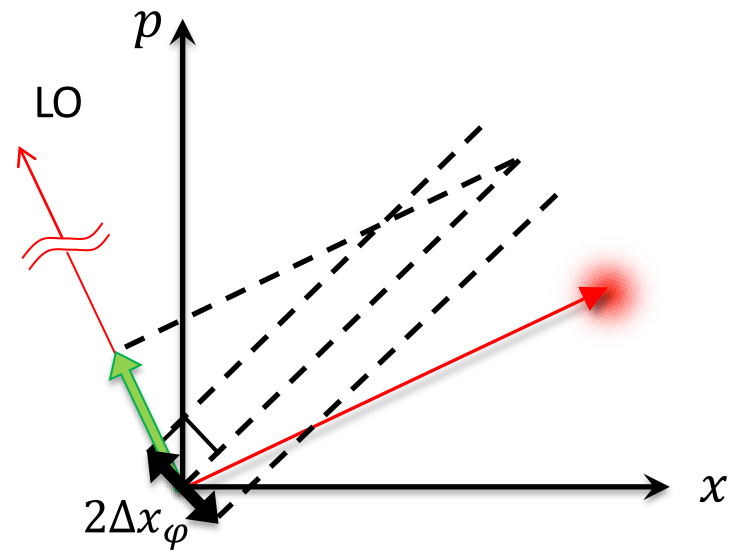
$$\bar{I} = 2|\alpha|\cos(\varphi - \Phi_{LO})$$

$$\Delta I^2 = \Delta x_{\varphi}{}^2$$

Adaptive Homodyne Measurement



PM: Phase Modulator
LO: Local Oscillator



$$\Phi_{LO} = \varphi' + \frac{\pi}{2}$$

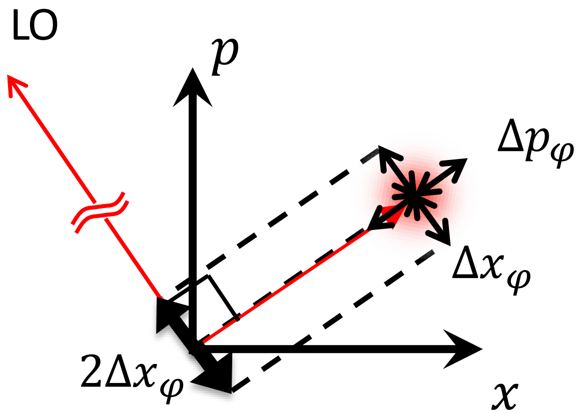
$$\bar{I} = 2|\alpha|\cos(\varphi - \Phi_{LO})$$

$$\Delta I^2 = \Delta \mathrm{x}_{\varphi}{}^2$$

$$\bar{I} = 2|\alpha|\sin(\varphi - \varphi') \approx 2|\alpha|(\varphi - \varphi')$$

$$\Delta I^2 = \Delta \mathrm{x}_{\varphi}{}^2$$

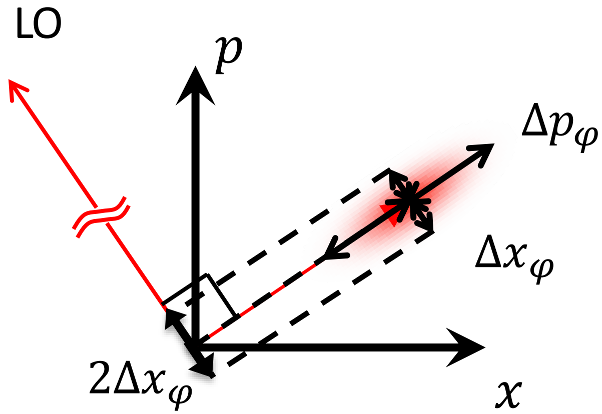
Phase-Squeezed State



Coherent state

$$\Delta x_\varphi \cdot \Delta p_\varphi = \frac{1}{4}$$

$$\Delta {x_\varphi}^2 = \Delta {p_\varphi}^2 = \frac{1}{4}$$



Phase-squeezed state

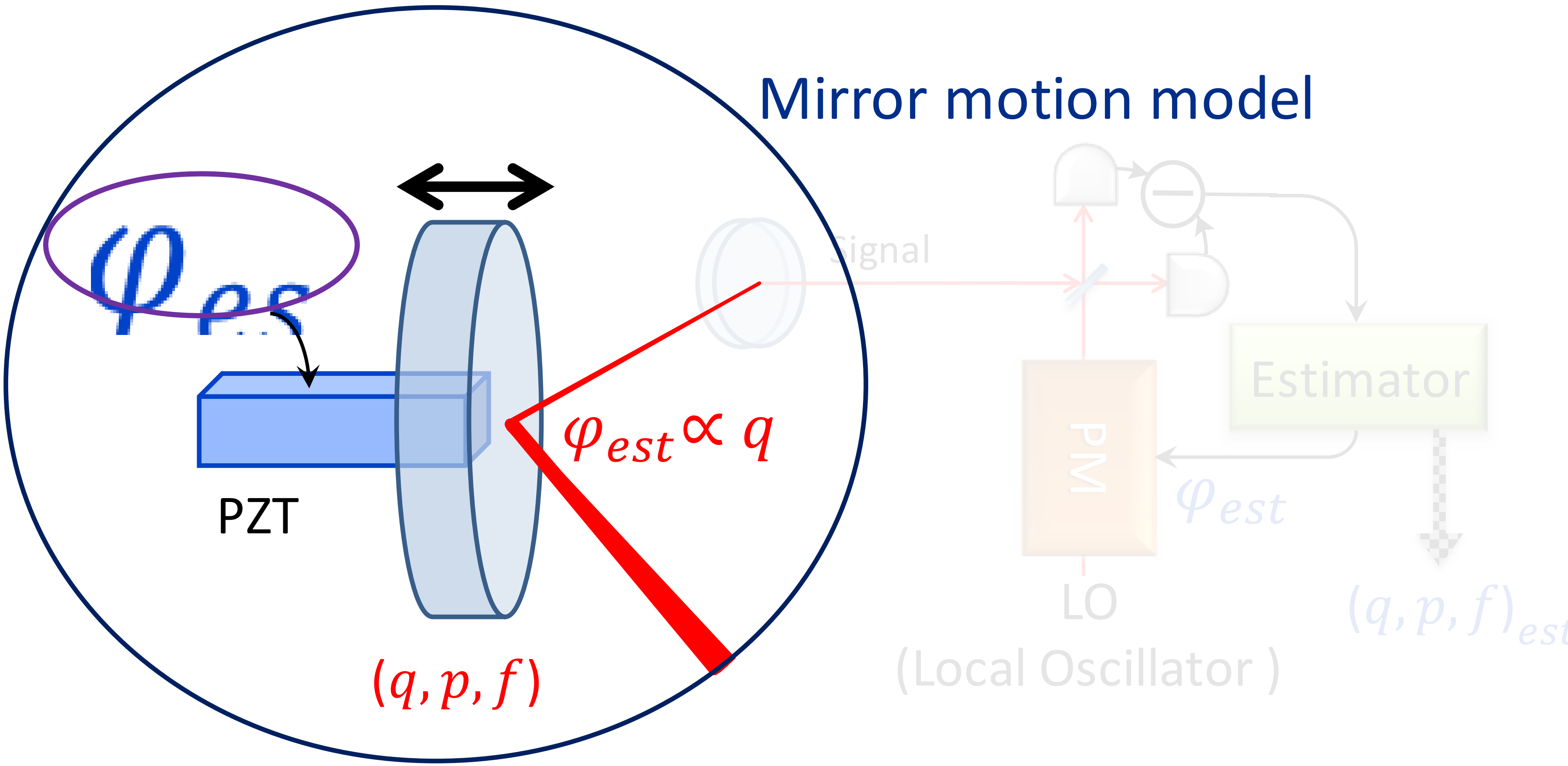
$$\Delta x_\varphi \cdot \Delta p_\varphi = \frac{1}{4}$$

$$\Delta {x_\varphi}^2 = \frac{1}{4} e^{-2r}, \quad \Delta {p_\varphi}^2 = \frac{1}{4} e^{+2r}$$

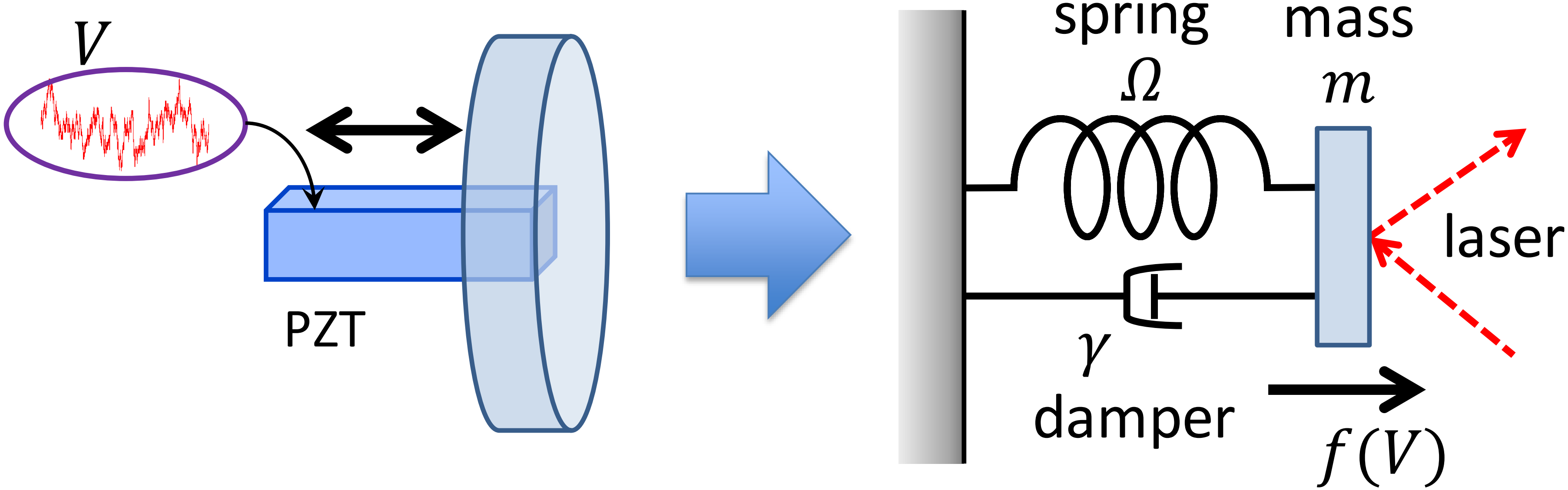
Purpose

◆Estimate Position, Momentum, and applied Force of a Mirror

Mirror motion model



Mirror Motion Model



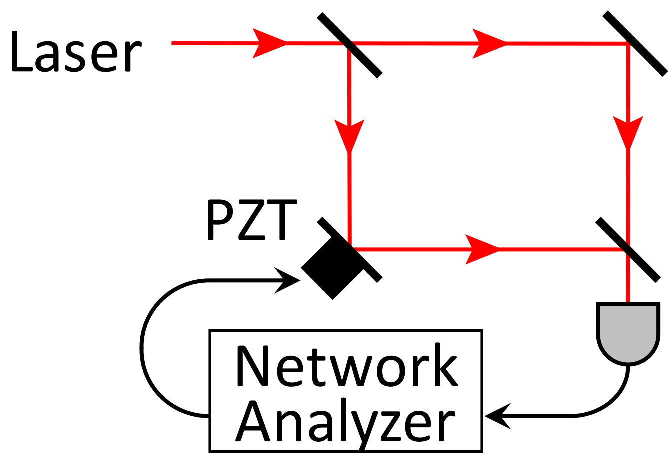
Spring-mass-damper model

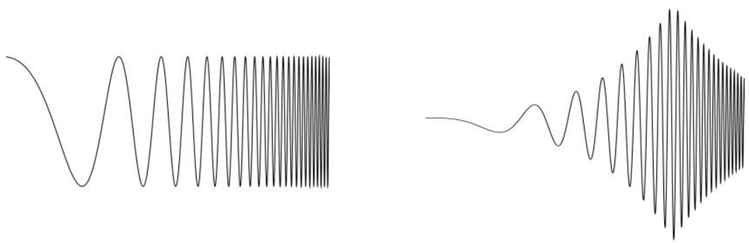
⇒Motion Equation:

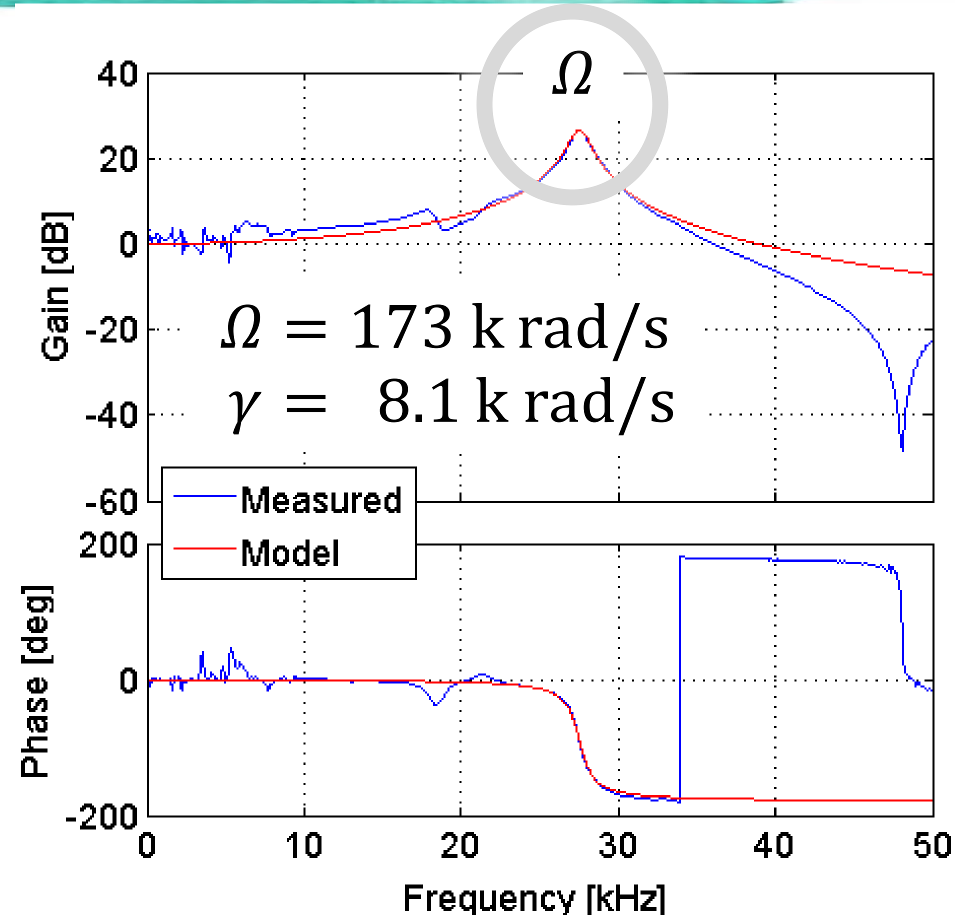
$$\begin{cases} \dfrac{dp}{dt} = -m\Omega^2 q - \gamma p + f \\ f = \beta V \end{cases}$$

$$m = 5.88 \times 10^{-4}\ [\mathrm{kg}]$$
$$\beta = 1.99 \times 10^{-1}\ [\mathrm{N/V}]$$

Measurement of Model Parameter



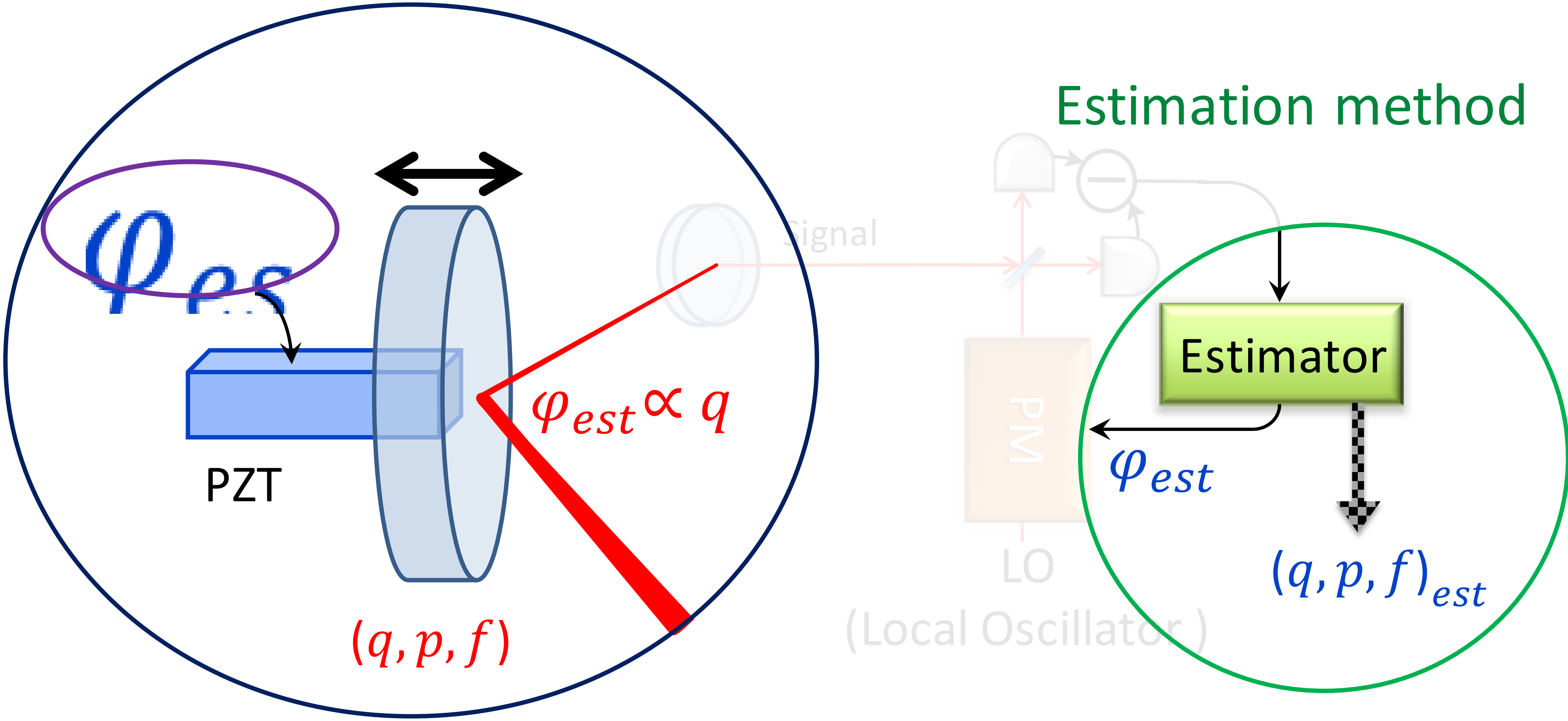




Use the Measured Transfer function and Voltage applied to PZT to compare estimated (q, p, f) to the true values

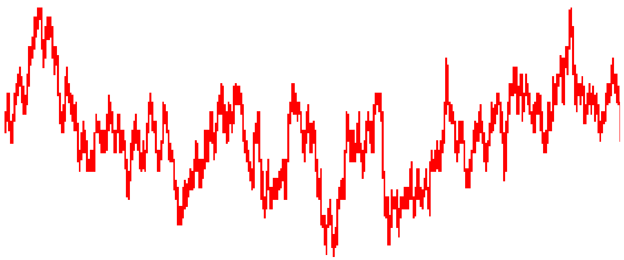
Purpose

◆Estimate Position, Momentum, and applied Force of a Mirror



Estimation Method

◆Applied force: random walk bound in zero



$$\frac{df}{dt} = -\lambda f + \sqrt{\kappa}\underline{w(t)}$$

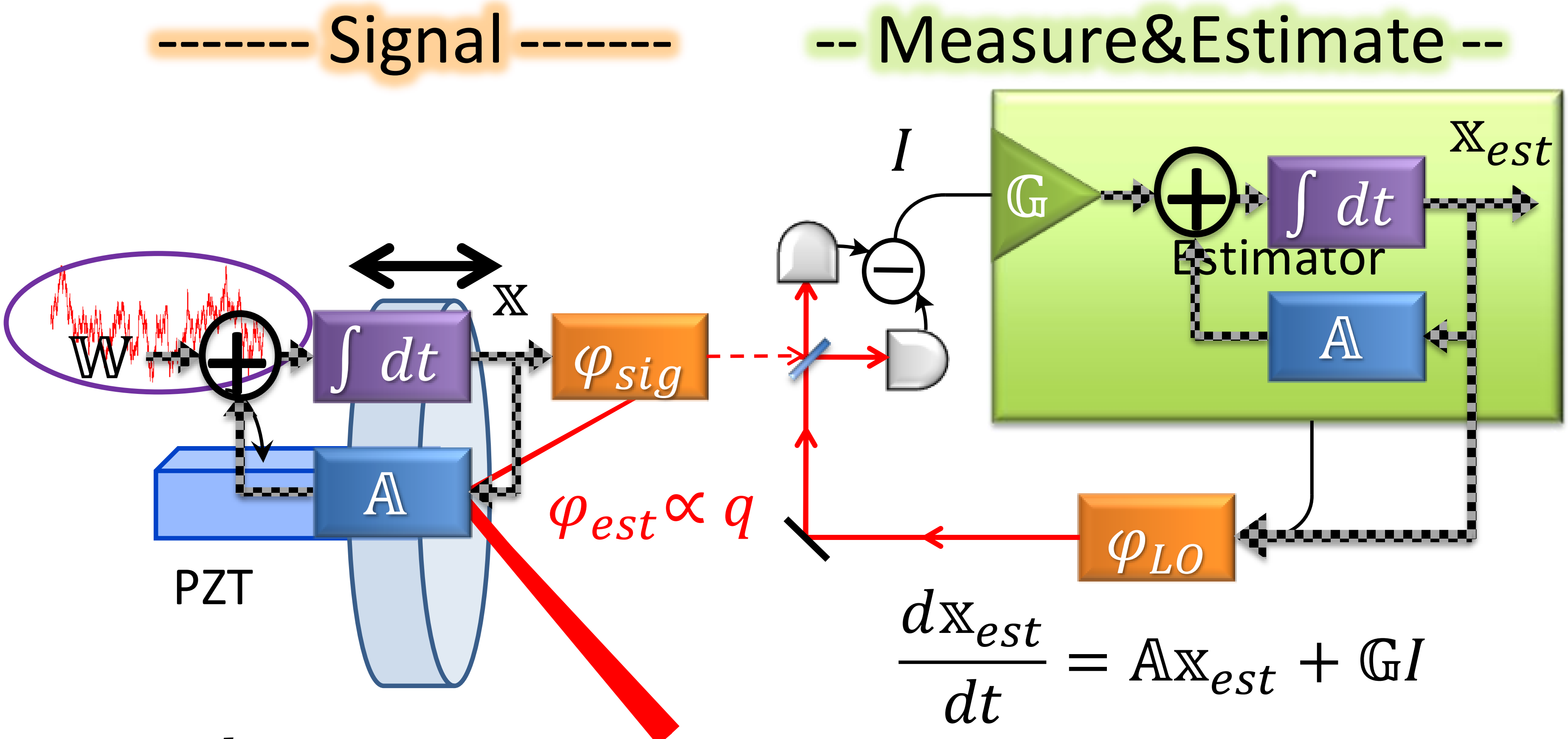
White Gaussian Noise

◆Time evolution of the system

$$\frac{d}{dt}\begin{pmatrix} q \\ p \\ f \end{pmatrix} = \begin{pmatrix} 0 & 1/m & 0 \\ -m\Omega^2 & -\gamma & 1 \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} q \\ p \\ f \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \sqrt{\kappa}w(t) \end{pmatrix}$$

$$\begin{pmatrix} q \\ p \\ f \end{pmatrix} \equiv \mathbb{x}, \quad \begin{pmatrix} 0 & 1/m & 0 \\ -m\Omega^2 & -\gamma & 1 \\ 0 & 0 & -\lambda \end{pmatrix} \equiv \mathbb{A}, \quad \begin{pmatrix} 0 \\ 0 \\ \sqrt{\kappa}w(t) \end{pmatrix} \equiv \mathbb{W}$$

Optimal Feedback Filter(1)



$$\frac{d\mathbb{x}_{est}}{dt} = \mathbb{A}\mathbb{x}_{est} + \mathbb{G}I$$

$$\frac{d\mathbb{x}}{dt} = \mathbb{A}\mathbb{x} + \mathbb{W}$$

Optimal Feedback Filter(2)

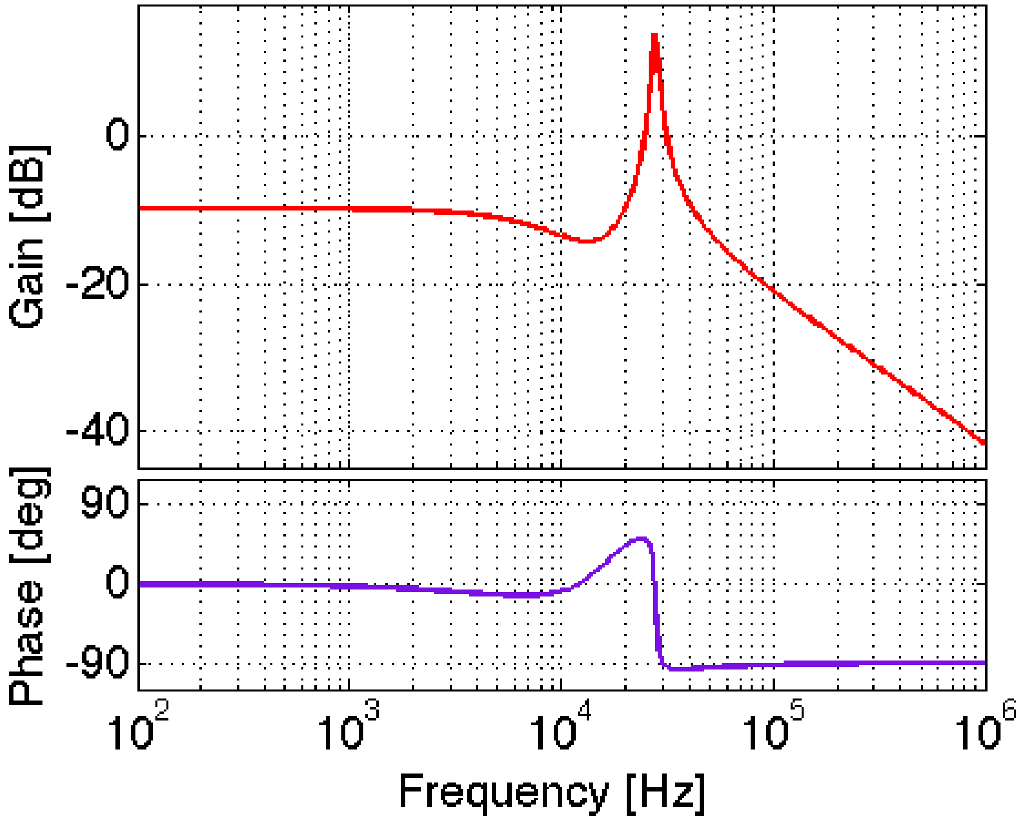
$$\frac{d\mathbb{x}_{est}}{dt} = \mathbb{A}\mathbb{x}_{est} + \mathbb{G}I$$

Fourier Transform

$$\mathbb{x}_{est} = (j\omega\mathbb{I} - \mathbb{A})^{-1}\mathbb{G}I$$

- Position
- Momentum
- Force

Estimator of Position

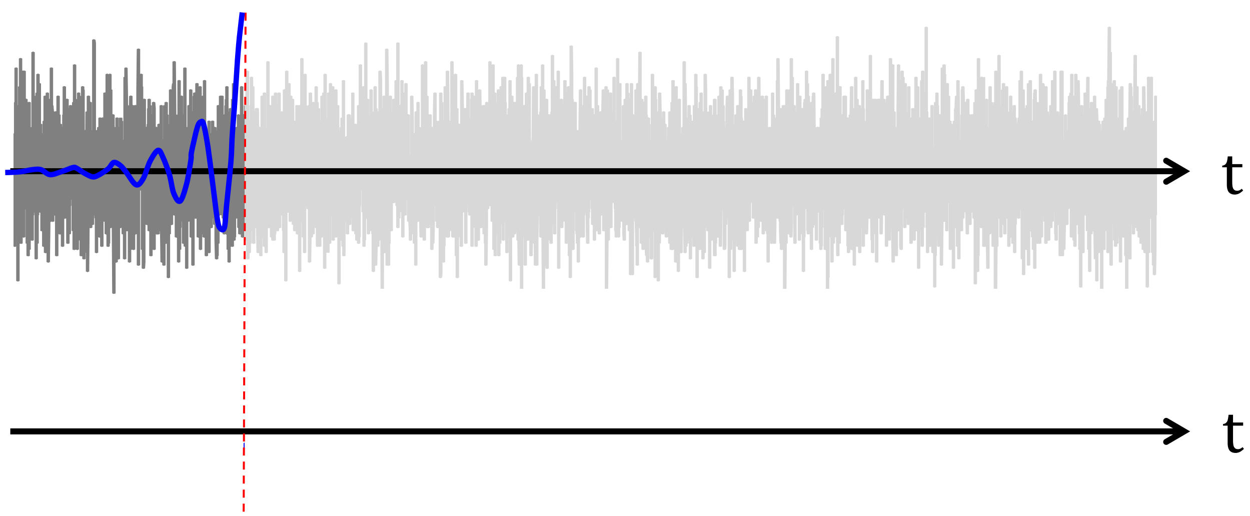


Optimal Feedback Filter(3)

$$\mathbb{x}_{est} = (j\omega\mathbb{I} - \mathbb{A})^{-1}\mathbb{G}I$$

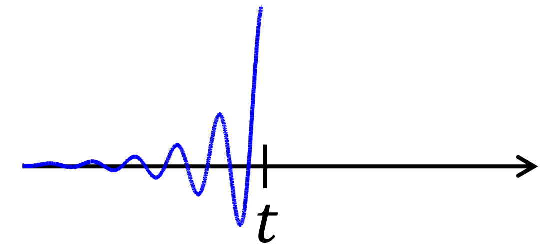
Inverse Fourier Transform

$$\mathbb{x}_{est}(t) = \int_{t_0}^{t} \mathbb{H}(t)\, I(t-\tau)$$

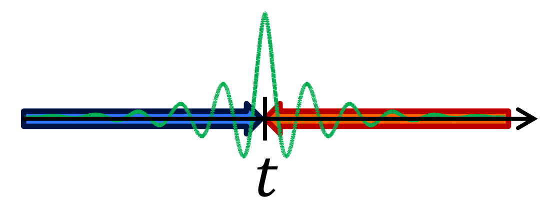


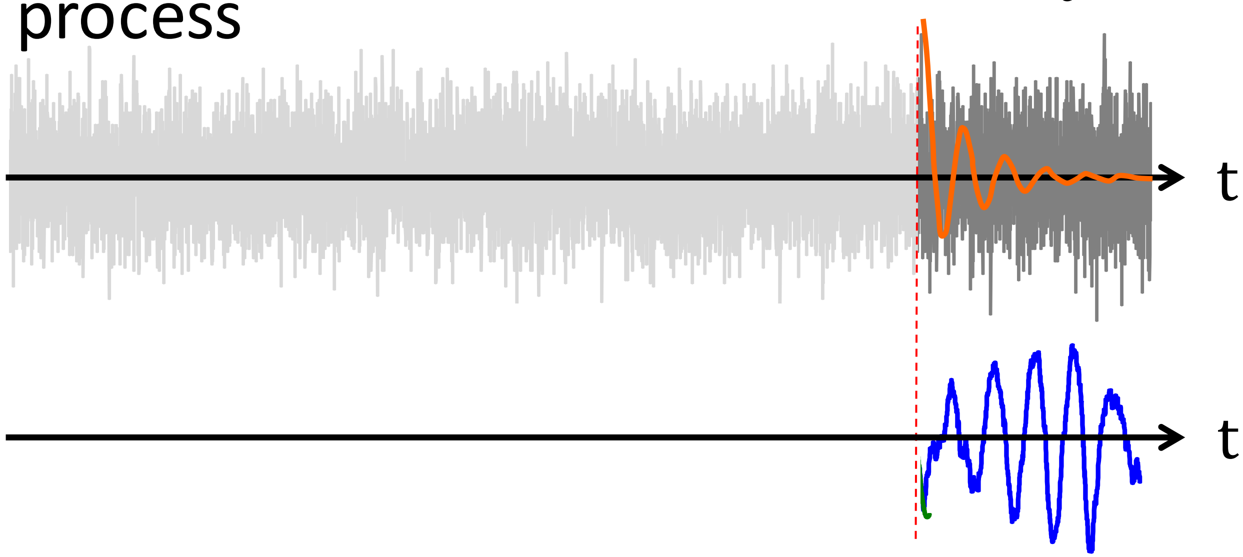
Estimation Method

- ◆Filtering (only past)
  - ○ real time
  - × less precise



- ◆Smoothing (past and future)
  - ○ more precise
  - × post process





Outline

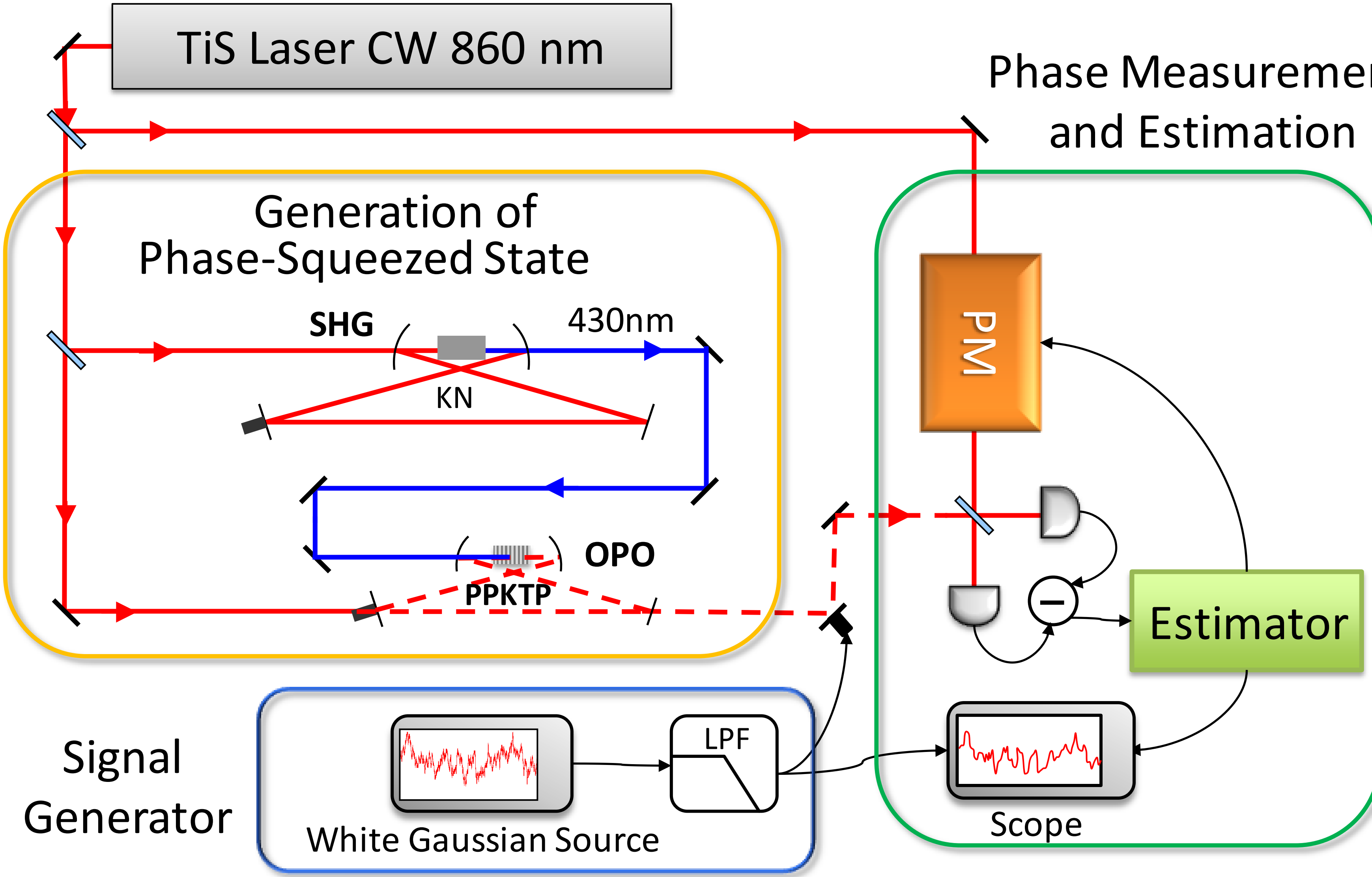
1. Introduction

2. Theory

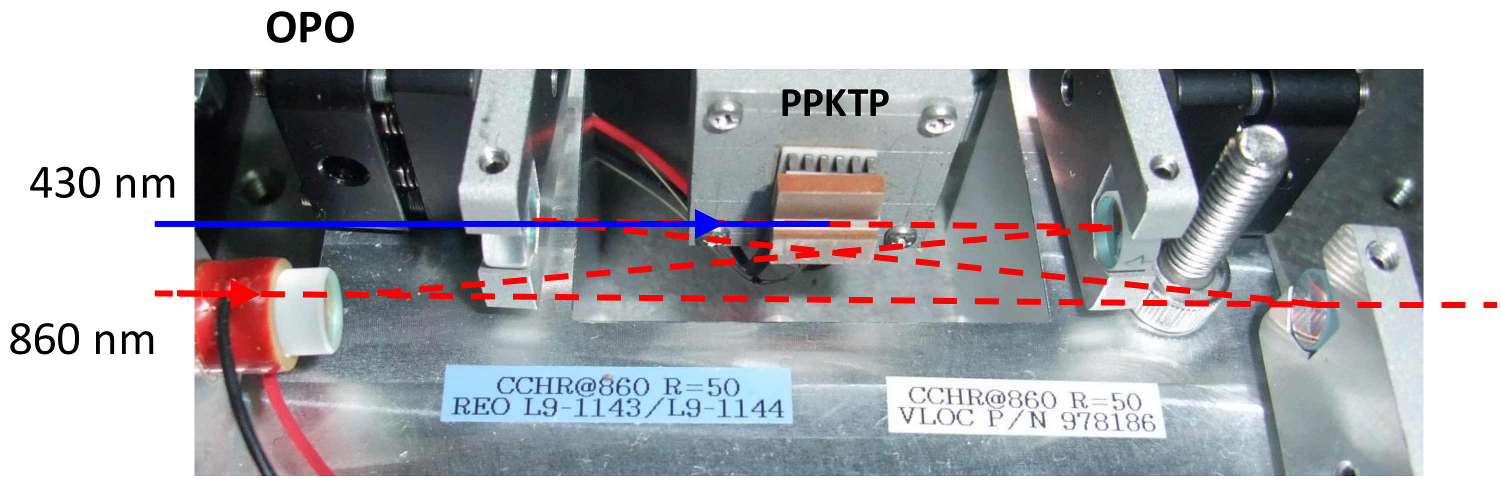
3. Experiment
   - κ dependence
   - $|\alpha|^2$ dependece
   - Phase-Squeezed State

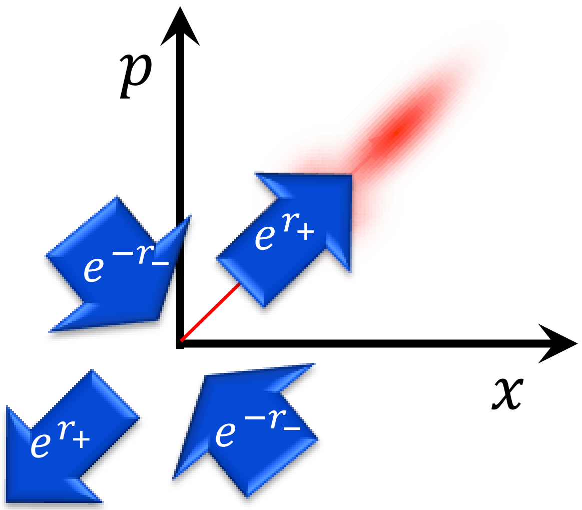
4. Summary

Experiment Setup



Generate Phase-Squeezed States



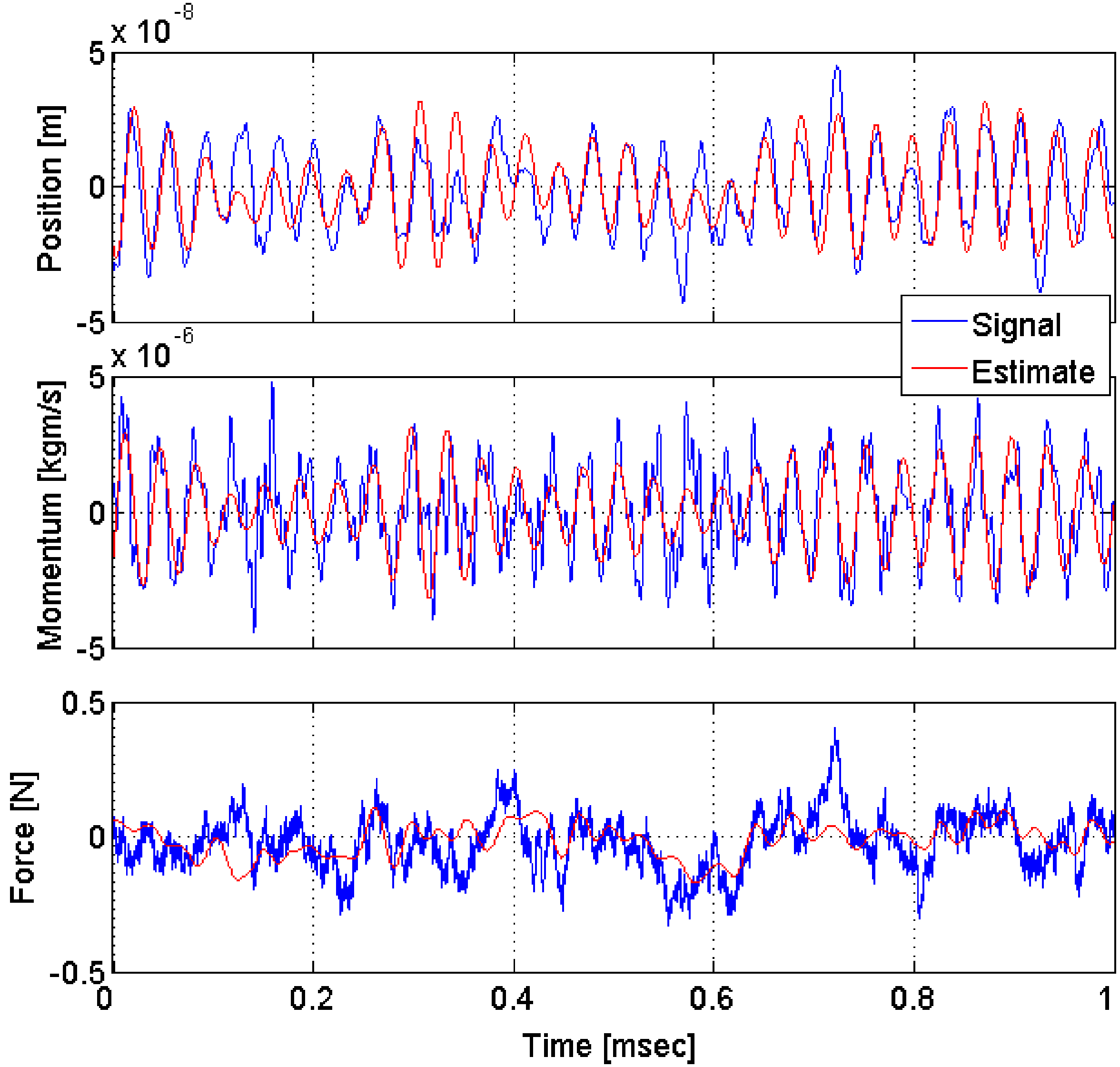


Squeeze Level

$e^{-2r} = 0.47$(-3.3 dB)

OPO: Optical Parametric Oscillator
PPKTP:Periodically Polled KTP

Time Domain Results



$|\alpha|^2 = 1 \times 10^6\ \mathrm{sec}^{-1}$

$\kappa \quad = 3 \times 10^9\ \mathrm{N^2 sec^{-1}}$

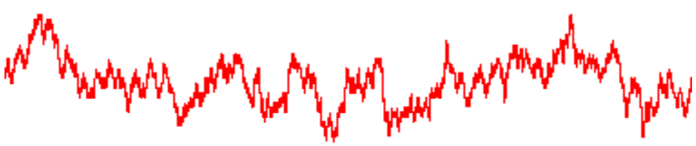
Coherent state

Smoothing method

Experiment Setup

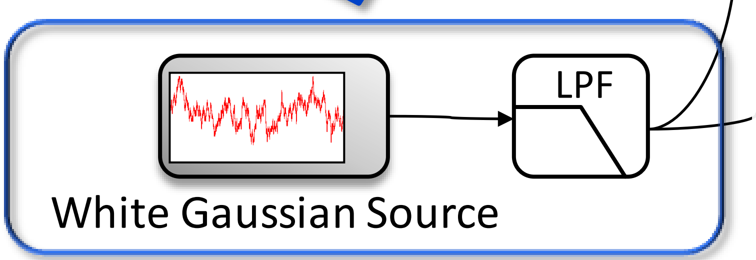
$|\alpha|^2 = 1 \times 10^6\ \mathrm{sec}^{-1}$
Coherent state

$$\frac{df}{dt} = -\lambda f + \sqrt{\kappa} w(t)$$



λ = 58 k rad/s

Signal Generator

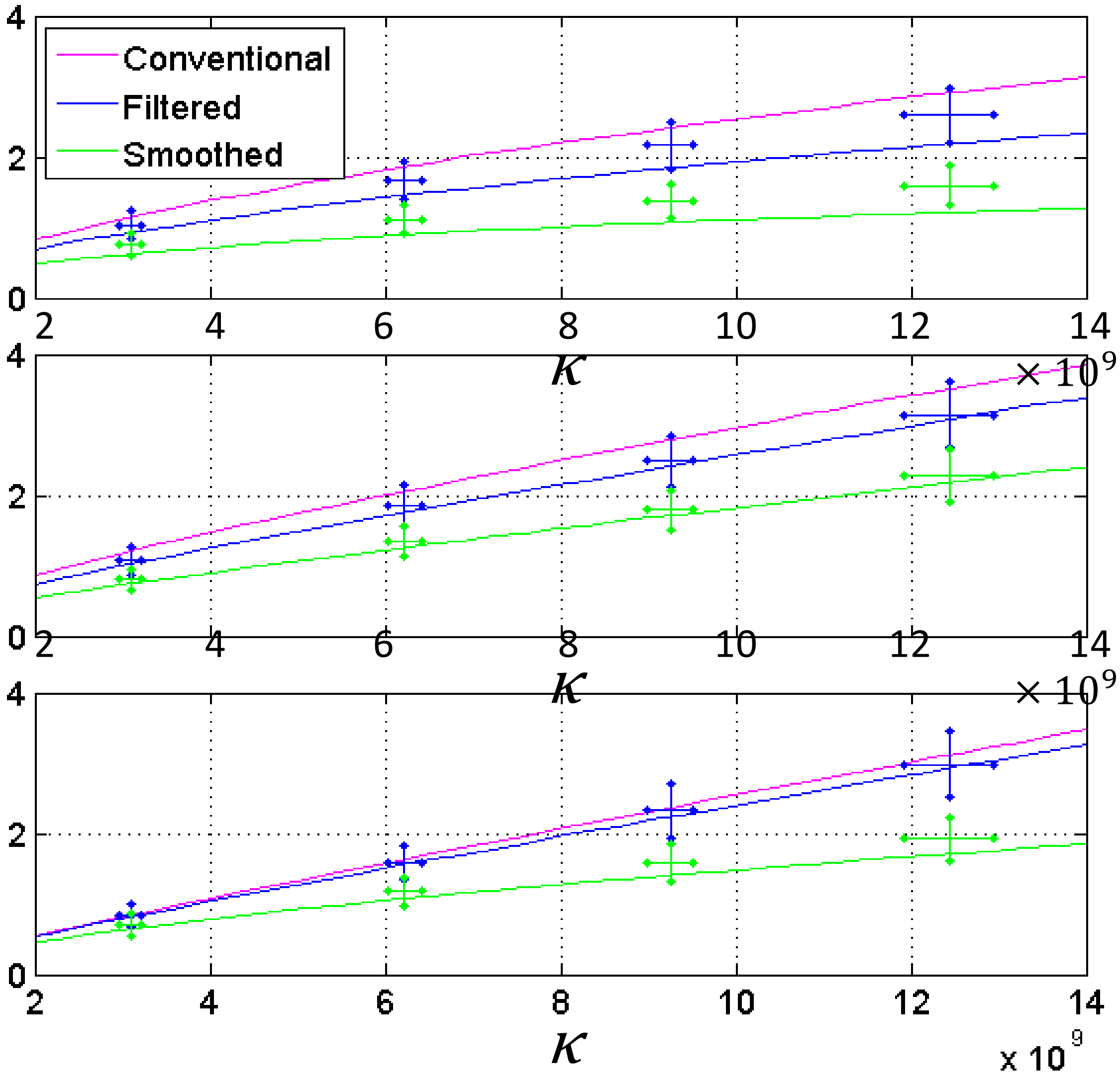


Signal-Amplitude (κ) Dependence

Position Variance [x10^{-16} m^2]

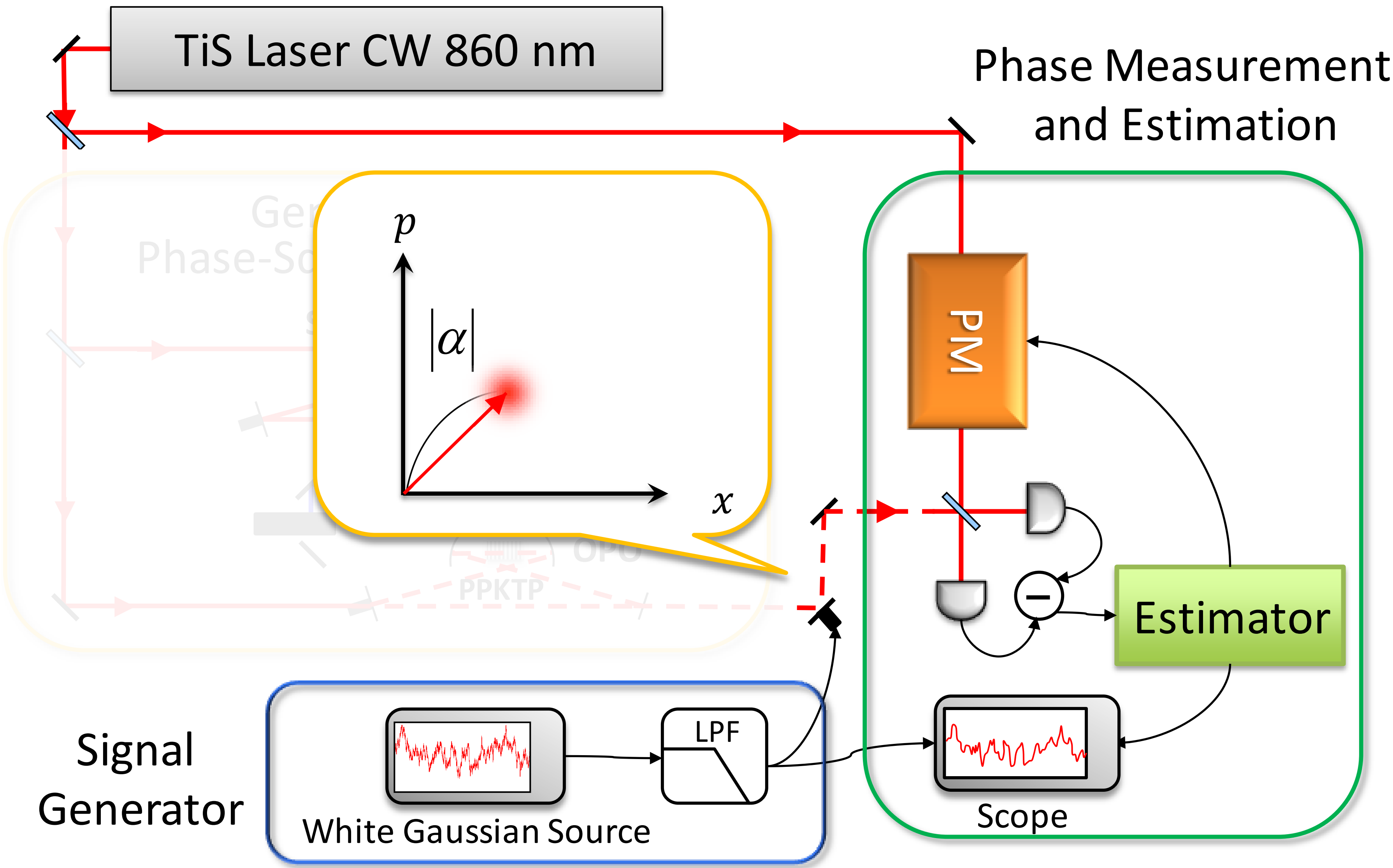
Momentum Variance [x10^{-12} kg^{2}m^2/s^2]

Applied Force Variance [x10^{-2} N^2]



Experiment Setup

$\kappa = 3 \times 10^9\ \mathrm{N}^2\mathrm{sec}^{-1}$
Coherent state

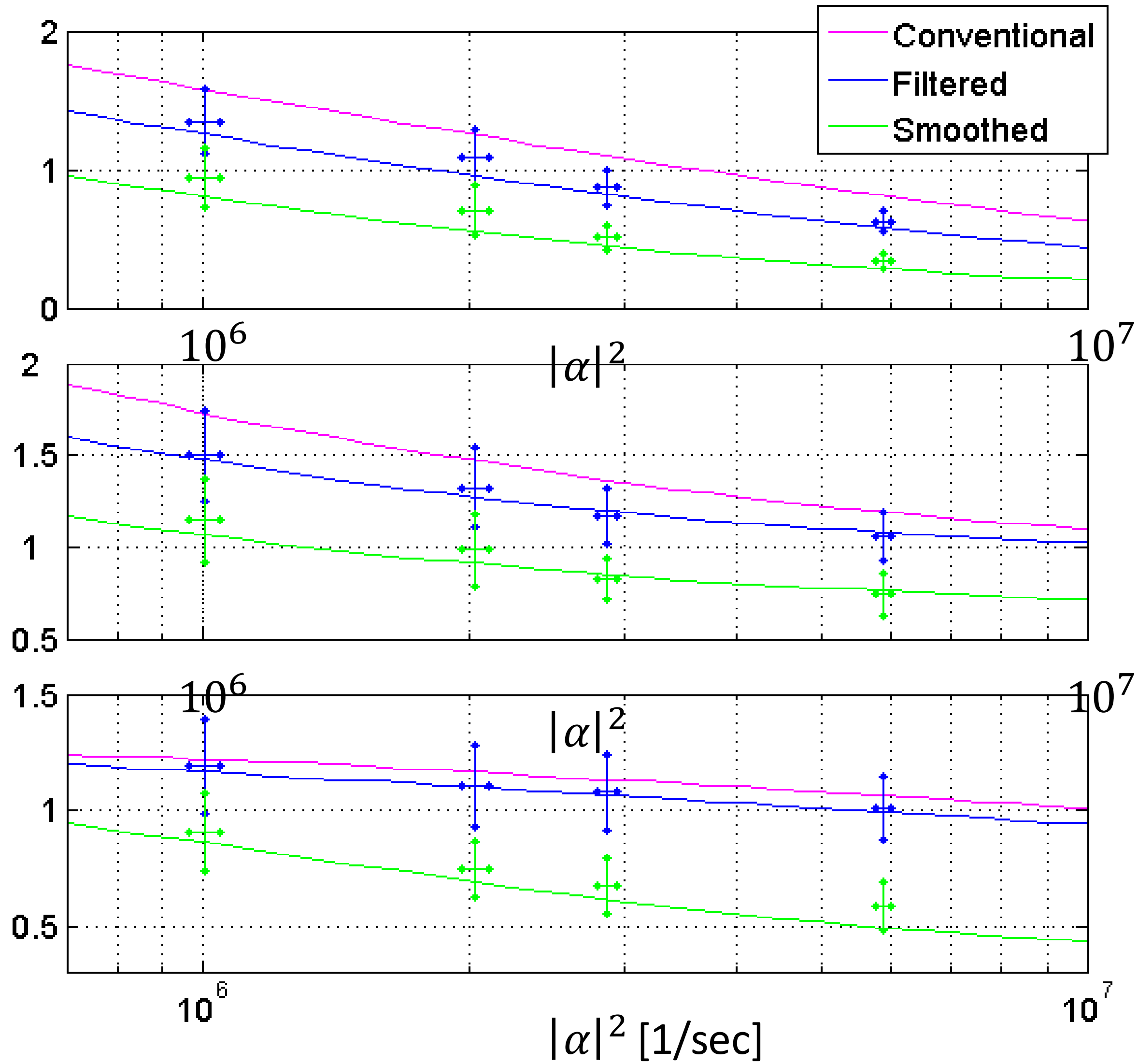


$|\alpha|^2$ Dependence

Position Variance $[\times 10^{-16}\ m^2]$

Momentum Variance $[\times 10^{-12}\ kg^2 m^2/s^2]$

Applied Force Variance $[\times 10^{-2}\ N^2]$



Kohjiro Iwasawa / Furusawa Lab. / Univ. of Tokyo

Outline

1. Introduction

2. Theory

3. Experiment

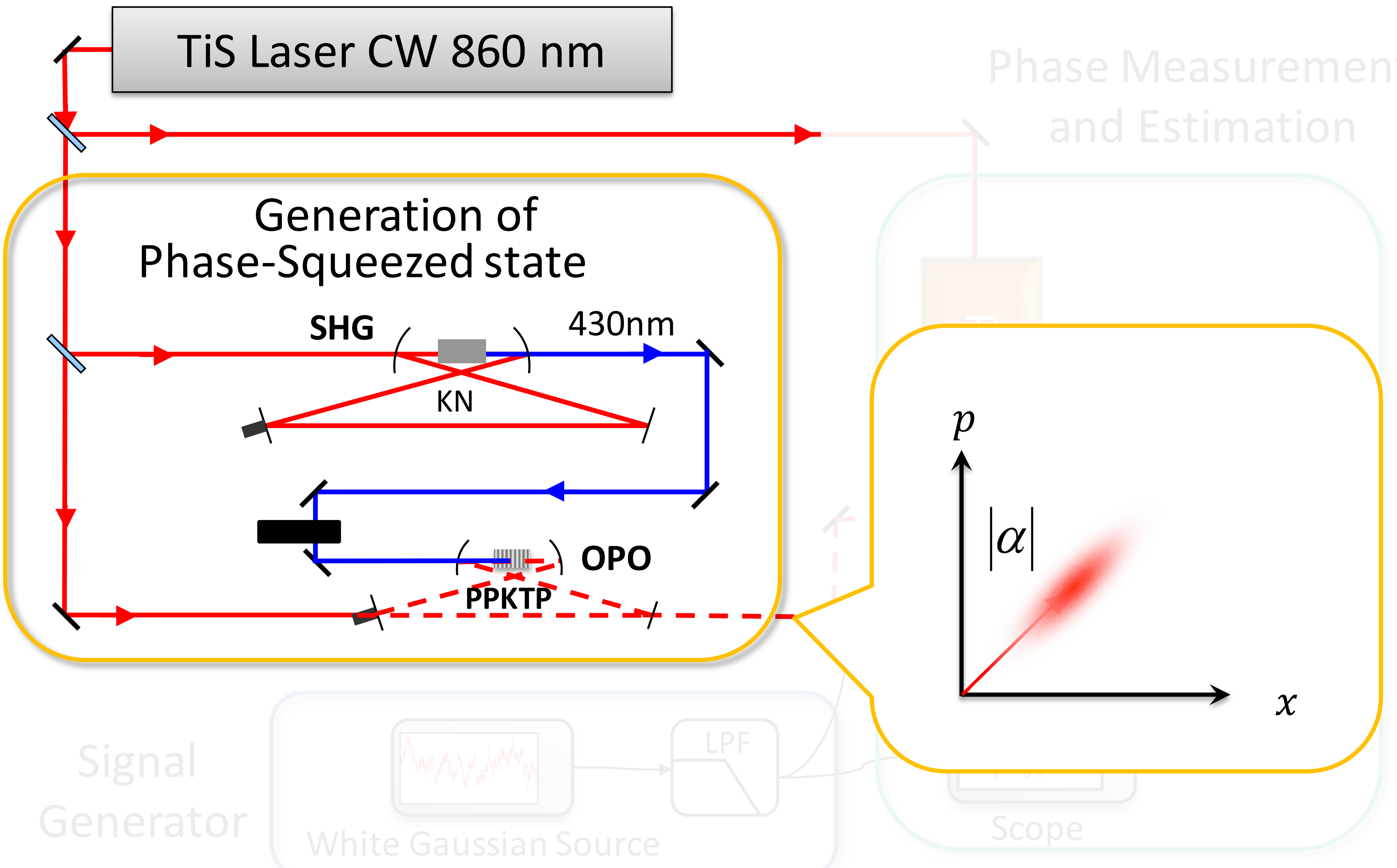
4. Summary

Experiment Setup

$\kappa = 3 \times 10^{9}\ \mathrm{N}^2 \mathrm{sec}^{-1}$

Phase-Squeezed state

Pump beam power=80mW

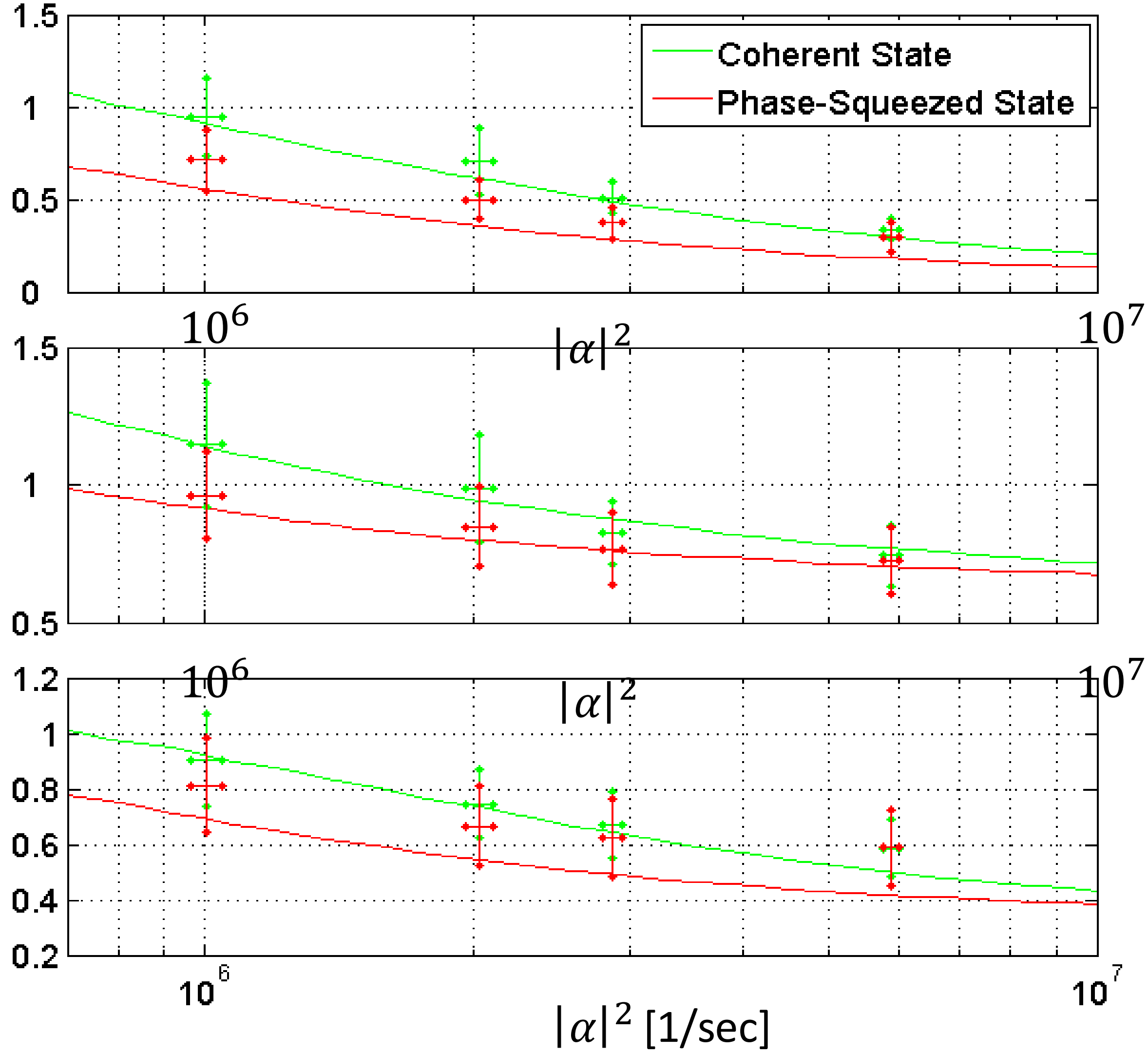


Results of Phase-Squeezed State

Position Variance
[x10^{-16} m^2]

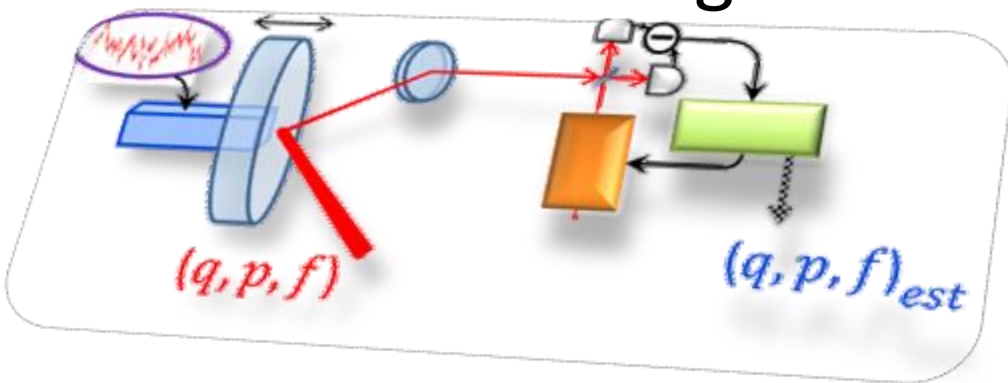
Momentum Variance
[x10^{-12} kg^{2}m^2/s^2]

Applied Force Variance
[x10^{-2} N^2]

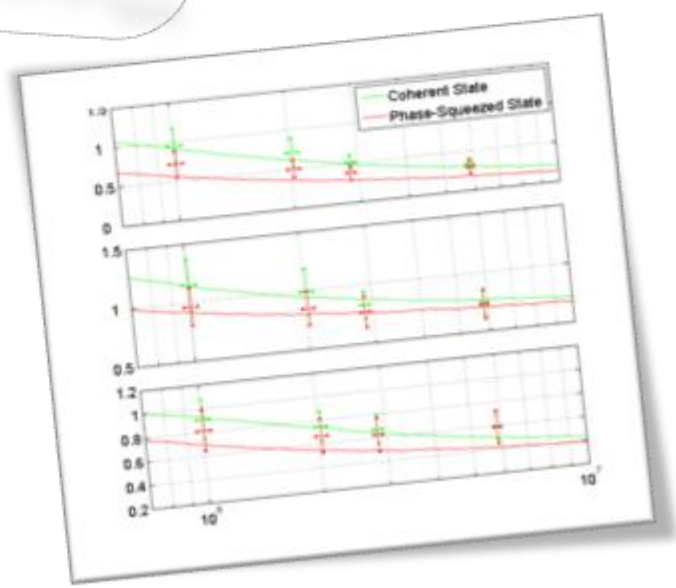


Summary

◆We demonstrated adaptive phase measurement for opto-mechanical sensing



◆We achieved better (q, p, f) estimation with smoothing and by using phase-squeezed states



Thank you for your attention