LCGT感度スペクトルに従う任意長時系列ノイズ生成シミュレーション

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  The method without IFFT
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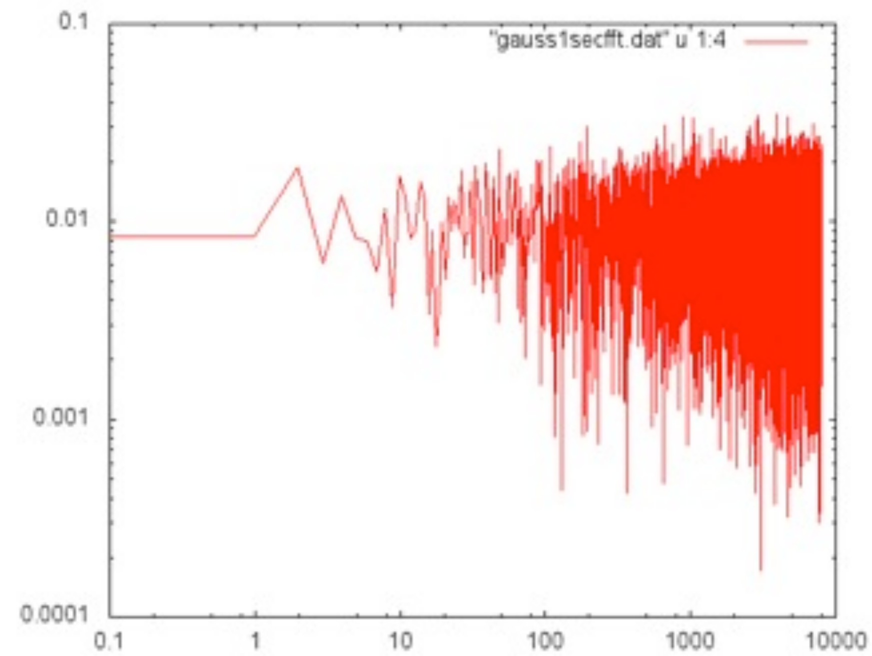
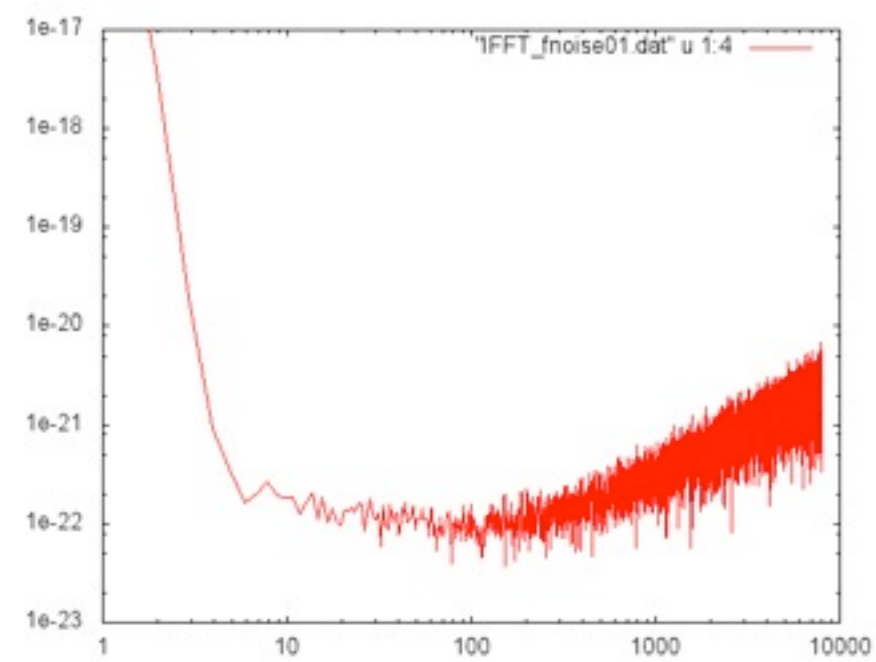
Introduction

Generate gaussian random noise of LCGT spectrum.

One method is LCGT noise make from white noise spectrum with IFFT.

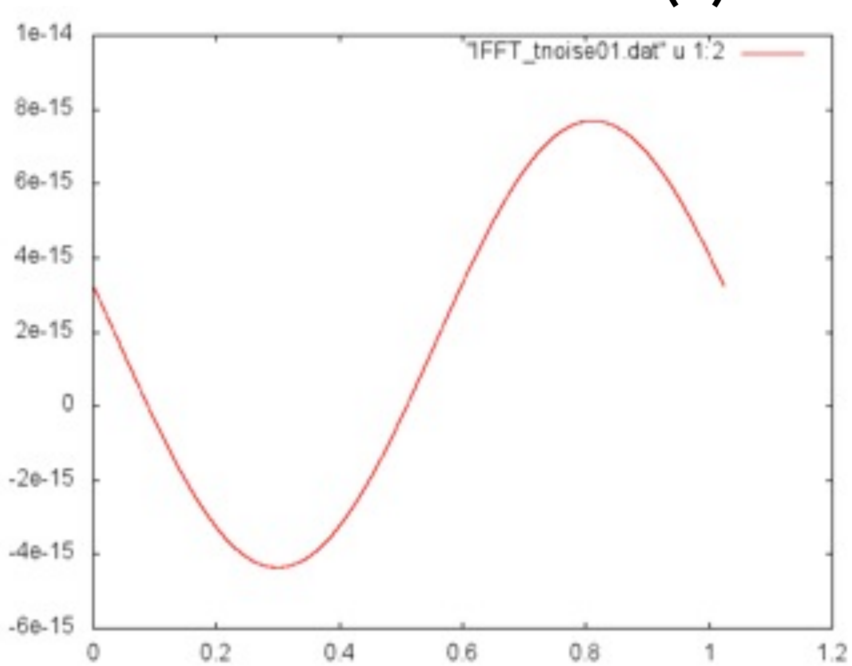


Transfer function of convert white noise to LCGT noise



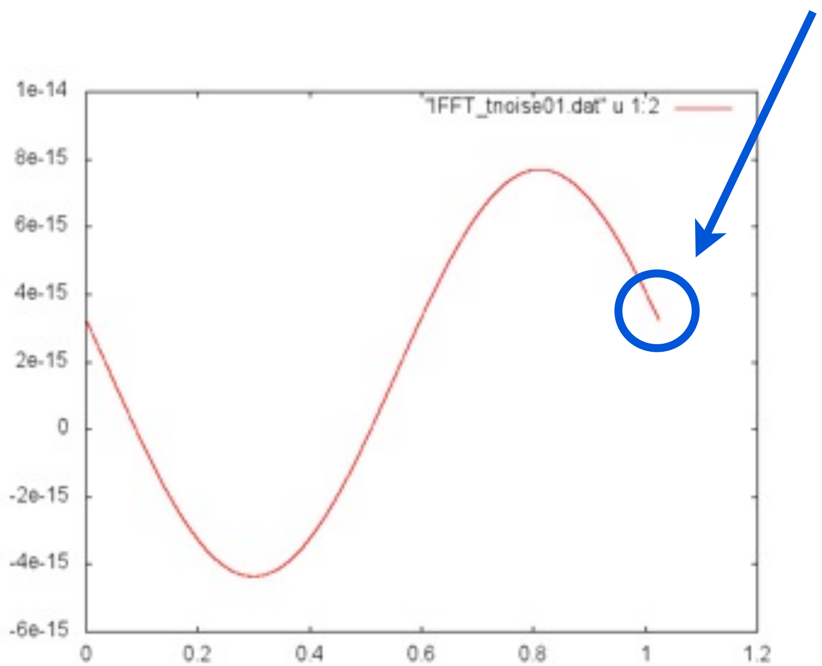
IFFT

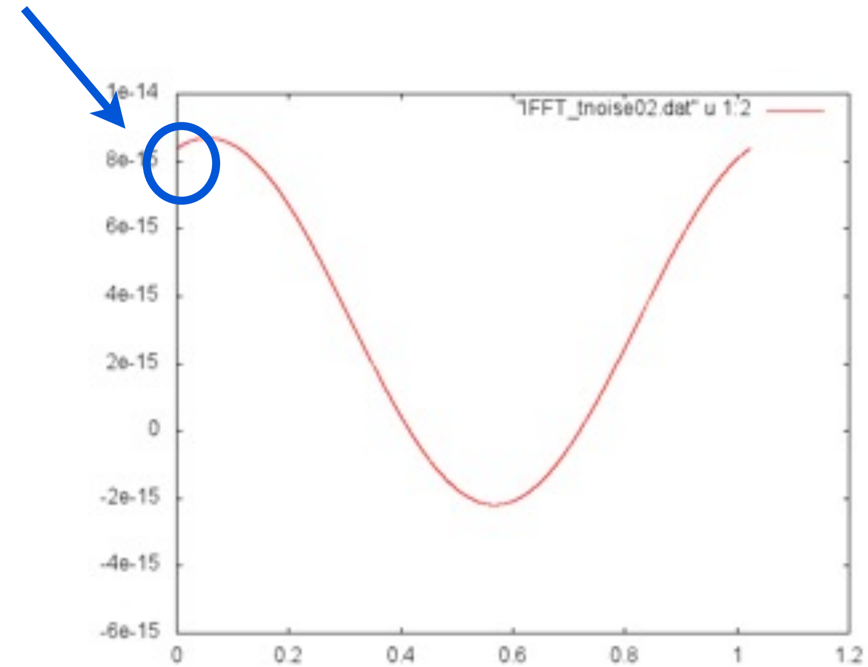
LCGT noise h(t)



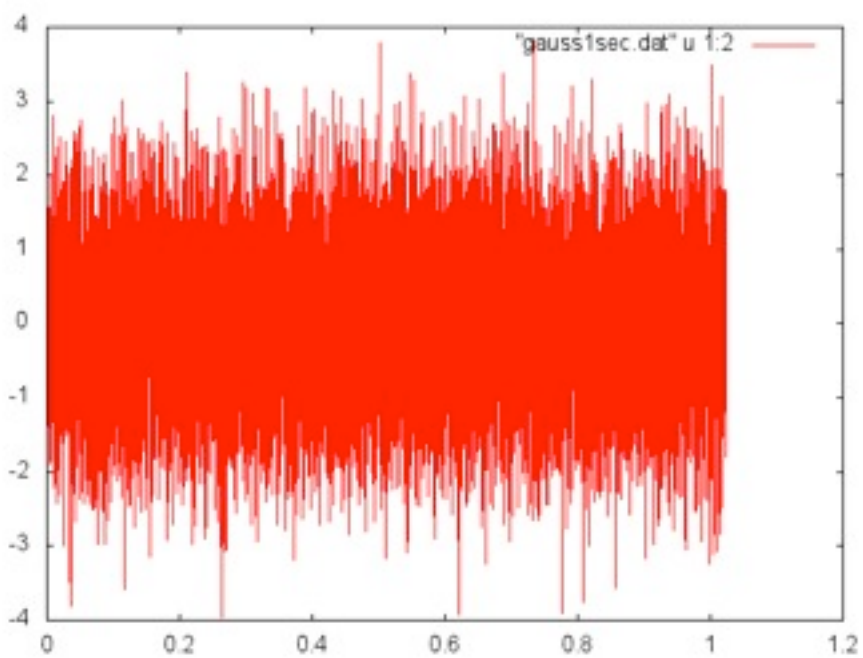
Introduction

Generated time series by IFFT have seem during datas.





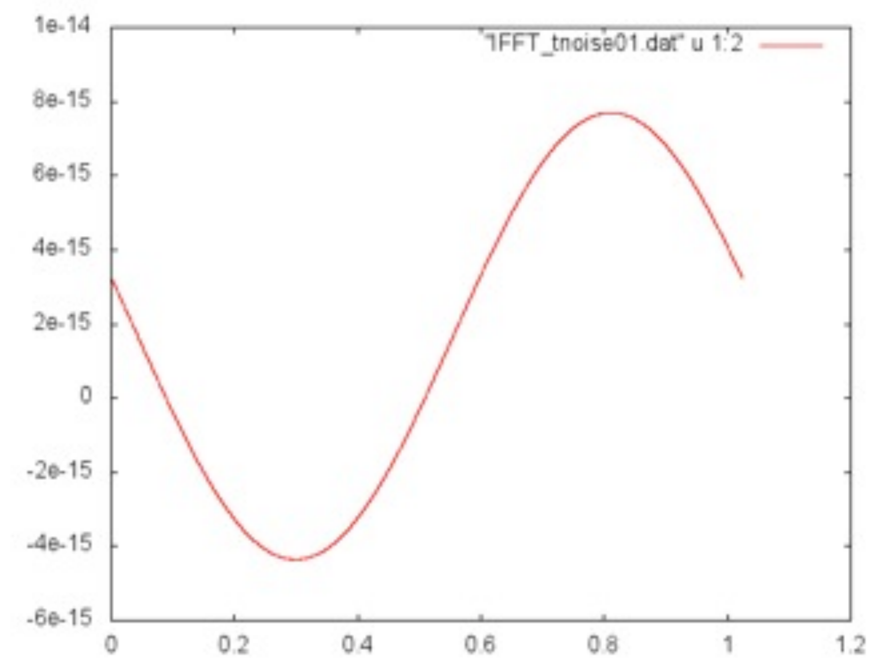
convert white noise w(t) to LCGT noise h(t) directly



gauss noise w(t)

$$x(t) = \frac{P(D)}{Q(D)} w(t)$$

$$\left(D = \frac{d}{dt} \right)$$



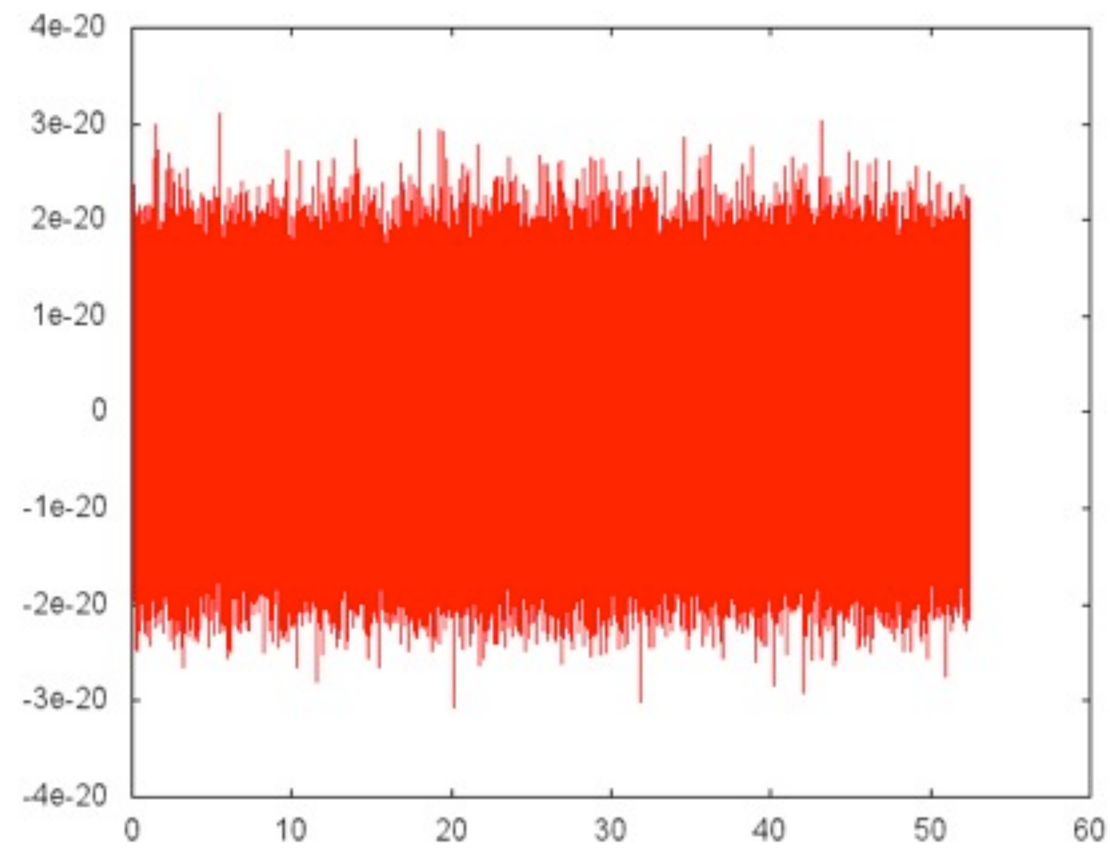
LCGT noise h(t)

Generation of random time series

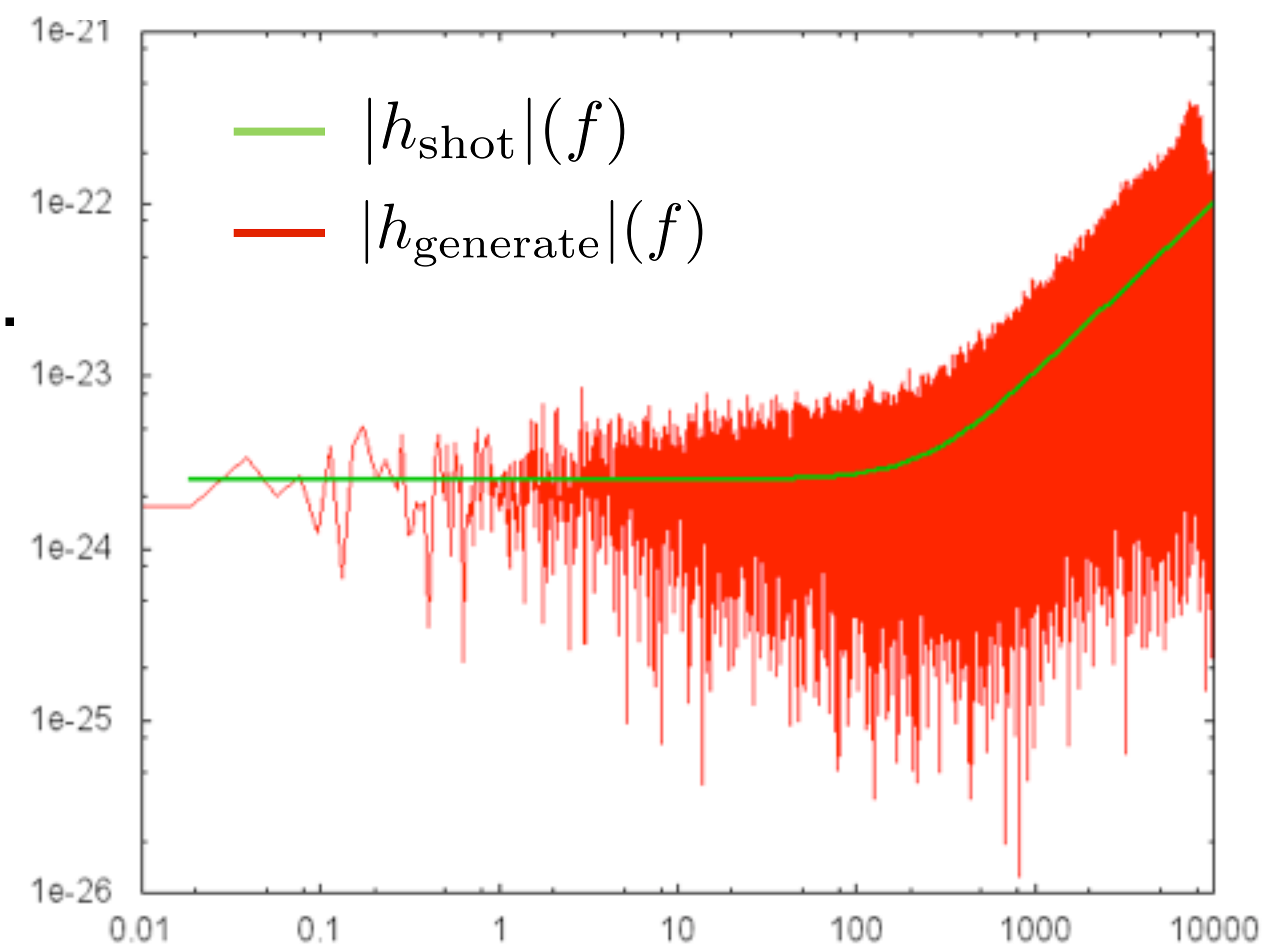
Generate random time series
from transfer function.

$$G(s) = \frac{5+s}{2.77235\times10^{18}s + 1.6554\times10^{14}s^2 + 4.555\times10^9 s^3 + \ldots}$$

“s” is complex frequency.



Power spectrum of time series
follow transfer function |G(f)|.



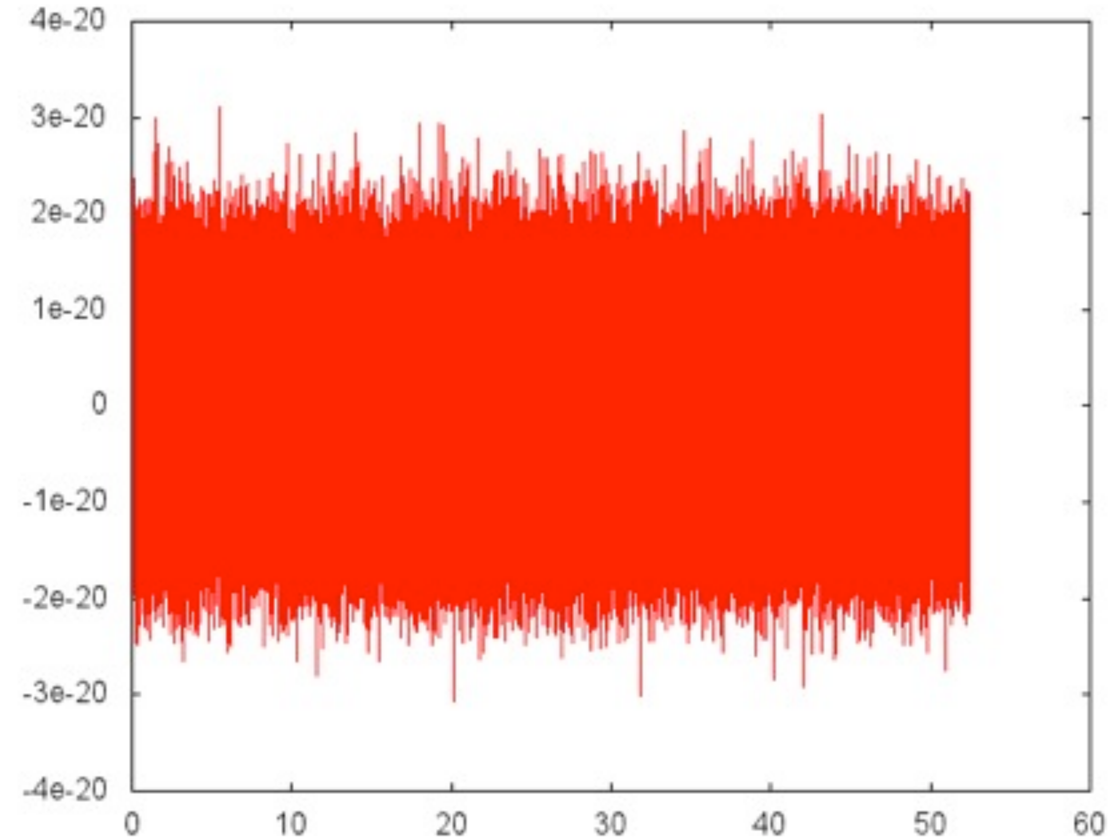
Right figures are example of
time series and fourier spectrum
of shot noise.

Generation of random time series

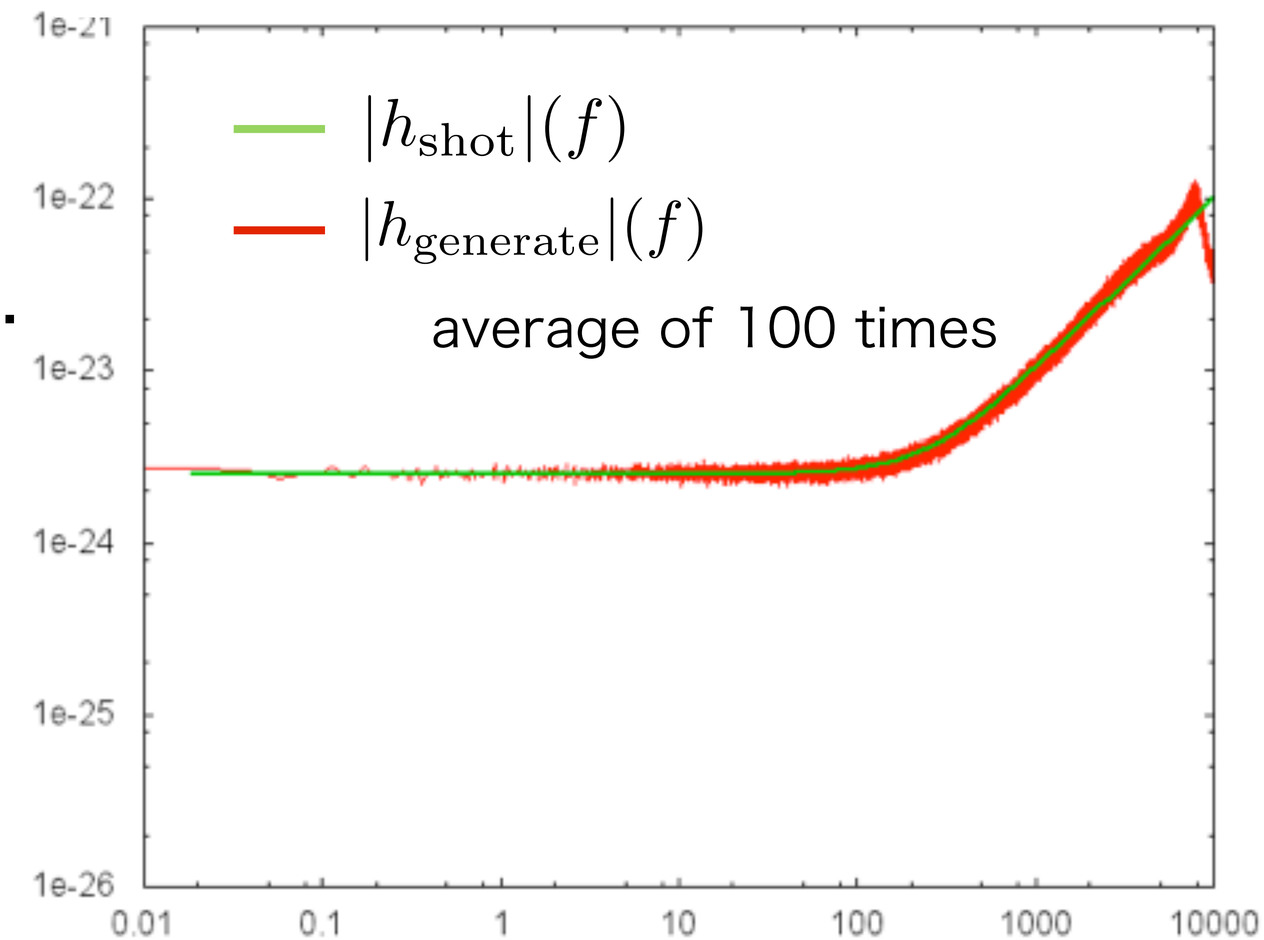
Generate random time series from transfer function.

$$G(s) = \frac{5+s}{2.77235\times 10^{18}s + 1.6554\times 10^{14}s^2 + 4.555\times 10^{9}s^3 + s}$$

"s" is complex frequency.



Power spectrum of time series follow transfer function |G(f)|.



Right figures are example of time series and fourier spectrum of shot noise.

Generation of random time series

time series x(t)

$$x(t) = \int_{-\infty}^{t} g(t-\tau)w(\tau)d\tau$$

g(t) : impulse response
w(t) : white noise
using GNU Scientific Library

power spectral density

$$S_x(\omega) = |G(\omega)|^2 S_w(\omega)$$

$$G(s) = \frac{P(s)}{Q(s)} = \frac{a_0 + a_1 s + \cdots + a_m s^m}{b_0 + b_1 s + \cdots + b_n s^n}$$

$$s = i\omega, n > m$$

x(t) can be written with differential operator.

$$x(t) = \frac{P(D)}{Q(D)} w(t) \qquad \left(D = \frac{d}{dt} \right)$$

$$\Leftrightarrow \quad \begin{array}{l} Q(D)\phi(t) = w(t) \quad \Rightarrow \quad b_0\phi + b_1\phi' + \cdots + b_n\phi^{(n)} = w(t) \\ x(t) = P(D)\phi(t) \end{array}$$

φ(t) is steady state solution

Generation of random time series

Require state vector z(t) to compute x(t).

$$z = [\phi(t) \quad \phi'(t) \quad \cdots \quad \phi^{(n-1)}(t)]^T$$

State vector z(t) satisfies the stochastic differential equation.

$$\frac{dz(t)}{dt} = Az(t) + f(t)$$

x(t) is solve with $x(t) = P(D)\phi(t)$

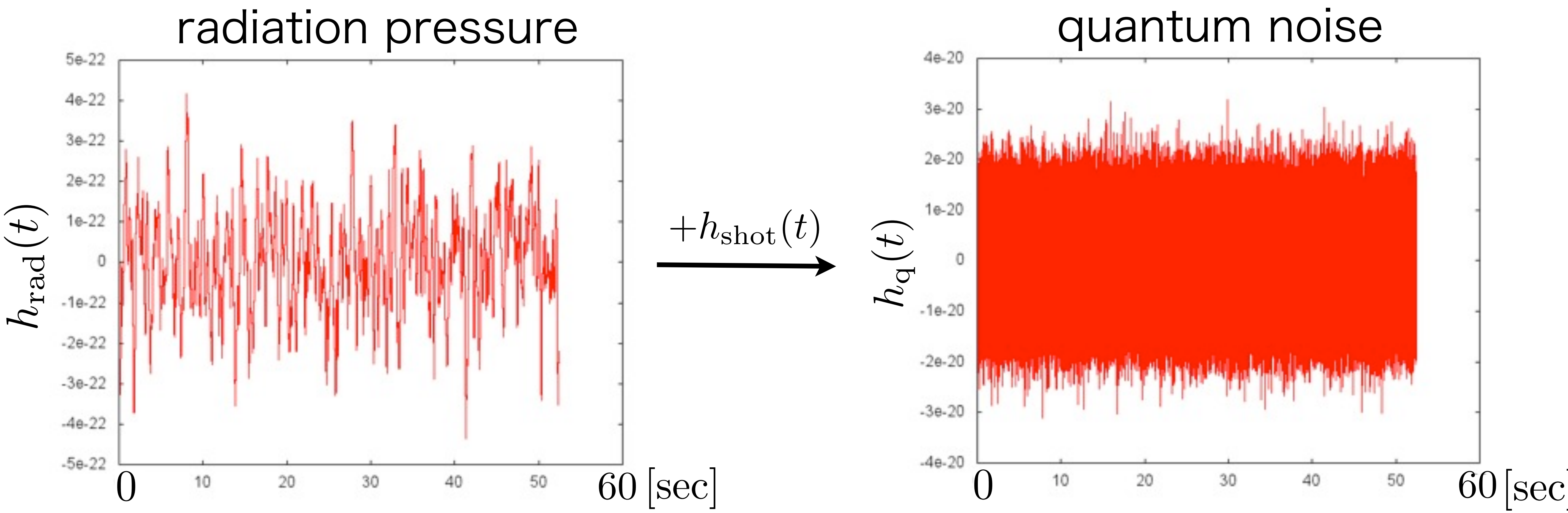
$$x(t) = a_0\phi + a_1\phi' + \cdots + a_m\phi^{(m)}$$

References

・S2-AEI-TN-3034

・SIAM review, Volume 7, Issue 1, page 68–80, 1965

Generation of random time series



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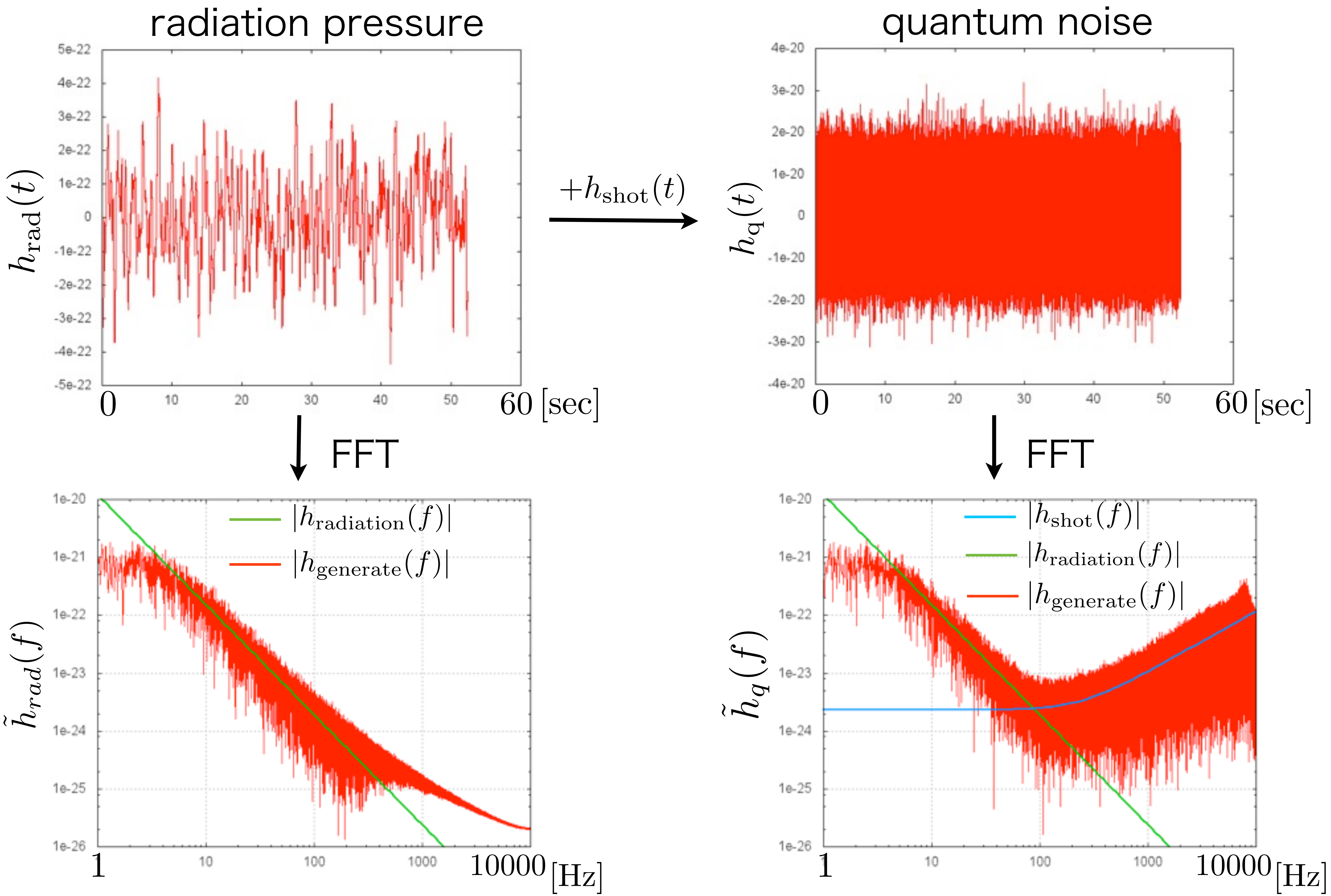
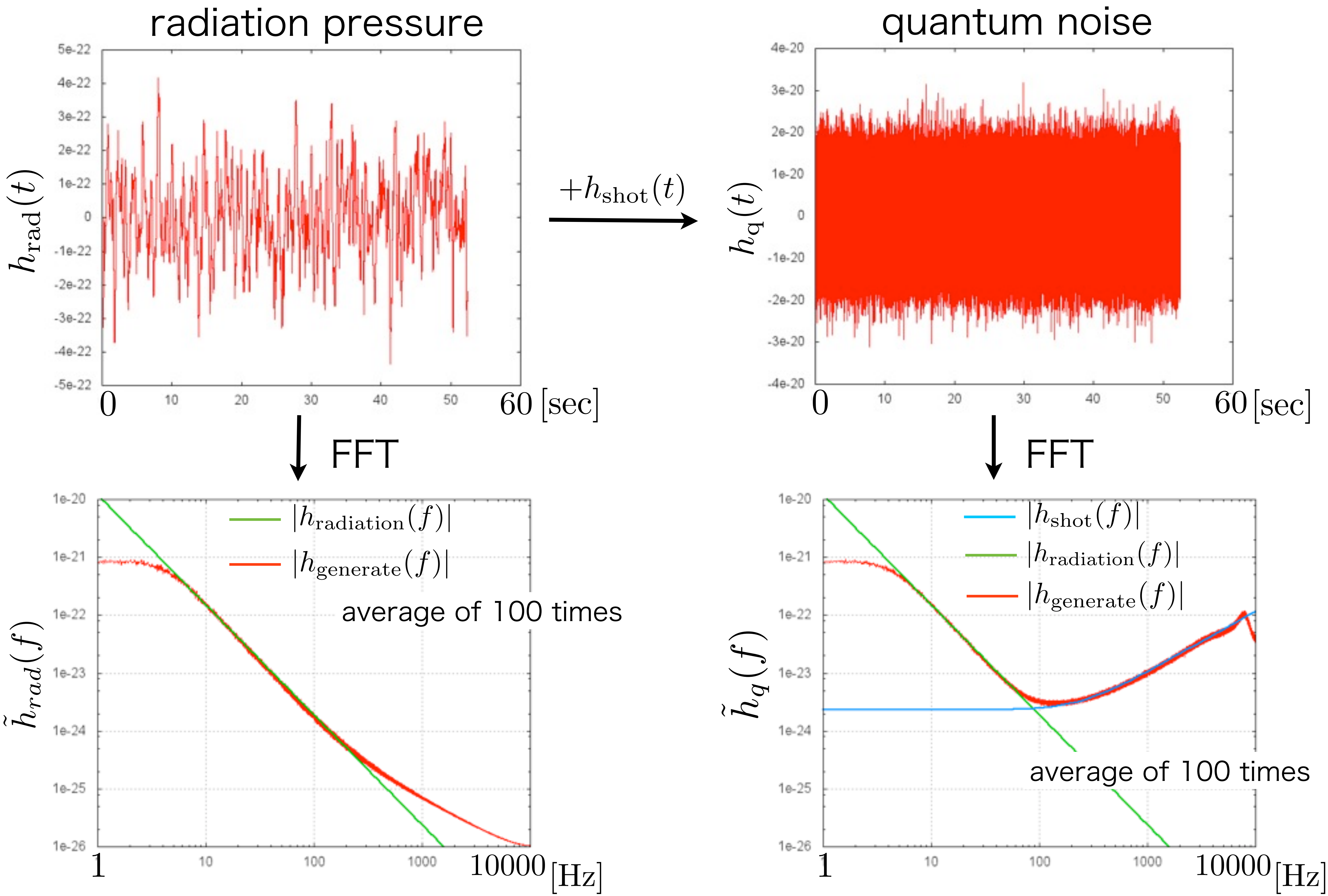
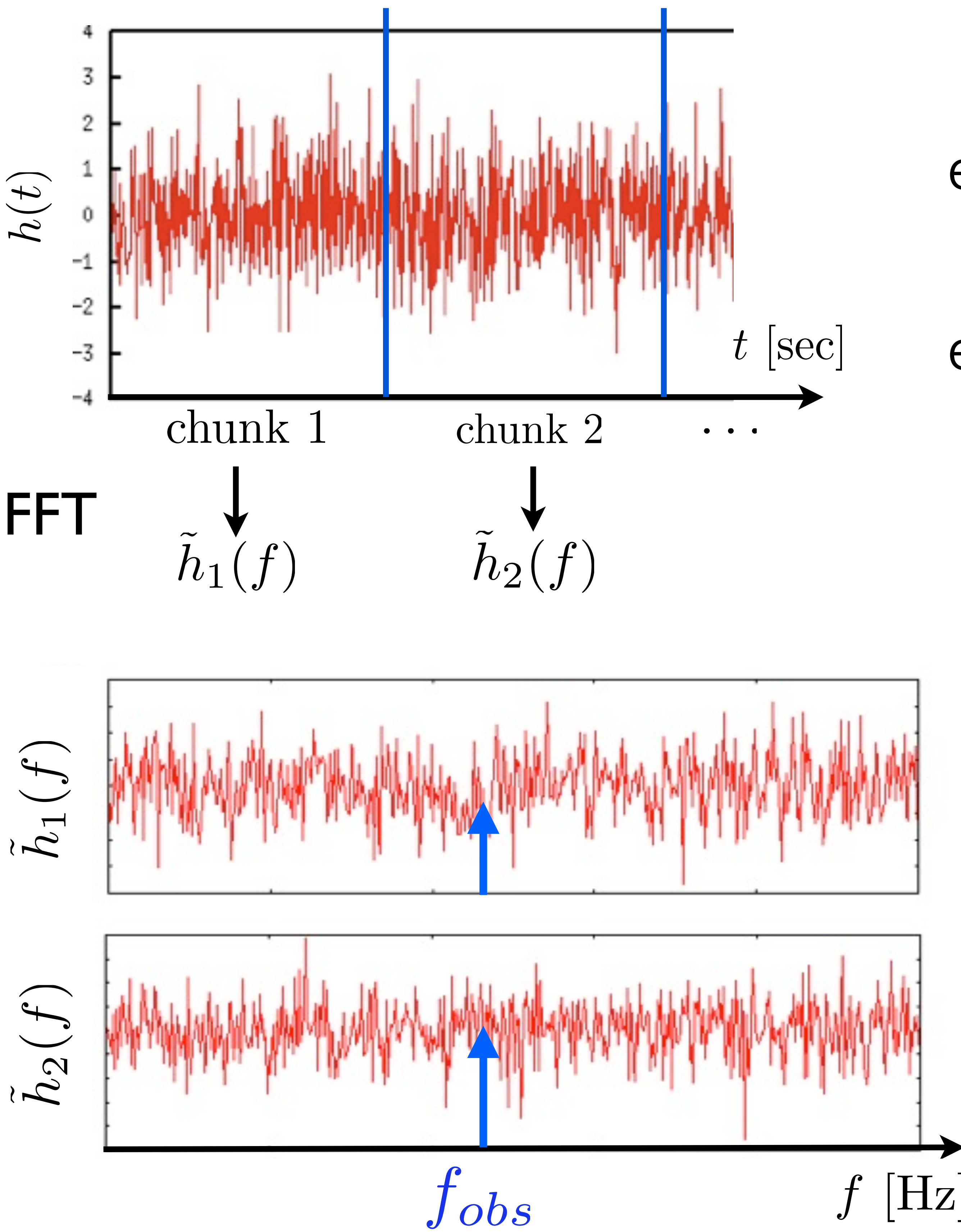


Generation of random time series

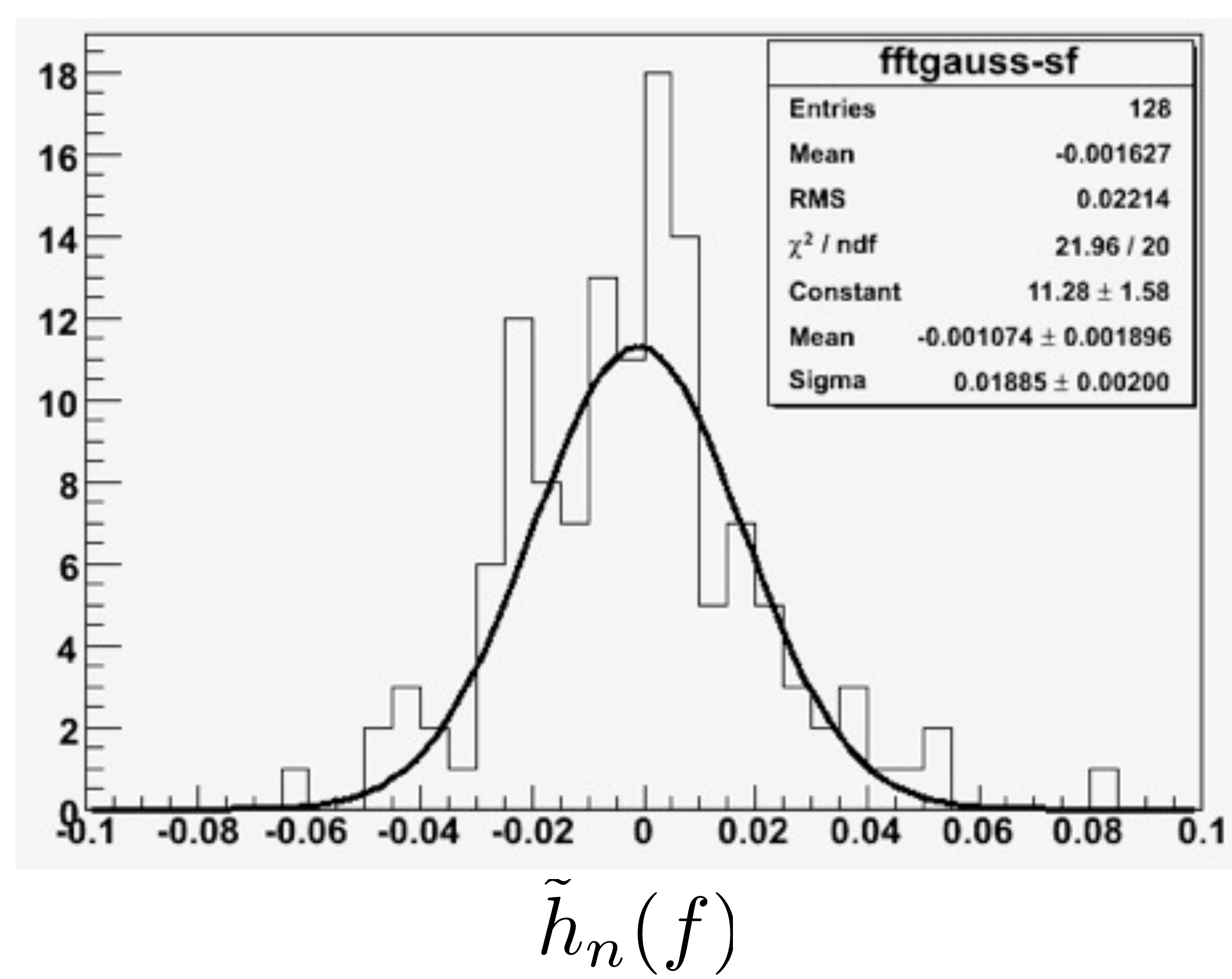


Gaussianity check



Time series divide chunks and each chunk do FFT.

Arbitrary frequency line up from each FFT data.



Kolmogorov-Smirnov test (K-S test)

null hypothesis “generated noise allow gaussian distribution”

statistic test $D = \max |F_n(x) - F(x)|$

$F_n(x)$: cumulative density function of sample

$F(x)$: cumulative density function of theoretic

significance probability $P_{KS}(D) = Q_{KS}(\sqrt{N}D)$ N: sample number

$$Q_{KS}(\lambda) = 2\sum_{j=1}^{\infty}(-1)^{j-1}\exp(-2j^2\lambda^2) \qquad \lambda = \sqrt{N}D$$

1) $P_{KS}(D) < \alpha$ α: significant level

reject null hypothesis : Not gaussian

2) $P_{KS}(D) \geq \alpha$

not reject null hypothesis

Kolmogorov-Smirnov test (K-S test)

null hypothesis "generated noise allow gaussian distribution"

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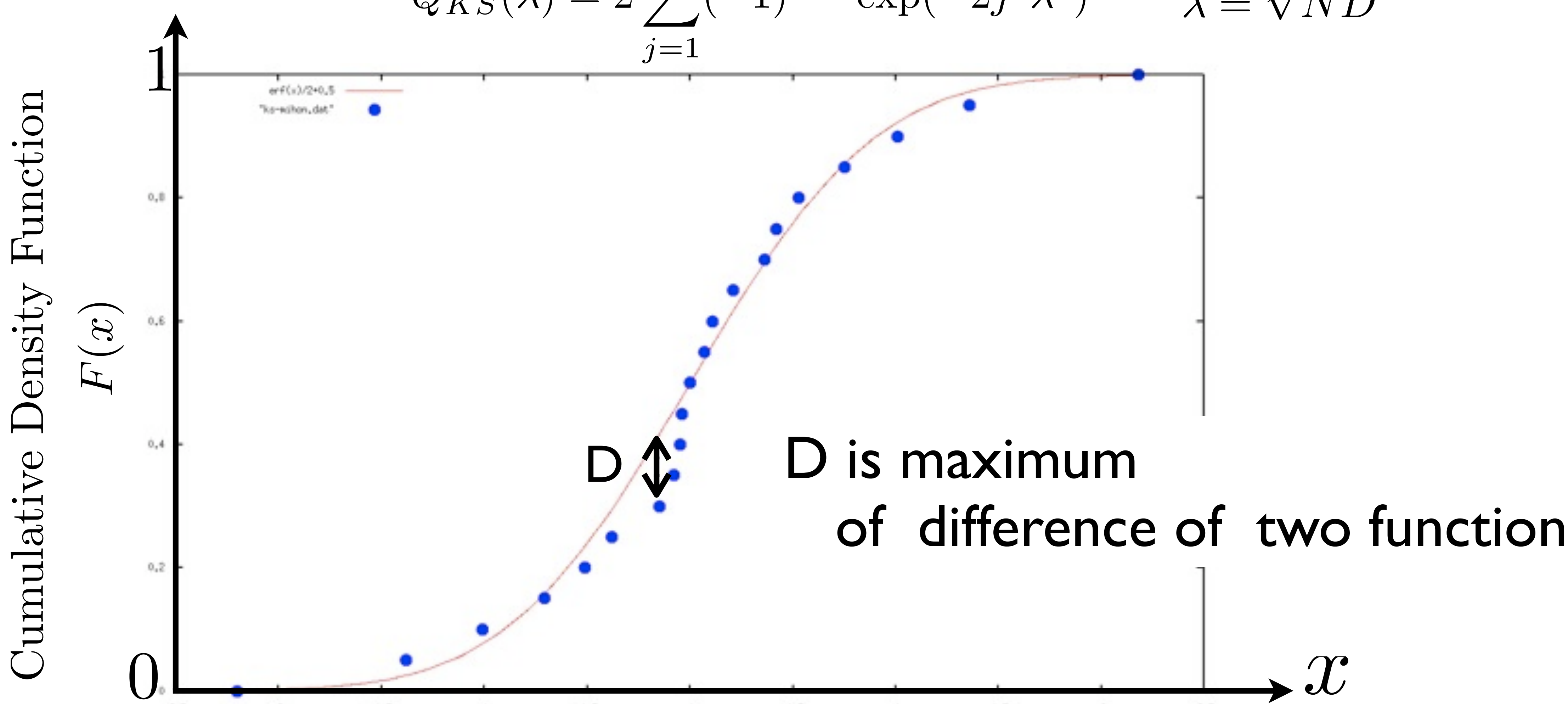


Anderson-Darling test (A-D test)

statistic test $$W_n = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln F(X_i) + \ln\{1 - F(X_{n-1-i})\}\right]$$

$F(x)$: cumulative density function of theoretic

$X_i (i = 1, 2, ..., n) \quad X_1 \leq X_2 \leq ... \leq X_n$: sample

significance probability $\lim_{n\to\infty} P(W_n > x) = 1 - A(x)$ n: sample number

$$A(x) = \frac{\sqrt{2\pi}}{x}\sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(i+1/2)}{\Gamma(1/2)\ j!}(4i+1)\exp\left(-\frac{(4i+1)^2\pi^2}{8x}\right)\int_0^{\infty}\exp\left(\frac{x}{8(\omega^2+1)} - \frac{(4i+1)^2\pi^2\omega^2}{8x}\right)\mathrm{d}\omega$$

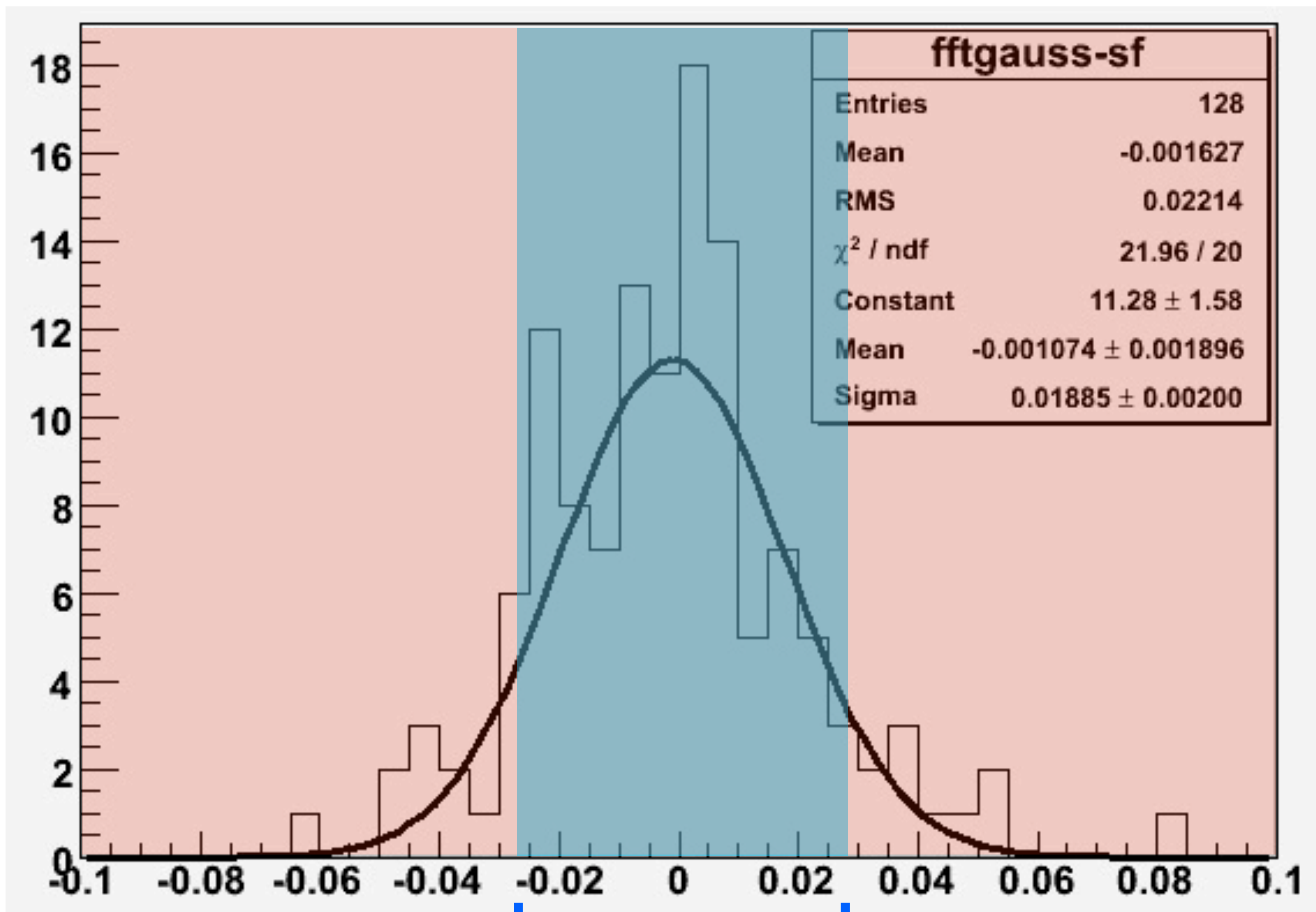
We can't use A(x) when W_{n} is too large

rejection limit: $W_{\mathrm{limit}} = 3.979$ **corresponding** $\alpha = 0.01$

Gaussianity check

K-S test : depend strongly on central part of distribution

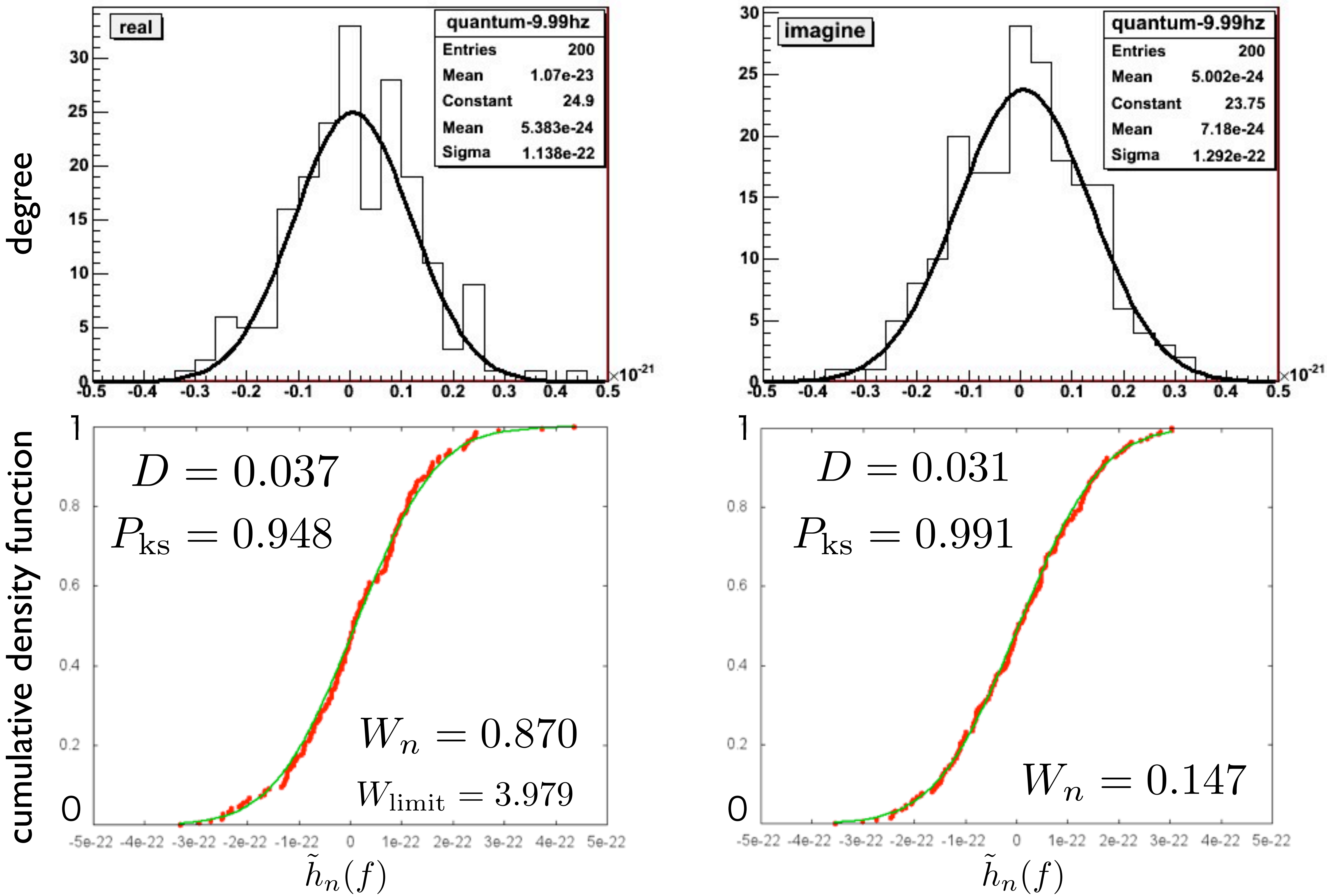
A-D test : depend on central part and tail of distribution



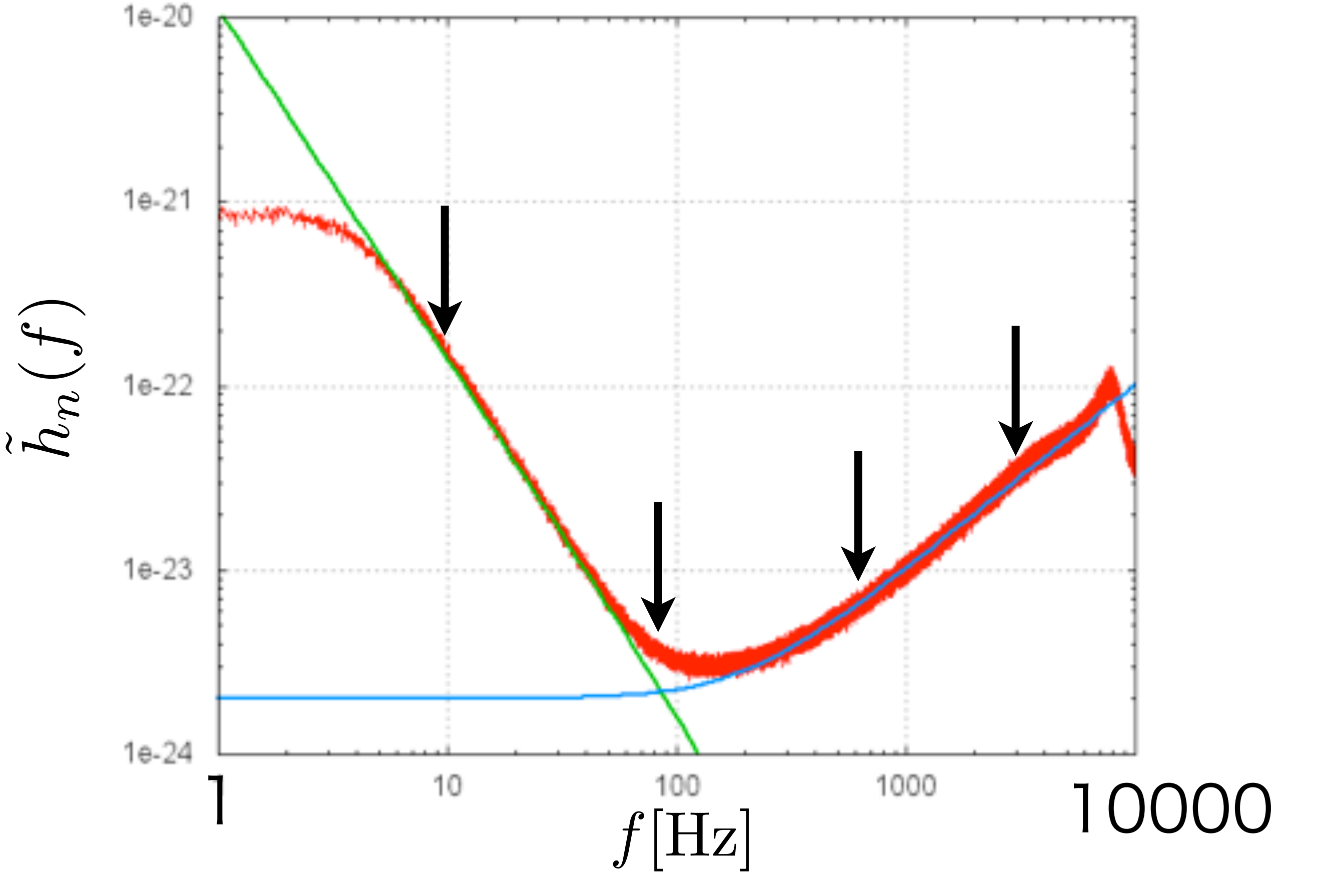
K-S test

A-D test

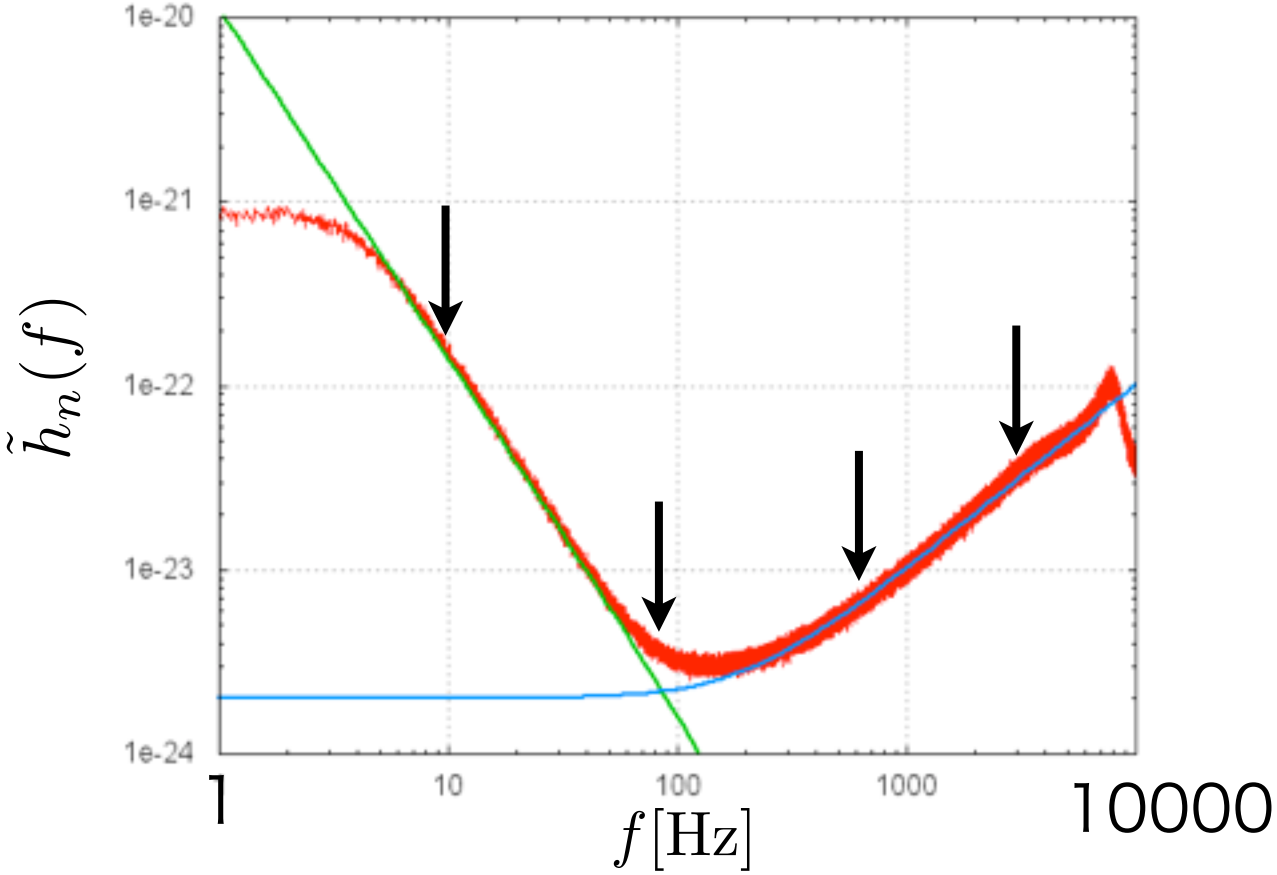
Gaussianity check about 10Hz

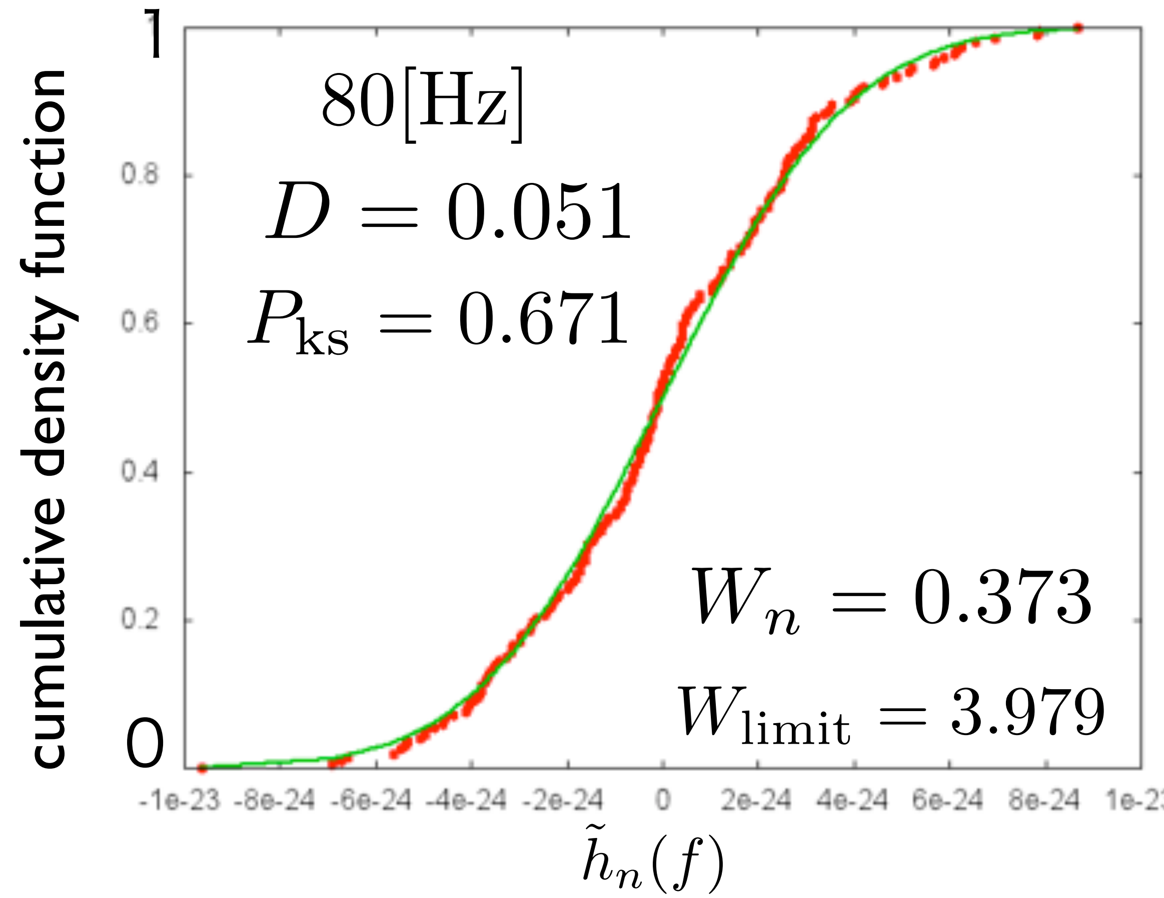


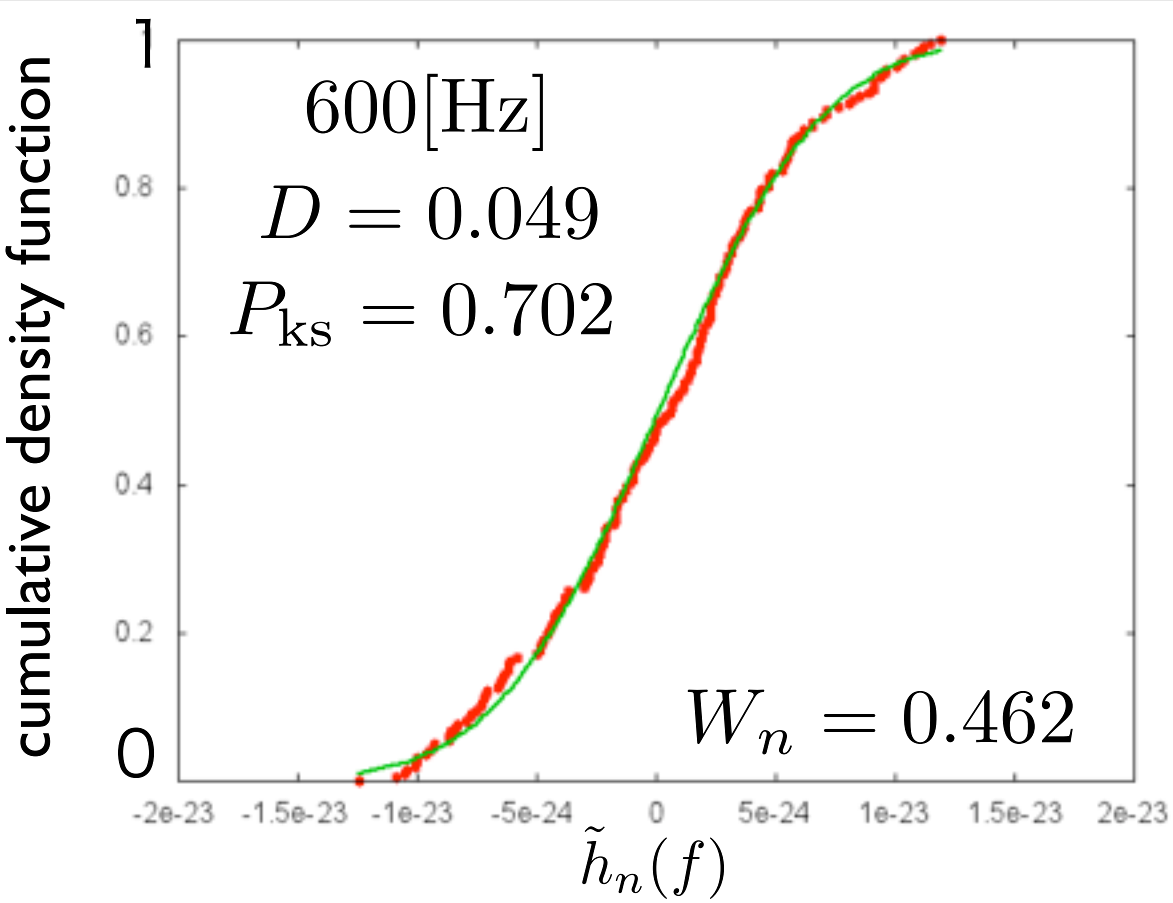
Gaussianity check　　another frequency

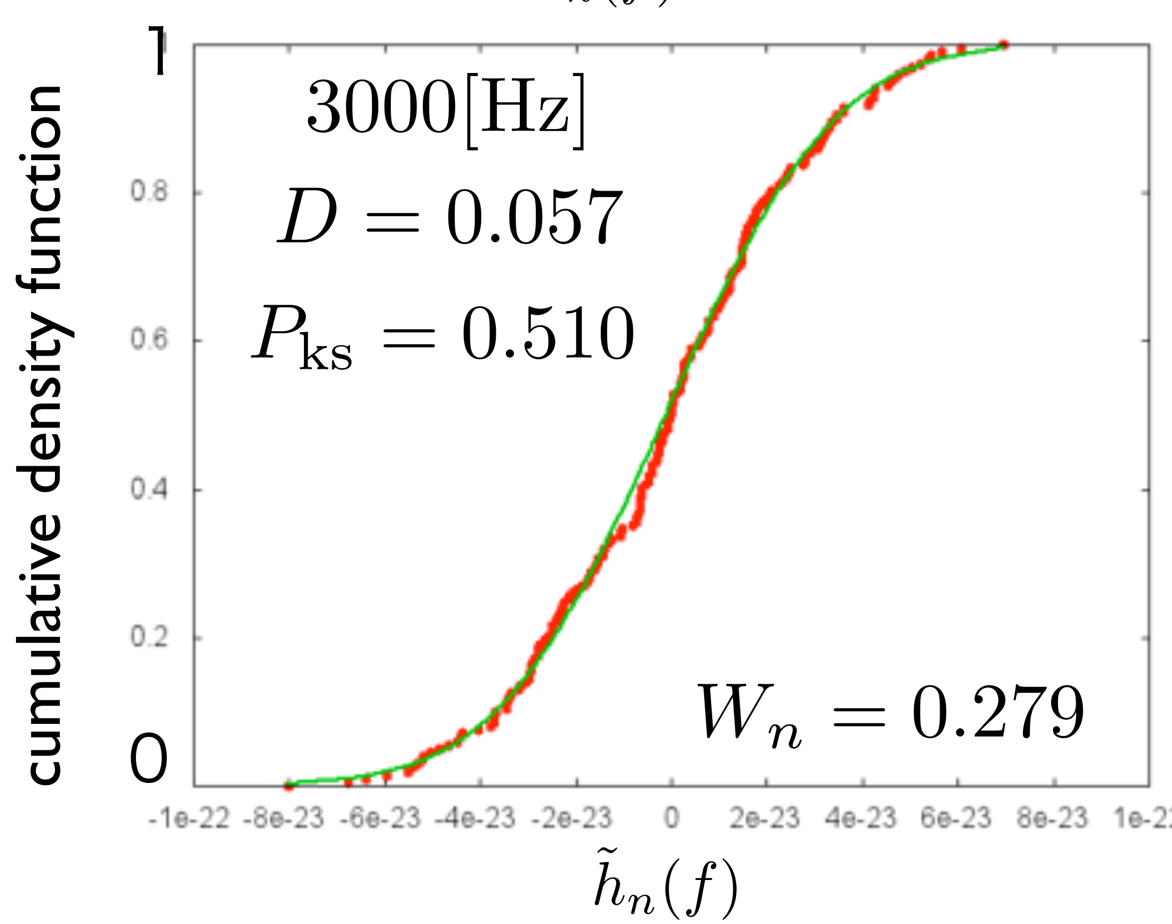


Gaussianity check　another frequency



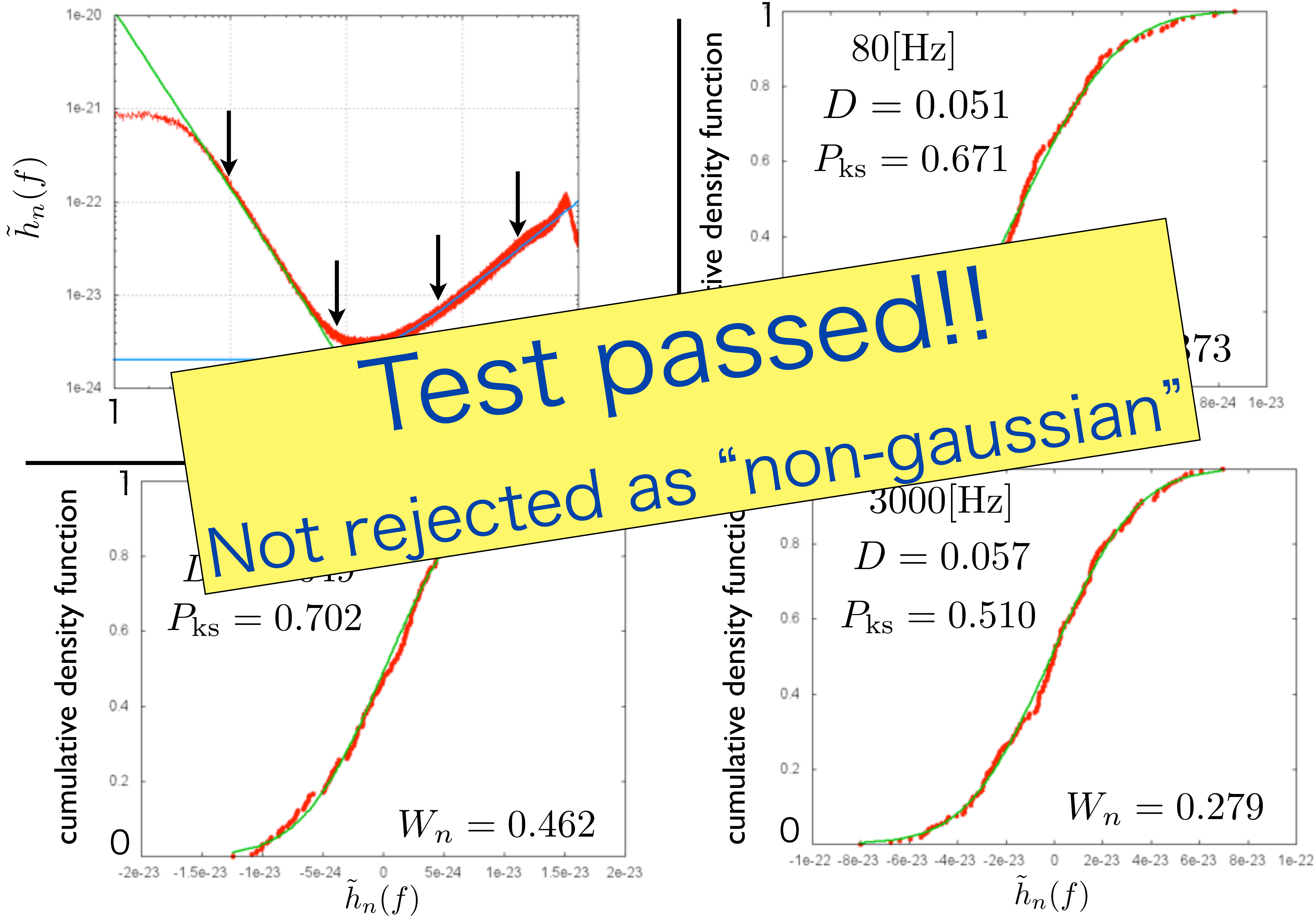






Gaussianity check

another frequency



Test passed!!

Not rejected as “non-gaussian”

Summary and Future

Summary

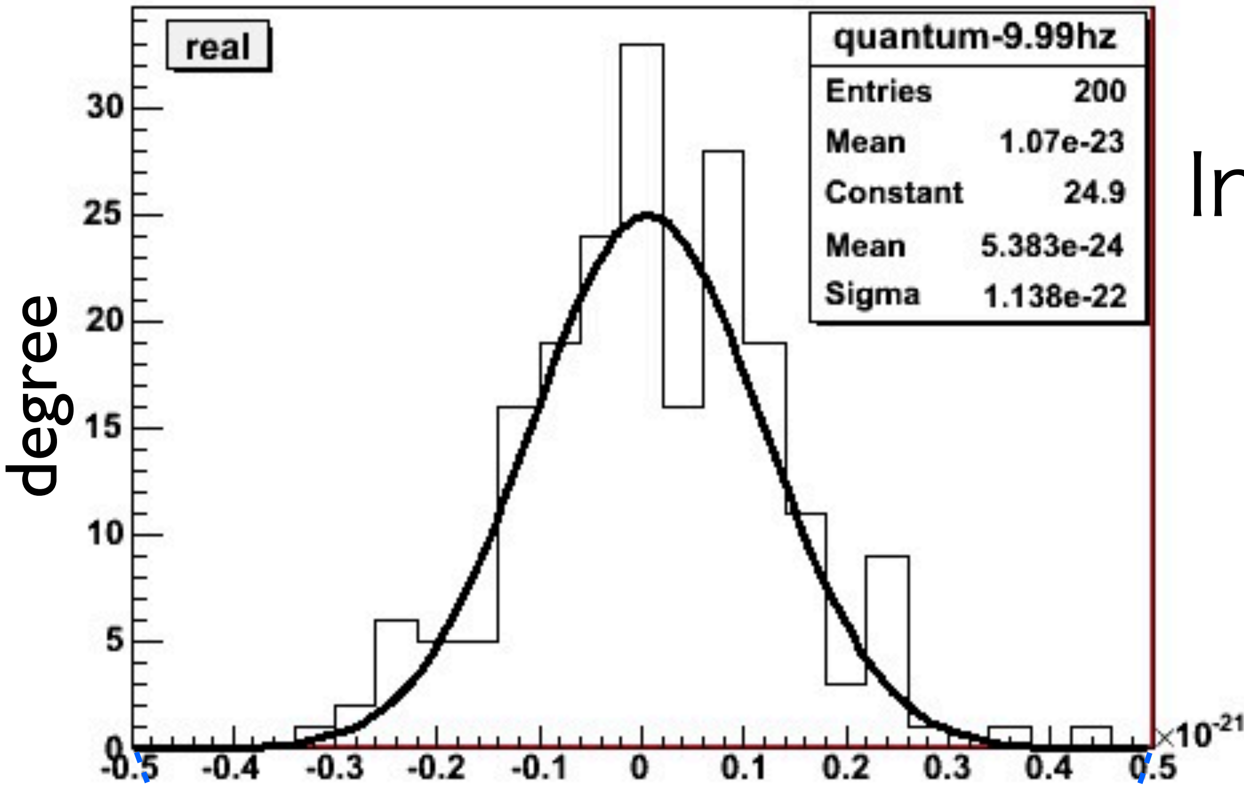
- Generate gaussian random noise
  of LCGT spectrum
- Check gaussianity of generated noise

Future

- Innovate non-linear and/or non-stationary noise
- frame data for analysis simulation

Appendix

Appendix

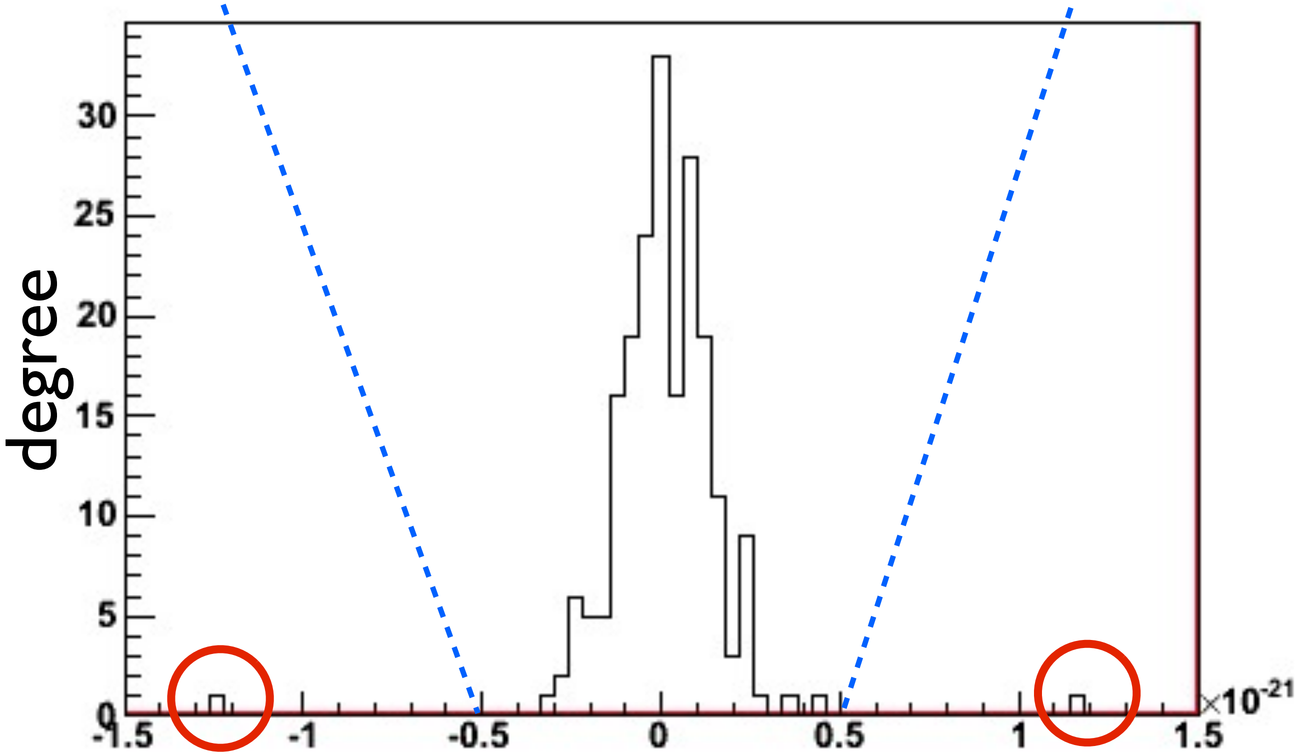


Injecte abnormal value (about 10σ)

Before injection

$D = 0.037$
$P_{\mathrm{ks}} = 0.948$

$W_n = 0.870$



After injection

$D = 0.107$
$P_{ks} = 0.017$

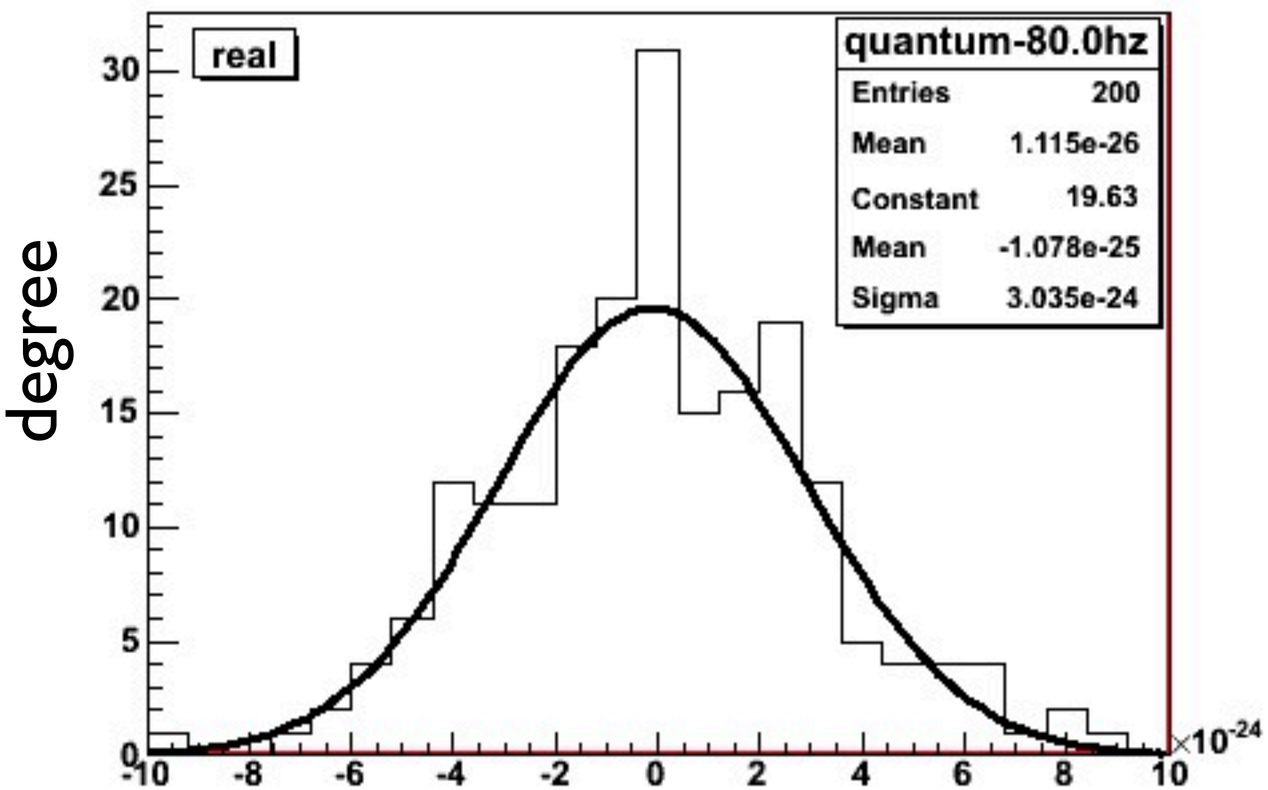
$W_n = 5.051$
$(\alpha = 0.01 \quad W_n = 3.979)$

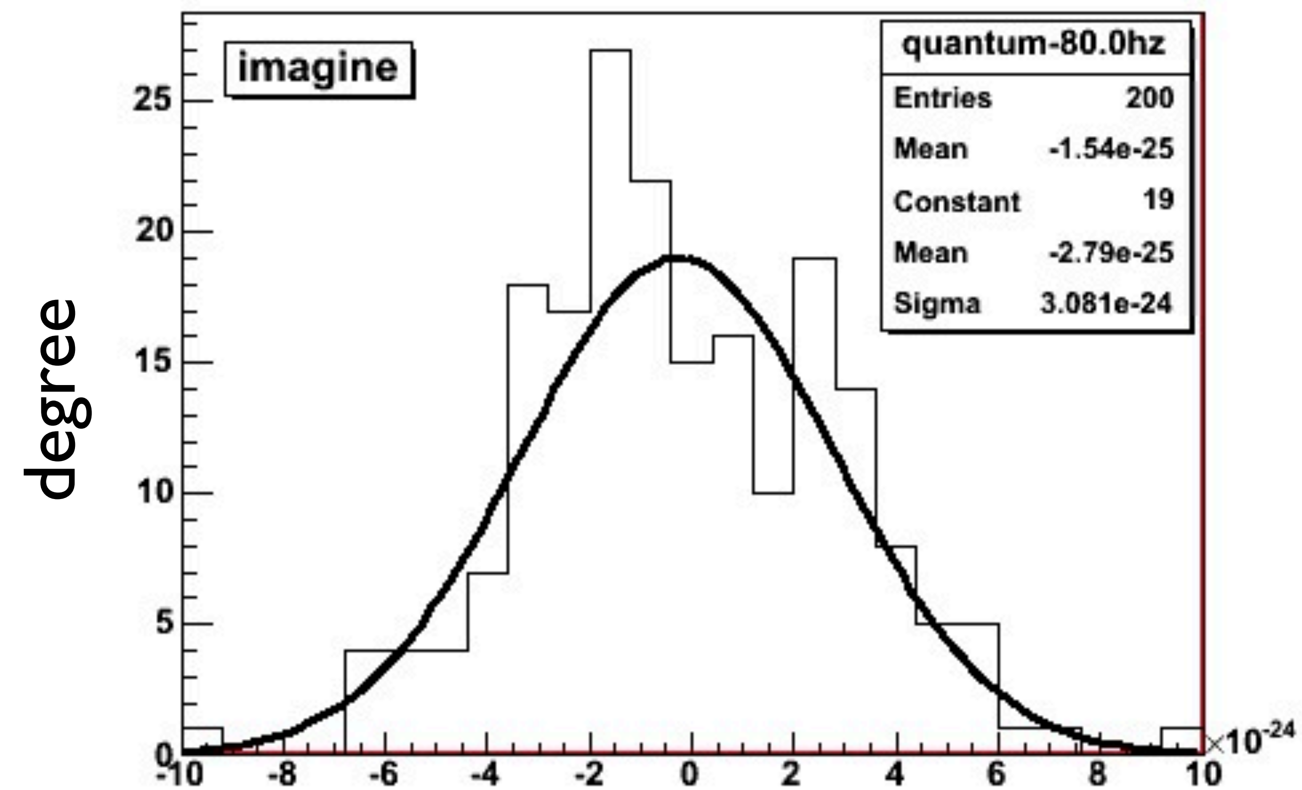
not reject
by K-S test

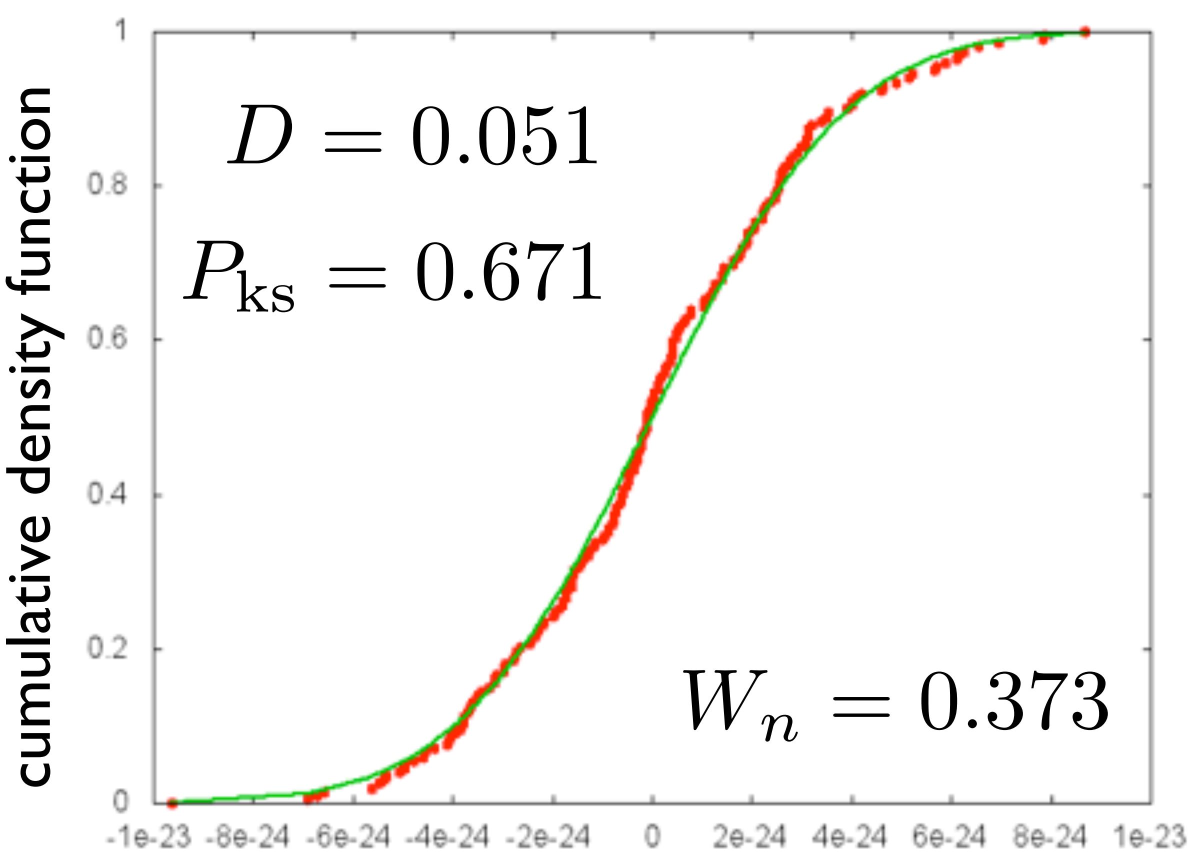
rejected
by A-D test

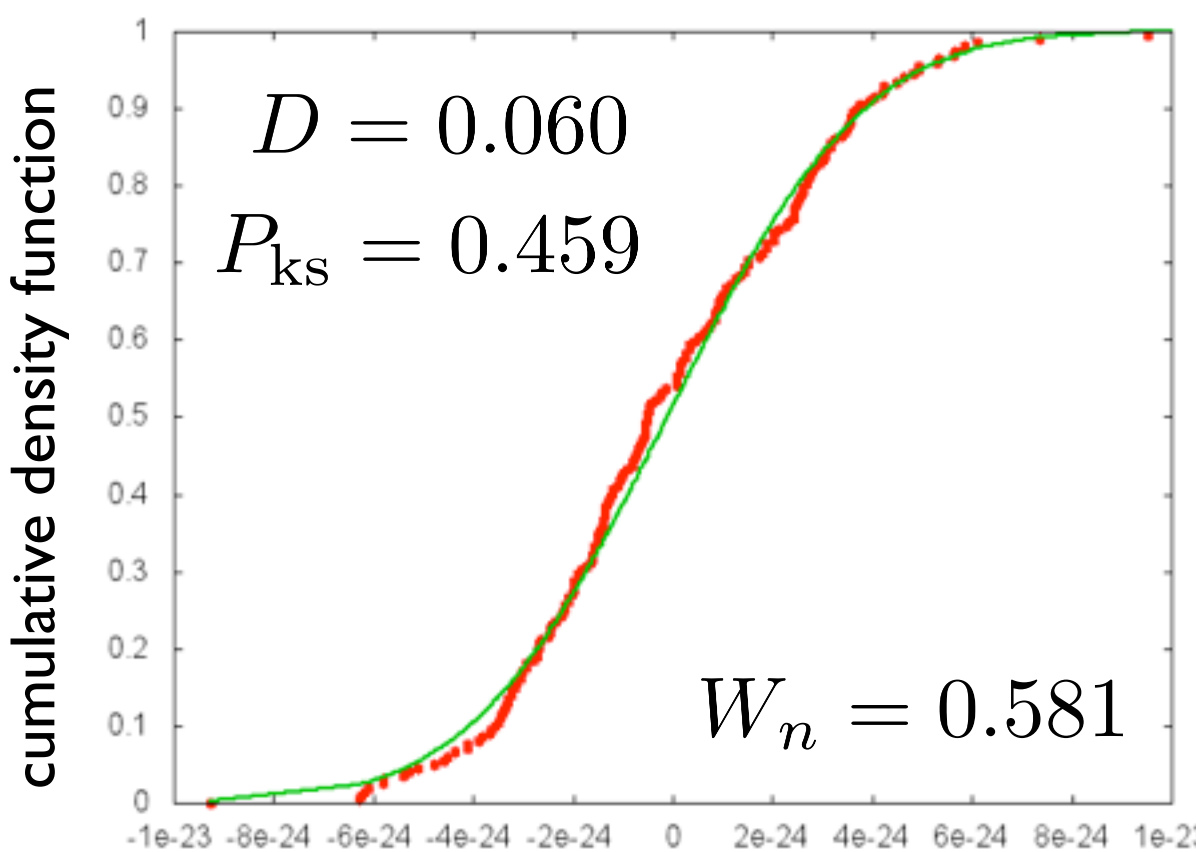
Appendix

about 80Hz



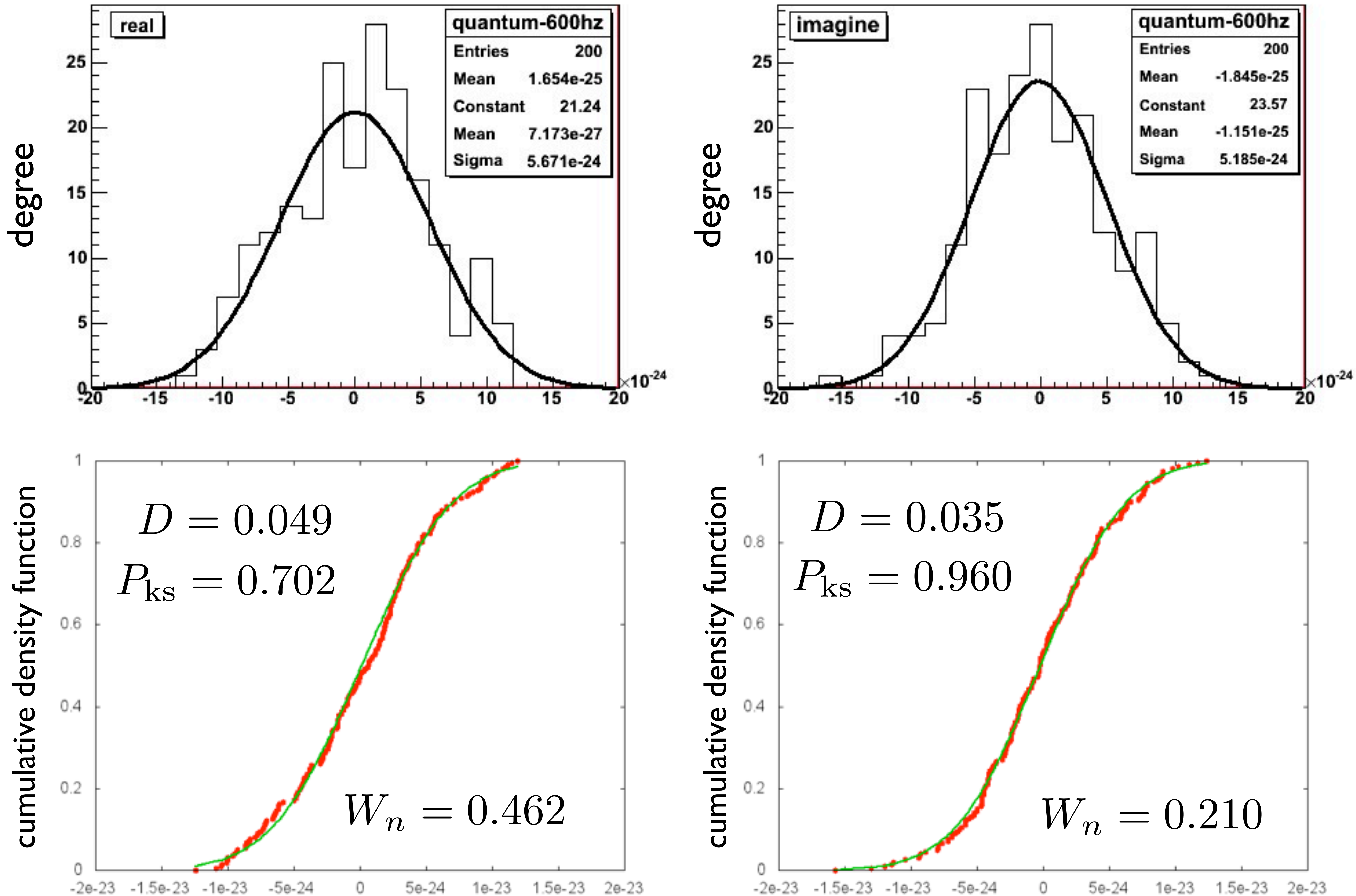






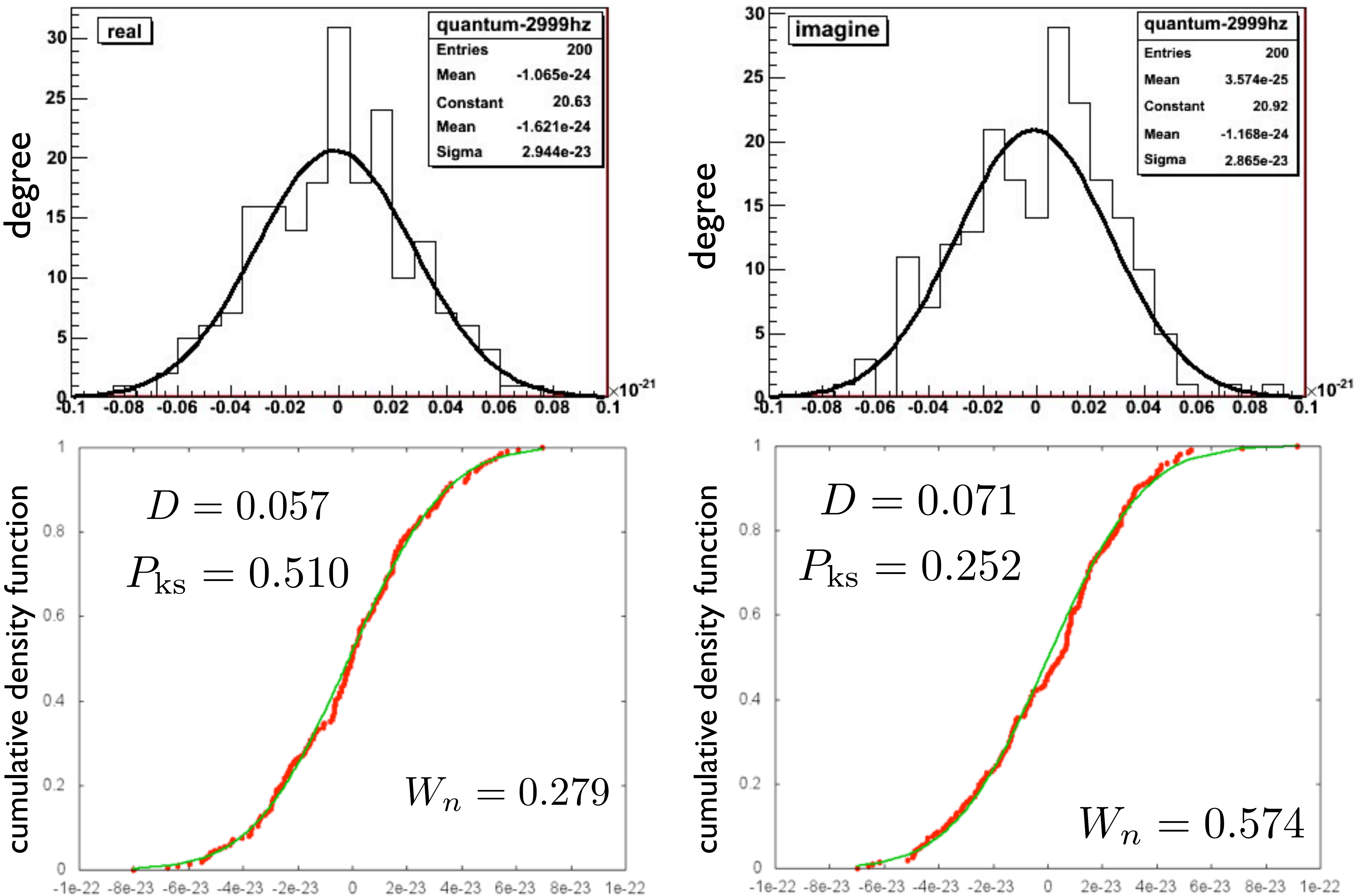
Appendix

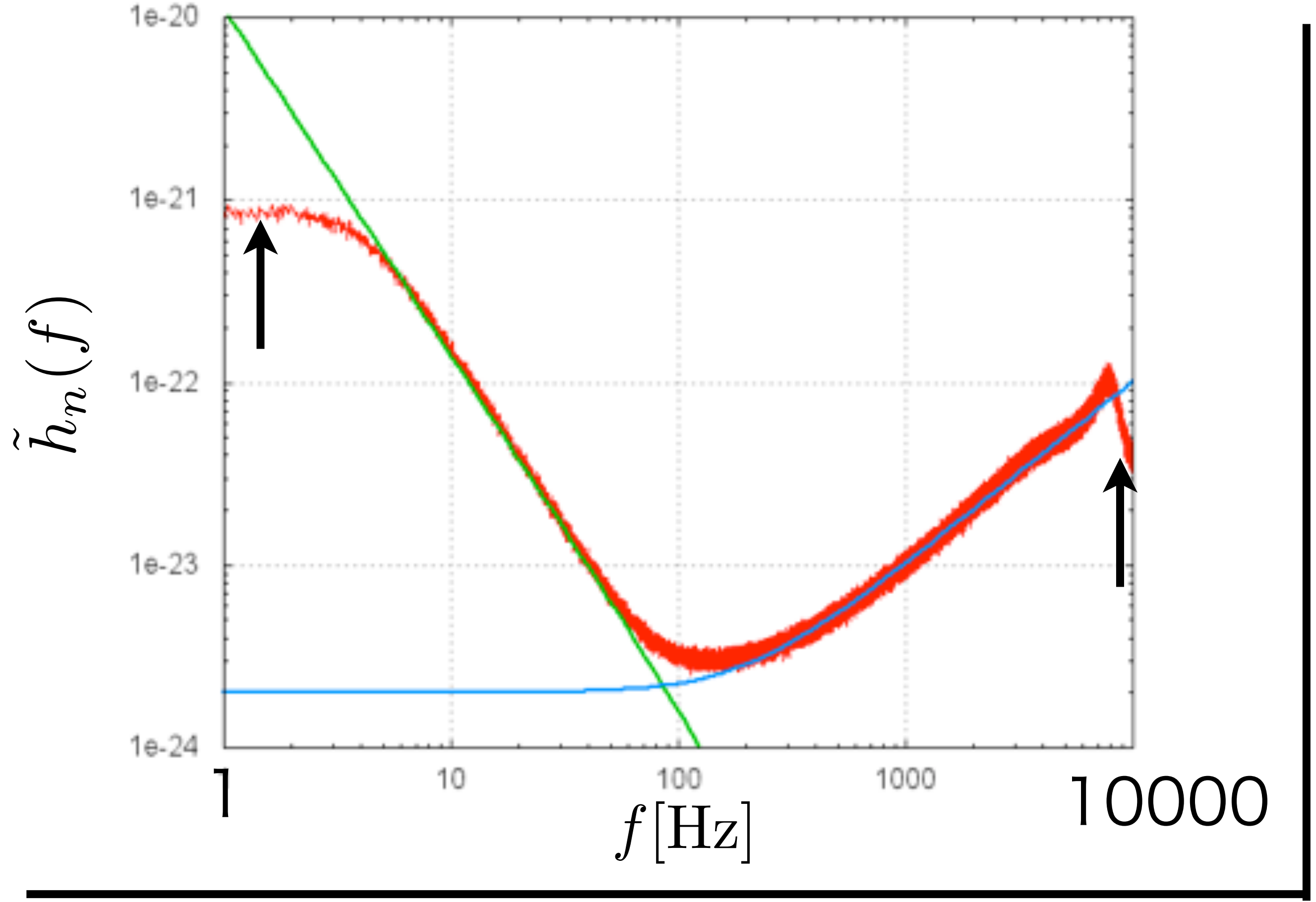
about 600Hz

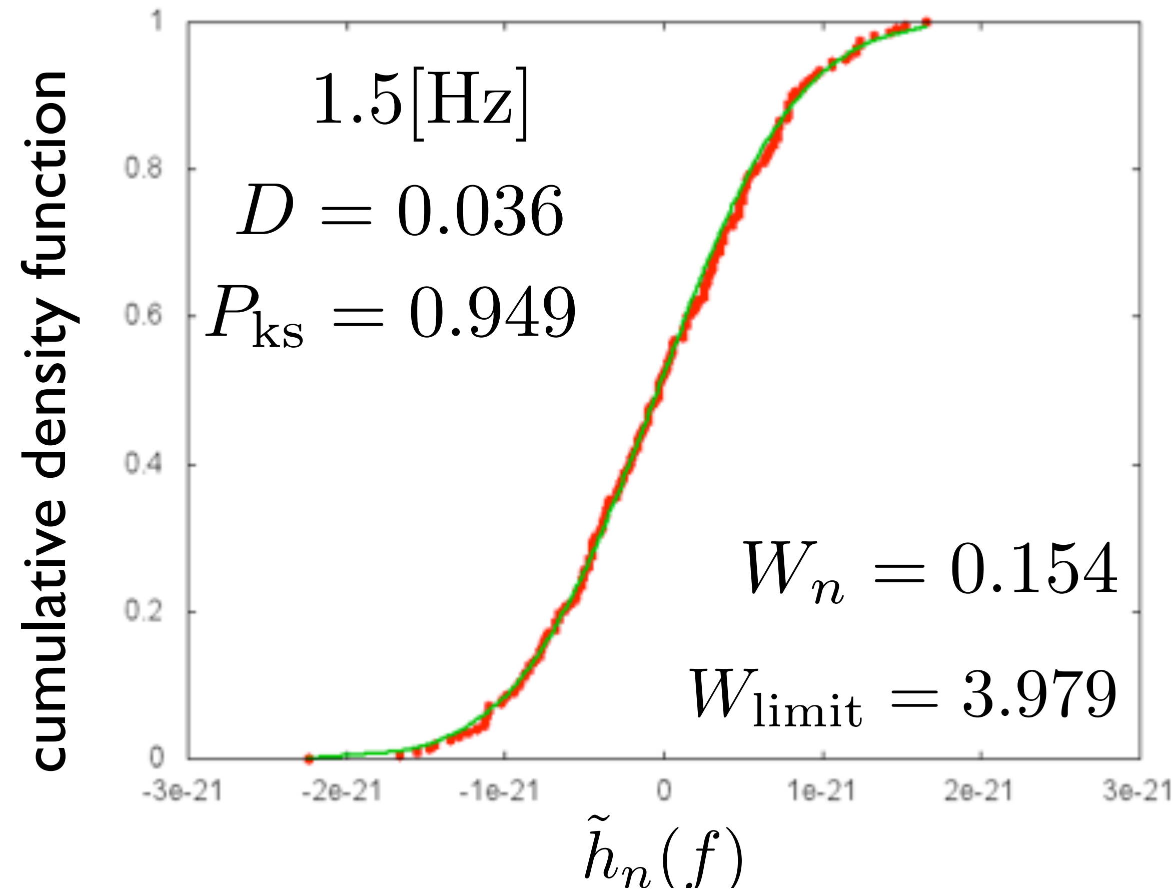


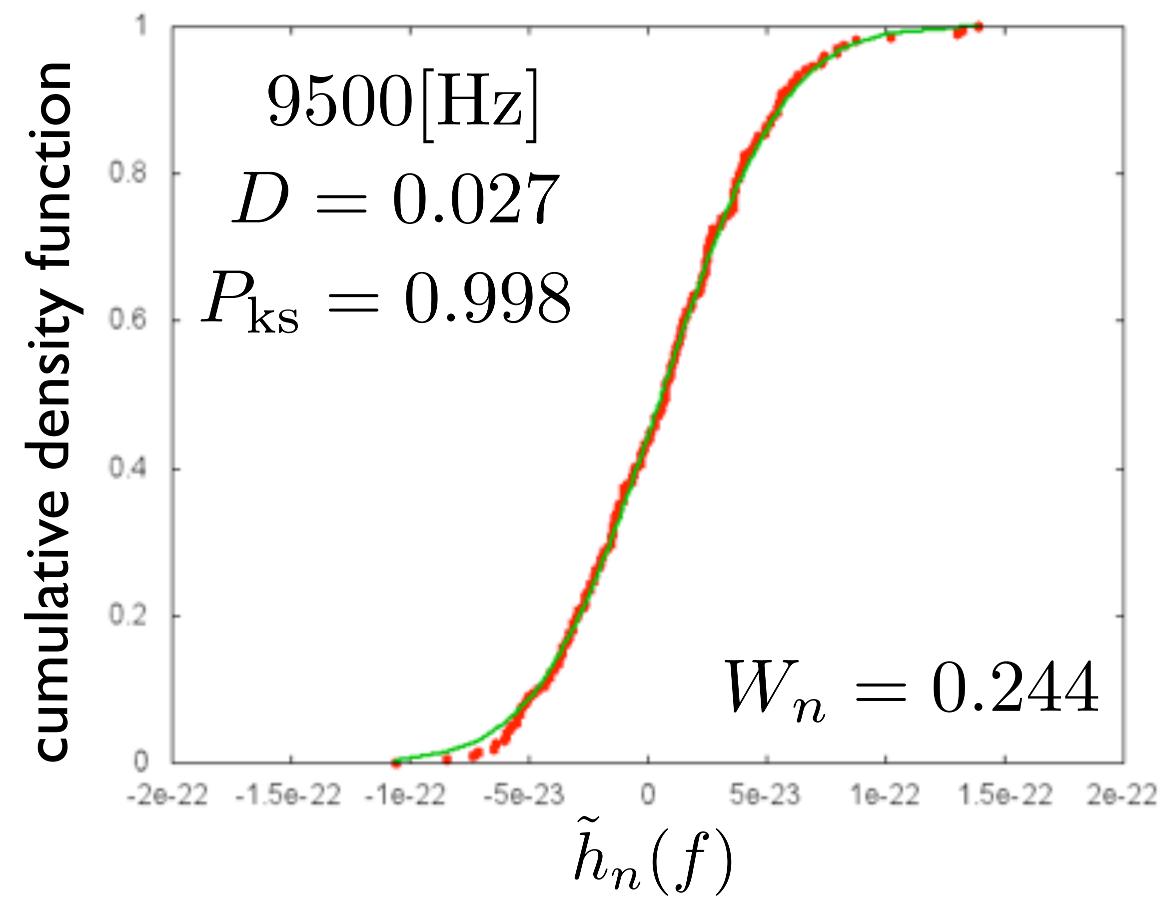
Appendix

about 3000Hz









Generated noise also keeps gaussianity at non-controlled frequencies.