Folding PRC for bLCGT (in case of 1.6-1.9 km arm cavity)

Kazuhiro Agatsuma,

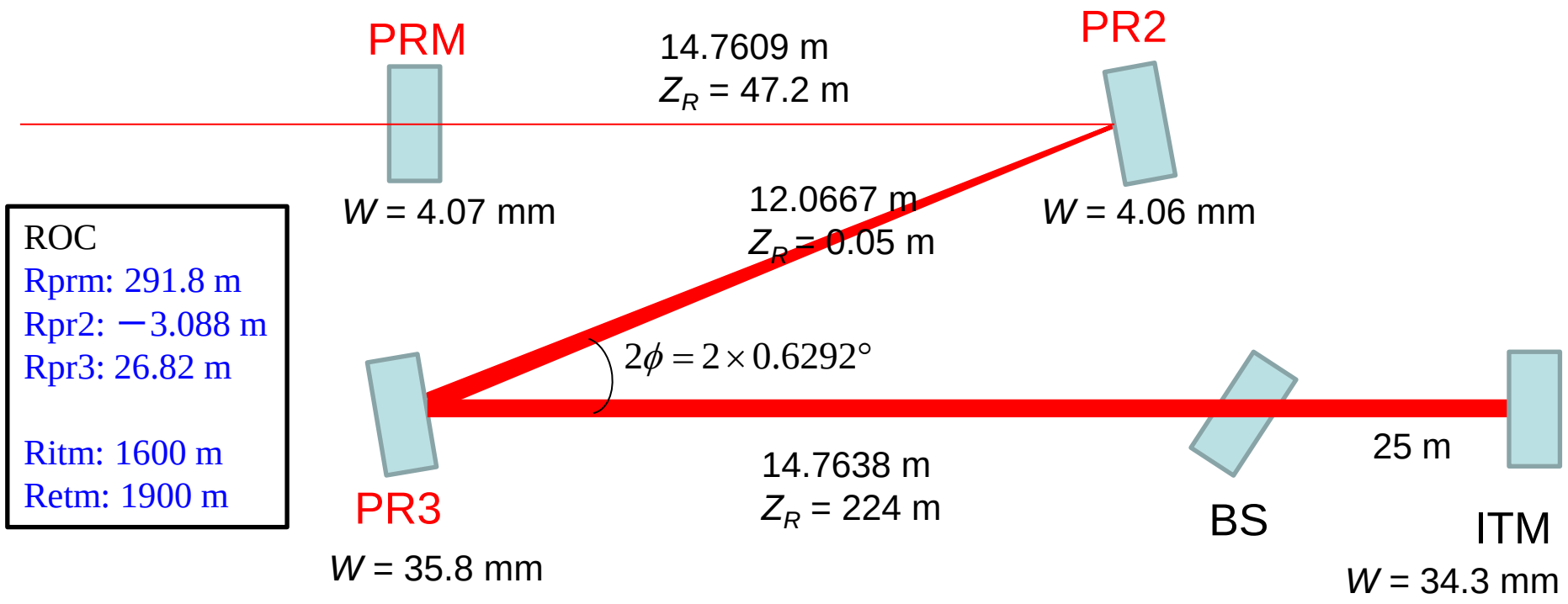
Chen Dan

Contents

- Design of the Power Recycling Cavity (Radius of Curvature etc.)
- Mode miss match by ROC error of the PRC mirrors
- Recovery of the mode match by changing the position of PRC mirrors
- Degeneracy with higher order mode including effect of the astigmatism

Design of PRC

Beam propagation from arm cavity to PR3 ⇒ PR2 ⇒ PRM



ROC
Rprm: 291.8 m
Rpr2: −3.088 m
Rpr3: 26.82 m

Ritm: 1600 m
Retm: 1900 m

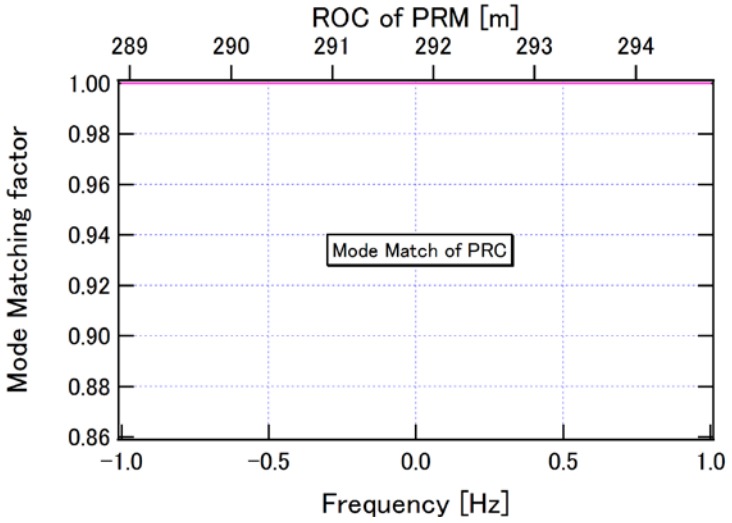
Design concept

- Gouy phase shift from ITM to PRM is 20° <= refer to AdLIGO
  Small gouy phase shift cause degeneracy with HOM
  Large gouy phase shift cause small WFS signal
- Beam radius at PRM close to it at PR2 <= thermal lens effect

⇒ thin parallel beam to thick one

Mode-Matching Factor of PRC

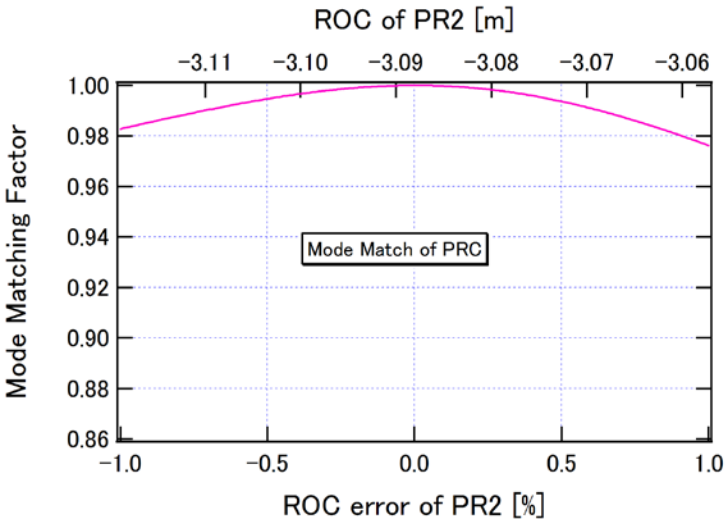
Possible ROC error is 1%



14.7609 m

Z_R = 47.2 m

PR2

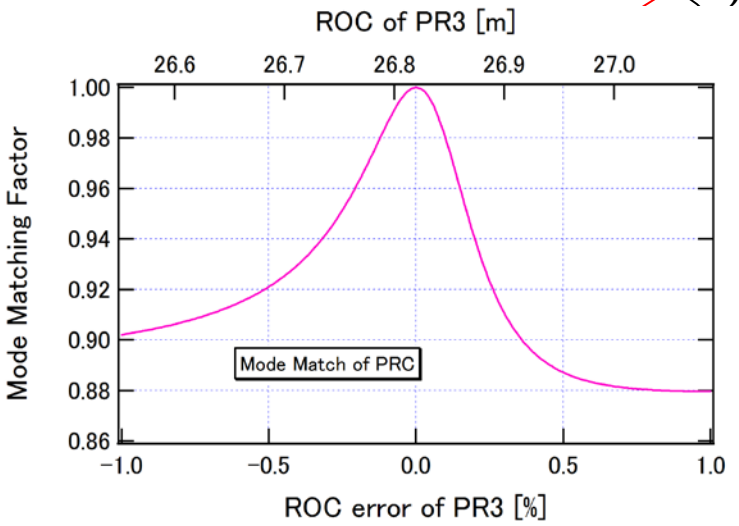


12.0667 m

Z_R = 0.05 m

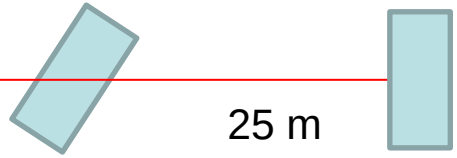
PR3

$2\phi = 2 \times 0.6292°$



14.7638 m

Z_R = 224 m



ROC error of PR3 is serious!

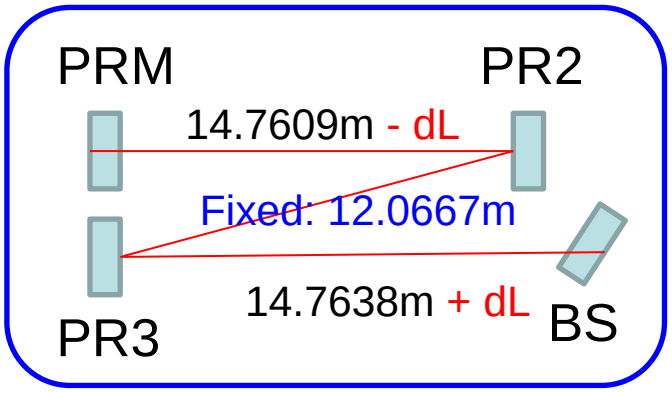
Patterns of changing mirror position

ROC error of the PR3 cause a mode-miss match of PRC.

Is it possible to recover the mode match by changing the position of the mirrors as the length of the cavity is kept?

There are three patterns bellow.

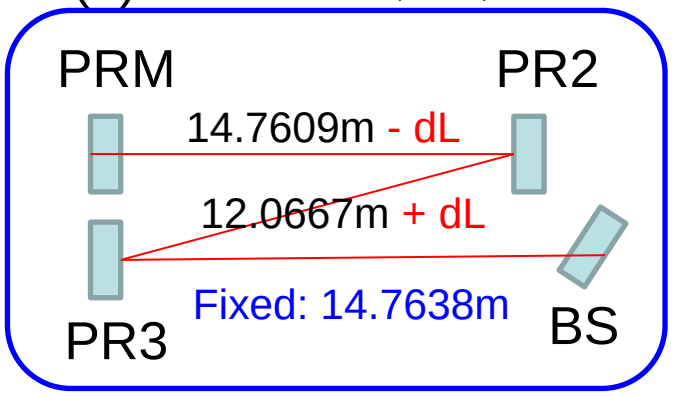
(1). PR2-PR3 fixed



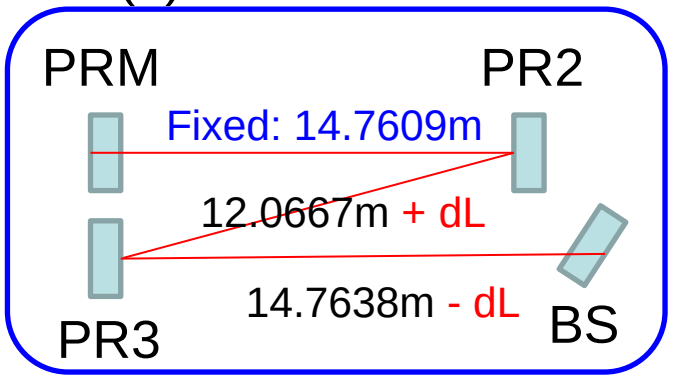
PRM-PR2: Z_R = 47.2 m
PR2-PR3: Z_R = 0.05 m
PR3-ITM: Z_R = 224 m

Patterns are classified whether change of the length PR2-PR3 is included or not.

(2). PR3-ITM（BS）fixed

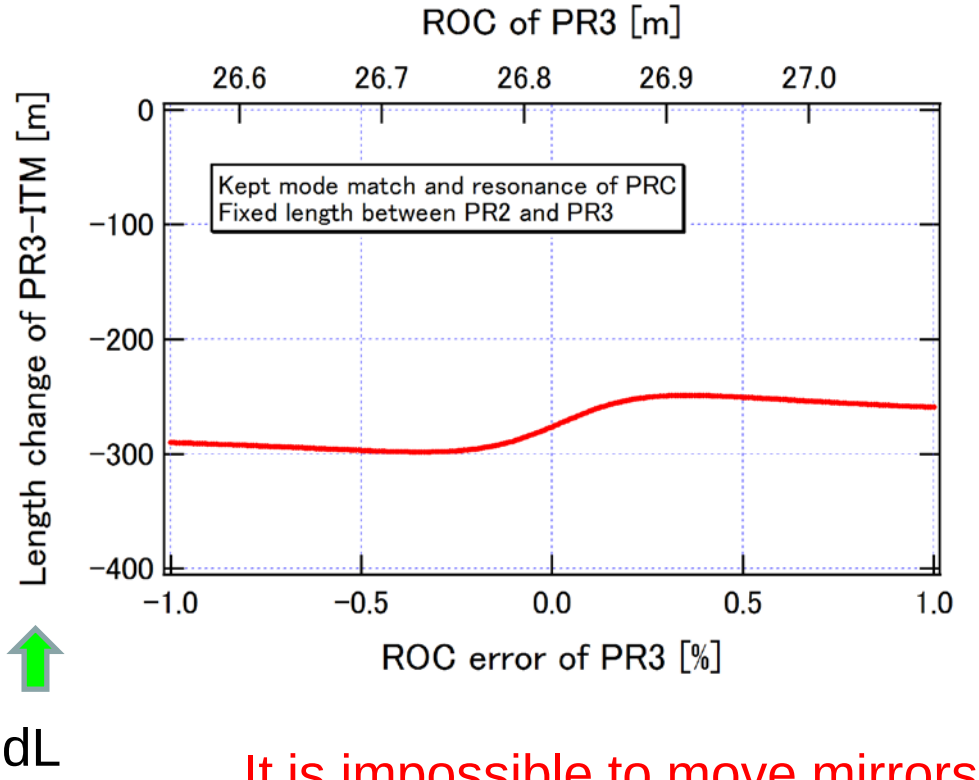


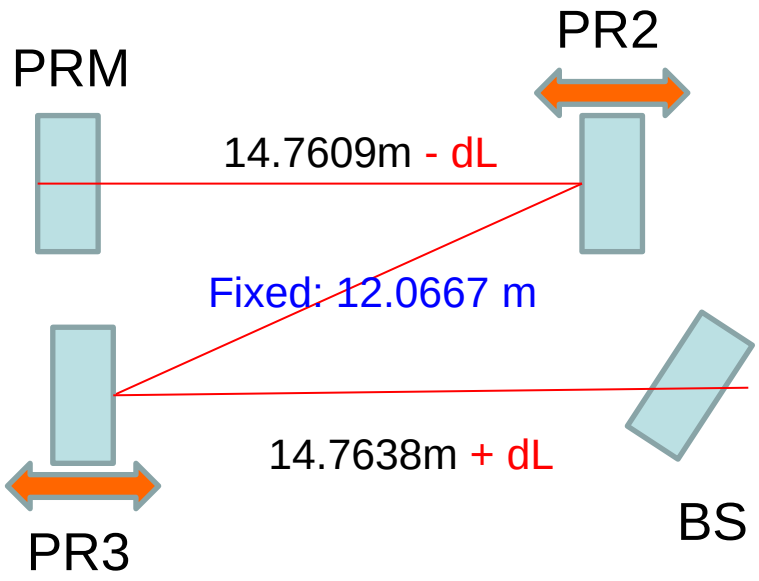
(3). PRM-PR2 fixed



PR3 error cancel (1)

To recover the mode match
=> adjusting ROC of wave front at PRM to ROC of PRM by changing the length PR3-ITM, PRM-PR2
(fixed parameters: ROC of PRM (292 m), Length of PRC and PR2-PR3)

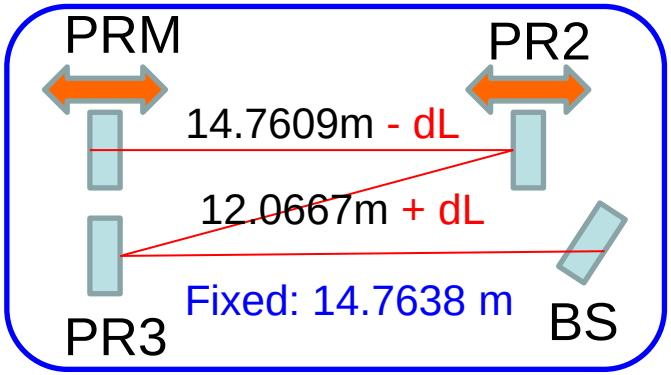


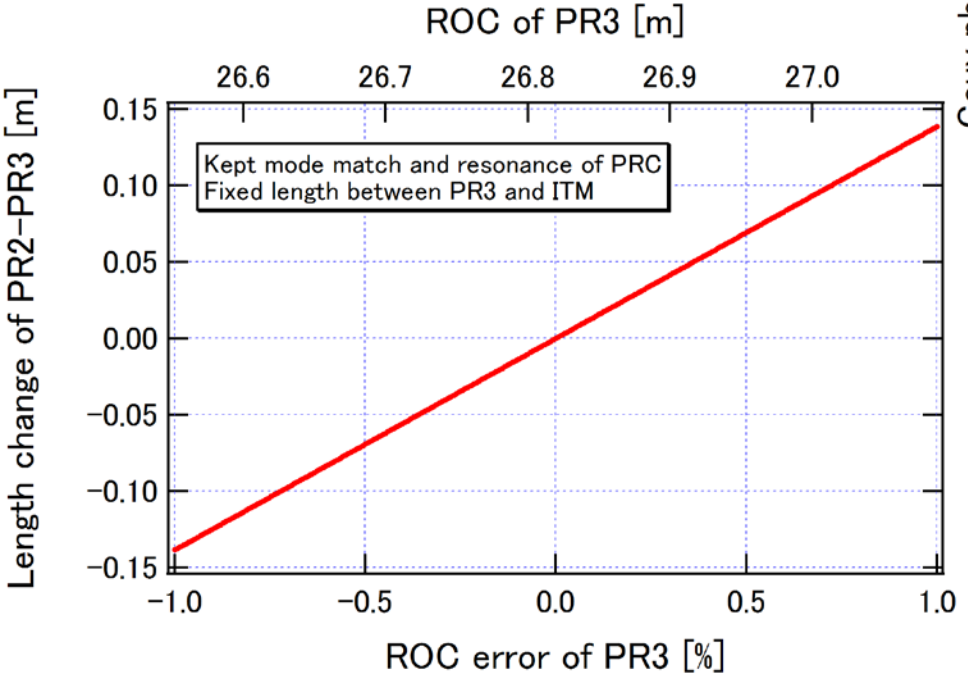


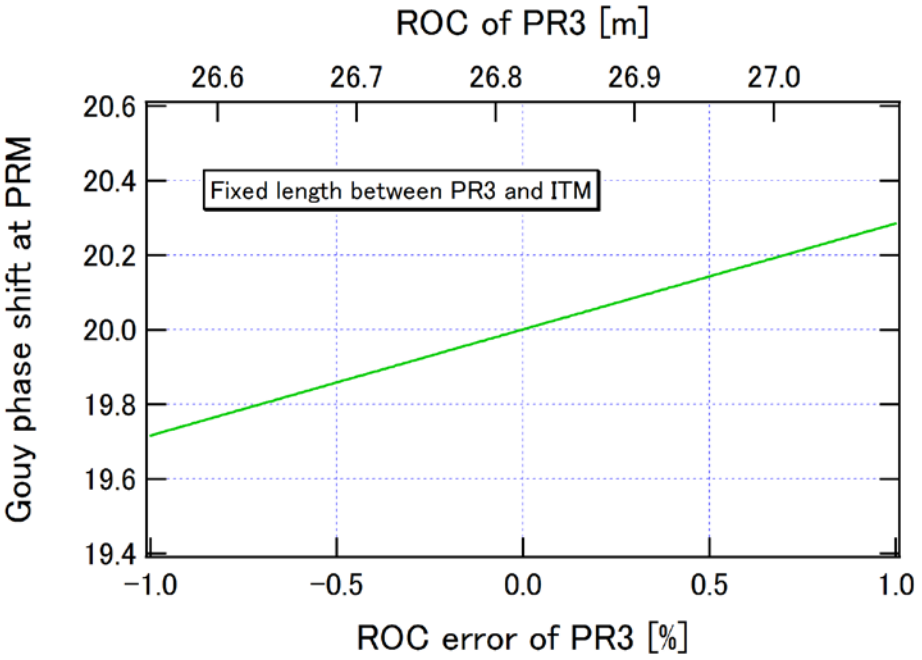
It is impossible to move mirrors by 300 m!
⇒ Pattern (1) is ruled out

PR3 error cancel (2)

Adjusting ROC of wave front at PRM to ROC of PRM by pattern (2).
(fixed parameters: ROC of PRM (292 m), Length of PRC and PR3-ITM)



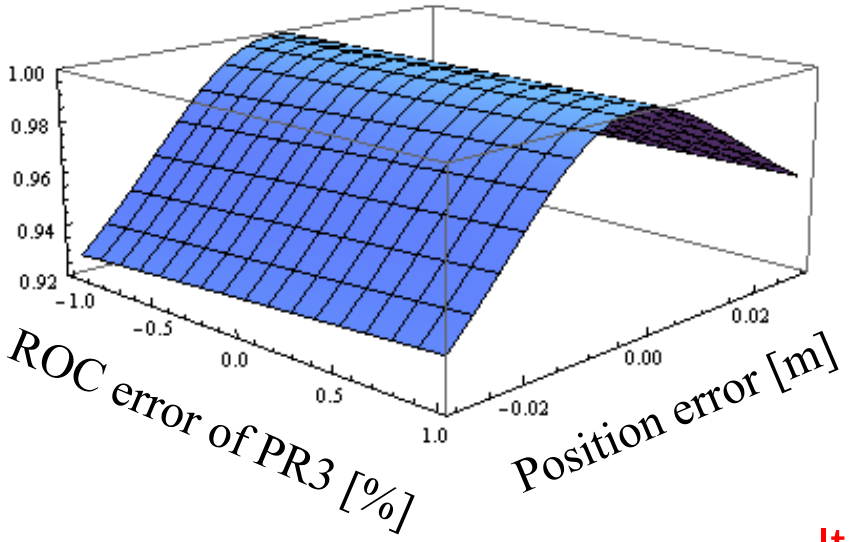


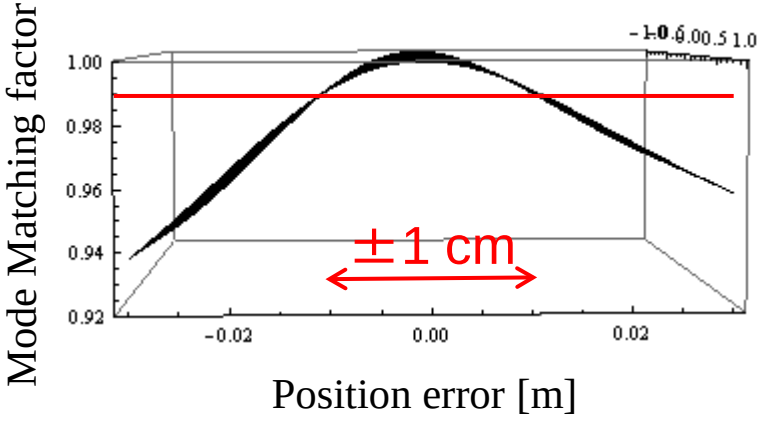


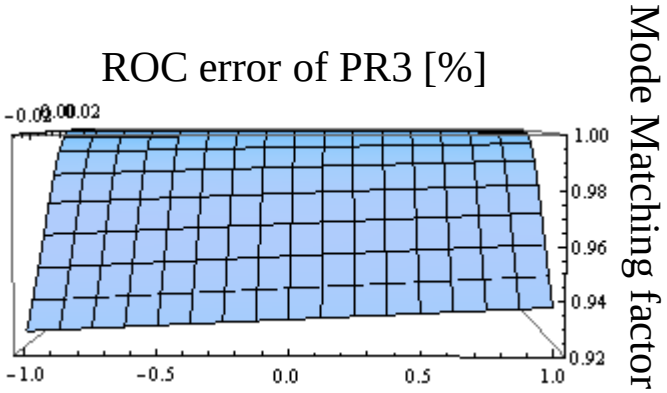
- It is possible to recover the mode match up to almost 100% by changing length ±14 cm.
  ⇒ PRM position need to move 28 cm.
- Gouy phase aberration is ±0.3°
- Pattern (3) is comparable with (2).

Position Error Effect

How much is the accuracy of a position shift to recover the mode match?
⇒ Calculation of the mode-matching factor as position error (±3cm) occur in the pattern (2).







It is possible to recover the mode match up to 99% by holding error of changing length by less than 1 cm.

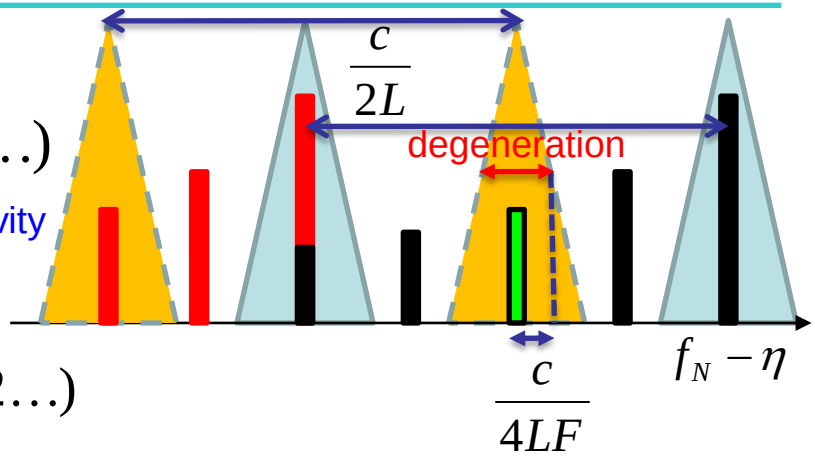
Q. Can we move vacuum tanks of PRC over 28 cm?
A. Yes! (by Takahashi-san)

PRC Degeneracy with Higher Mode

Condition of resonance with HOM (half way)

$$k_0 L - (n+m+1)\eta = \frac{\pi}{2} + N\pi \quad (N = 0, 1, 2\ldots)$$

Reflection from the arm cavity



In the frequency region (half way)

$$f_N - \frac{c}{2\pi L}(n+m+1)\eta = \frac{c}{2L}\left(N + \frac{1}{2}\right) \quad (N = 0, 1, 2\ldots)$$

The degeneracy happens when a center frequency of HOM enter a region of line width of the fundamental mode.
The condition of degeneracy for a cavity with finesse *F* is

$$\frac{c}{2\pi L}(n+m)\eta = \frac{c}{4L} \pm \frac{c}{4LF}$$

k_0 : Wave number of laser
L : Cavity length
η : Gouy phase shift (one way)
n, m : Order of the HG
F : Finesse

Astigmatism cause a difference of Gouy phase η between two planes.

Tangential plane : $f_{ix} = f_i \times \cos\phi_i$ → η_x
Sagittal plane : $f_{iy} = f_i / \cos\phi_i$ → η_y

f : Focus length
ϕ : Folding angle
i : PR2 or PR3

$$n\eta_x + m\eta_y = \frac{\pi}{2} \pm \frac{\pi}{2F}$$

Degeneracy with Higher Mode

$$n\eta_x + m\eta_y = \frac{\pi}{2} \pm \frac{\pi}{2F} \text{ and } \frac{3\pi}{2} \pm \frac{\pi}{2F} \Rightarrow 1.41\sim1.73,\ 4.56\sim4.87$$

$$\phi = 0.6292^\circ = 0.011[\text{rad}]$$
$$F = 10, \quad \eta \approx 20^\circ$$

n＼m	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.35	0.69	1.04	1.39	1.74	2.08	2.43	2.78	3.13	3.47
1	0.35	0.70	1.05	1.39	1.74	2.09	2.44	2.78	3.13	3.48	3.82
2	0.70	1.05	1.40	1.74	2.09	2.44	2.79	3.13	3.48	3.83	4.18
3	1.05	1.40	1.75	2.09	2.44	2.79	3.14	3.48	3.83	4.18	4.53
4	1.40	1.75	2.10	2.45	2.79	3.14	3.49	3.83	4.18	4.53	4.88
5	1.75	2.10	2.45	2.80	3.14	3.49	3.84	4.19	4.53	4.88	5.23
6	2.10	2.45	2.80	3.15	3.49	3.84	4.19	4.54	4.88	5.23	5.58
7	2.46	2.80	3.15	3.50	3.85	4.19	4.54	4.89	5.23	5.58	5.93
8	2.81	3.15	3.50	3.85	4.20	4.54	4.89	5.24	5.59	5.93	6.28
9	3.16	3.50	3.85	4.20	4.55	4.89	5.24	5.59	5.94	6.28	6.63

PRC does not degenerate with HOM in a order of (n+m)<10

(because the phase is between (n+m) = 4 and 5)

The split of HOM (n+m) is negligible in case of the folding angle $\phi = 0.6292^\circ$

η~20° is important

⇒ 4th order mode start to degenerate from η = 20.2° , (max is 22.5°)

⇒ 5th order mode start to degenerate from η = 19.8° , (max is 18°)

Folding Angle and HOM

Calculation for degeneracy by changing the folding angle of PRC

Folding angle to degenerate (n+m) = 4

(n, m)	(4, 0)	(3, 1)	(2, 2)	(1, 3)
(degree)	3.01	3.81	5.23	7.30

$\frac{\pi}{2} \pm \frac{\pi}{2F}$: including line width

1.00°

The 4th order mode start to degenerate if the PRC have a folding angle over 1°

⇒ Requirement for the folding angle is less than 1 degree.

Summary

For bLCGT configuration (1.6-1.9km arm cavity)

●Mode miss match of PRC by ROC error

- ROC error (1%) of PR2 reduce the mode match down to 98%
- ROC error (1%) of PR3 reduce the mode match down to 88%.

⇒ ROC error of PR3 is serious

●Recovery of mode match by changing position of mirrors

- It is possible to recover the mode match up to almost 100% by changing length of mirrors by ±14 cm for the ROC error (1%) of PR3.

⇒ PRM position need to move over 28 cm. It is possible.

⇒ Gouy phase aberration is ±0.3˚

- Error of correction length need to suppress within ±1 cm to recover the mode match up to 99%.

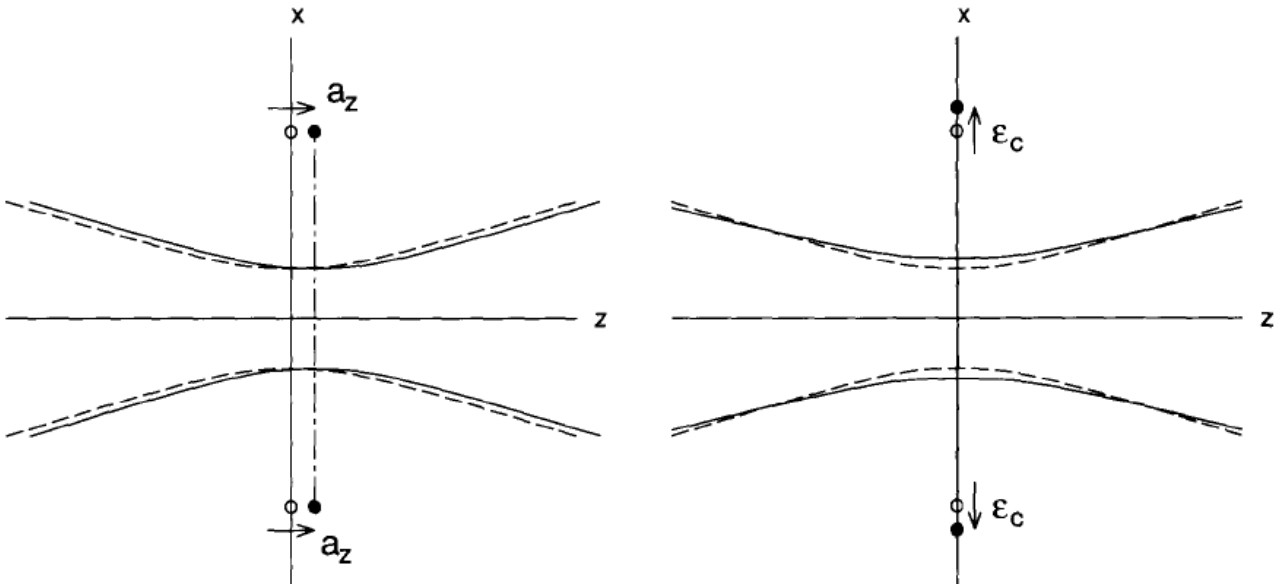
●Degeneracy with higher order modes

- PRC does not degenerate with HOMs in order of (n+m)<10.
- Requirement for the folding angle is less than 1 degree.

The similar result was obtained for iLCGT configuration (Flat-7km arm cavity)

Additional slide

Mode Matching Factor



$$\left\langle \psi_{00}^{\mathrm{LG}} \left| \hat{P}(a_z)\hat{S}(\epsilon_c) \right| \psi^{\mathrm{FG}} \right\rangle = \frac{2z_{\mathrm{R}}\sqrt{1+\epsilon_c/z_{\mathrm{R}}}}{2z_{\mathrm{R}}+\epsilon_c+\mathrm{i}a_z} \equiv MM$$

Accordance between the Rayleigh range, Z_R ,and the beam radius at the waist

<< Definition >>
Mode matching factor : MM^2
Mode miss-match : $(1 - MM^2)$