Torsional Mode Damping for the GAS Filter Chain

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1. About This Work

In the seismic attenuation system (SAS) for LCGT, a chain of GAS filters is planned to be used for the vertical seismic attenuation [1]. In the chain, each filter is suspended by a single wire. As a wire is very flexible in its torsional mode, good attenuation in the yaw mode is achieved by this configuration. However, as the energy dissipation of this mode is small (it means Q is high) and the resonant frequency is quite low, the filters will keep oscillating for a long time if this mode is excited. According to the experience in TAMA, it takes about few hours to stop the oscillation without any damping mechanism. During this time, the interferometer cannot be locked and we have to wait. Some damping mechanism should be prepared to avoid this situation.

We plan to damp this mode passively by eddy current. A disc with several permanent magnets attached on the bottom (I call it the "magnet disc" in the following sentence) is suspended from the filter 0, and damps the yaw motion of the filter 1 (see Fig. 2). The yaw motion of the filter 2, filter 3 and payload is also damped, if the stiffness of wires is set appropriately. What we have to do is to find the optimal magnet configuration and stiffness of wires so that the yaw motion is damped effectively in all filters.

As the magnet disc damps also the translational motion of filters (both in horizontal and vertical directions), it may worsen the attenuation performance of SAS in higher frequency (the transfer function may drop not f^{-2n} but f^{-2n+1}). So we have to design the magnet disc and its suspension carefully so that it does not spoil the performance of SAS.

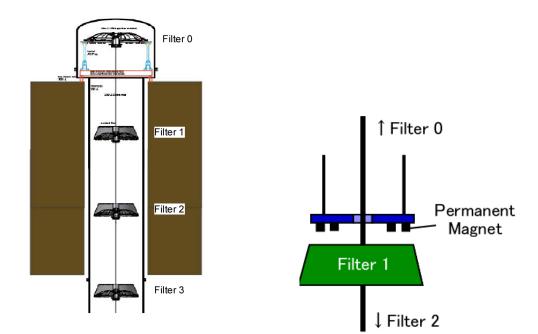


Figure 1: Current design of SAS [1]

Figure 2: Damping mechanism

In this work, I did a simple simulation of the yaw motion of the GAS filters in order to determine the damping coefficient of eddy current damping, stiffness of wires and the suspension style of the magnet disc. As the stiffness of the wire depends on its thickness, the diameters of the wires can be determined. I will propose a new wire design in this paper.

2. Model

To simulate the yaw motion of the filters, I constructed one dimensional "point-mass" model drawn in Fig. 3. For simplicity, the yaw motion of the inverted pendulum stage (supporting the filter 0) and the inner structure of the payload are neglected. As they have much higher resonant frequencies, we can regard them as rigid structures. A new filter, the control box, is inserted between the GAS filter chain and the payload in order to control the motion of the payload actively, just like the marionette in Virgo [2]. To control the motion of the payload in various degrees of freedom, the connection of the control box and the payload must be a single wire. So we must also take into account the torsional mode of this wire. Unfortunately the design of the control box has not fixed yet, so I deal with various possibilities of the moment of inertia of the control box.

To know the response of the system to external torque, impulsive torque is exerted to the payload. The torsional motion of the payload is sensed, and then the impulse response and the transfer functions (input: torque exerted to the payload, output: torsional angle of the payload) are calculated by using MATLAB Control System Toolbox. The inertial damping of the wire is neglected in this simulation

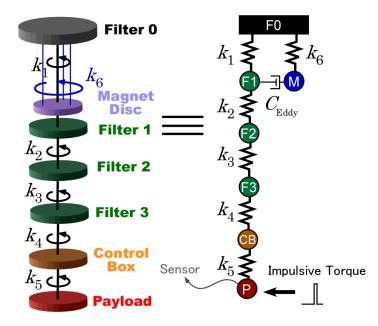


Figure 3: Simulation model

3. Torsional Stiffness of Wire

Torsional stiffness of a wire with the length L and the shear modulus G (77 GPa for maraging steel) is given as

$$k_{\phi} = \frac{GJ}{L}.$$

Here *J*, the torsional moment of inertia of a circular cross section is $J = \pi D^4/32$ with the wire diameter *D*[3]. As the stiffness is proportional to D^4 , the GAS filter chain gets soft rapidly in the yaw mode if we use thin wires. However, we cannot make the wires so thin because the tensile strength of the wires must be less than the yield tensile strength so that they do not break down. So the minimum diameter of a wire is determined by the weight of its load. The yield tensile strength of maraging steel is 1800 N/mm², but I set the maximum tensile strength 500 N/mm² with a safety factor of more than 3. The minimum diameter D_{min} of each wire is calculated as

$$(\pi/4) \times D_{\min}^2 \times 500 \text{ N/mm}^2 = M_{\text{load}} \times 9.81 \text{ m/s}^2.$$

The following table shows the calculated minimum diameter and stiffness of each wire. The mass of each GAS filter is assumed to be 87 kg.

#	Length [m]	Load [kg]	Diameter [mm]	Stiffness [Nm/rad]
1	2.1	561	> 3.74	> 0.70
2	2.1	474	> 3.44	> 0.50
3	2.1	387	> 3.11	> 0.34
4	2.1	300	> 2.74	> 0.20
5	1.0	240	> 2.45	> 0.27

Table 1: Wire parameters

4. How to Suspend the Magnet Disc

There are two ways to suspend the magnet disc, 6-wire or 3-wire (Fig. 4). The 6-wire suspension restricts the motion of the magnet disc in every 6 DOF (rigid connection). On the other hand, 3-wire suspension is flexible in the X, Y and yaw mode. Stiffness for the yaw mode of 3-wire suspension depends on the configuration of the wires. If attached points of wires and the center of disc are separated by R (see Fig. 4), the stiffness for the yaw mode is given as

$$k_{\phi} = \frac{MgR^2}{L} \,,$$

where *L* is the wire length and *M* is the mass of the disc. If we assume M = 30 kg and L = 2.0 m, the stiffness for the yaw mode in each *R* is calculated in Tab. 2. We can make it as flexible as a single wire suspension, or make it rigid.

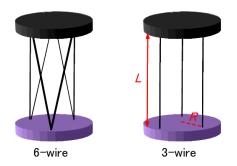


Figure 4: Two ways to suspend the magnet disc

R [m]	Stiffness [Nm/rad]
0.05	0.37
0.10	1.47
0.35	18.0

Table 2: Torsional stiffness of 3-wire suspension

5. Moment of Inertia

The moment of inertia of each filter is shown in Tab. 3. The value of the standard GAS filter 6.24 kgm² is estimated by G. Gennaro. The moment of inertia of the payload 5.61 kgm² is a sum of all the filters in the current design of the suspension system. As this is not a final design, there's a possibility of changing, but it will not change dramatically. As I mentioned in Sec. 1, the moment of inertia of the control box is not fixed yet, so I tested several values 2.0, 4.0 and 6.0 kgm². The value of the magnet disc 1.84 kgm² is estimated by assuming it as a disc with 30 kg mass and 70 cm diameter. Of course, this value has not fixed and can be changed.

Filter Name	Detail	Moment of Inertia [kgm ²]
Standard Filter	Filter 1~3	6.24
Payload	Platform	1.95
	Magnet Box	2.35
	Intermediate Mass	0.69
	Recoil Mass	0.45
	Test Mass	0.17
	Total	5.61
Control Box		2.0, 4.0, 6.0 (Not Fixed)
Magnet Disc		1.84 (Not Fixed)

Table 3: Moment of inertia list

6. Process to Find Optimal Parameters

There are a lot of parameters to be optimized, stiffness of wires $k_1 \sim k_5$, stiffness of the magnet disc suspension k_6 , damping coefficient of eddy current C_{Eddy} and moment of inertia of the magnet disc I_{m} . I tune these parameters by checking the transfer function and the impulse response of the system with slightly changing the values of parameters and find the optimal parameters that shorten the damping time most.

First, the spring constants of wires $k_1 \sim k_5$ and the damping coefficient C_{Eddy} are optimized, in the case of robust suspension of the magnet disc (large k_6). Then, the stiffness of the suspension k_6 and the moment of inertia of the magnet disc I_m are optimized. In the process of optimization, the stiffness of wires $k_1 \sim k_5$ are made as small as possible because a thinner wire is better for seismic attenuation (see Sec. 8).

7. Calculation Results and Optimal Parameters

The parameters optimized in the above-mentioned process are shown in Tab. 4-6 for each case of the moment of inertia of the control box ($I_c = 2.0, 4.0 \text{ and } 6.0 \text{ kgm}^2$). In any case, it turns out that robust suspension (large k_6) is better for the magnet disc. We cannot determine the moment of inertia of the magnet disc I_m from this simulation (as the suspension is robust, any value is OK).

The yaw mode of the payload is damped well when the motions of all the filters are coupled well. In the case of $I_c = 6.0 \text{ kgm}^2$, the moment of inertia of each filter is almost same, so we can make their yaw modes coupled well by making the stiffness of each wire same ($k_1 = k_2 = k_3 = k_4 = k_5$). On the other hand, when the moment of inertia of the control box is relatively small ($I_c = 2.0 \text{ and } 4.0 \text{ kgm}^2$), the wires above and below the control box must be softer than other wires ($k_4, k_5 < k_1, k_2, k_3$).

The transfer function and the impulse response of the system (input: torque exerted to the payload, output: torsional angle of the payload) for the optimized parameters are shown in Fig. 5-7. In any case, the damping time is less than 10 minutes.

Symbol	Value	Unit	Note
k_1	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_2	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_3	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_4	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_5	0.70	Nm/rad	L = 1.0 m, D = 3.11 mm
k_6	>10	Nm/rad	Robust suspension is better.
$C_{ m Eddy}$	5~10	kgm/(s rad)	

Table 4: Optimized parameters ($I_c = 6.0 \text{ kgm}^2$)

Symbol	Value	Unit	Note
k_1	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_2	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_3	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_4	0.50	Nm/rad	L = 2.1 m, D = 3.44 mm
k_5	0.27	Nm/rad	L = 1.0 m, D = 2.45 mm
k_6	>10	Nm/rad	Robust suspension is better.
CEddy	4~8	kgm/(s rad)	

Table 5: Optimized parameters ($I_c = 4.0 \text{ kgm}^2$)

Symbol	Value	Unit	Note
k_1	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_2	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_3	0.70	Nm/rad	L = 2.1 m, D = 3.74 mm
k_4	0.27	Nm/rad	L = 2.1 m, D = 2.94 mm
k_5	0.27	Nm/rad	L = 1.0 m, D = 2.45 mm
k_6	>10	Nm/rad	Robust Suspension
$C_{ m Eddy}$	4~8	kgm/(s rad)	

Table 6: Optimized parameters ($I_c = 2.0 \text{ kgm}^2$)

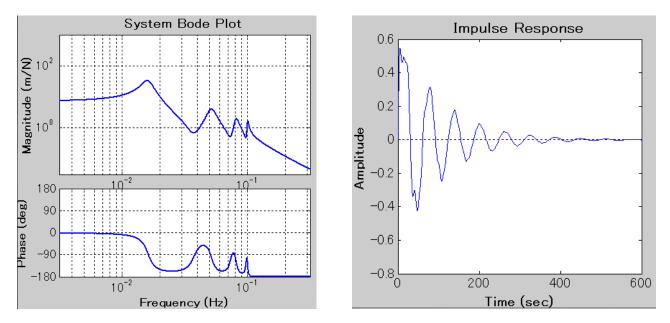


Figure 5: Transfer function and impulse response for optimized parameters ($I_c = 6.0 \text{ kgm}^2$)

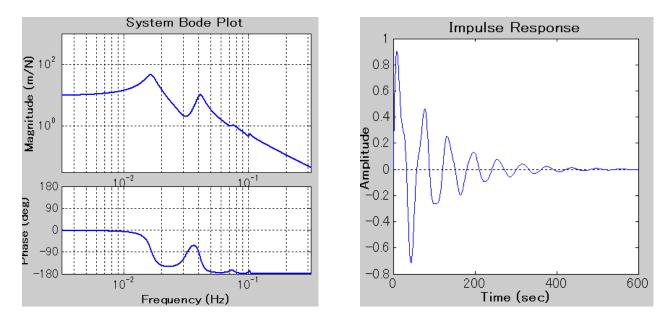


Figure 6: Transfer function and impulse response for optimized parameters ($I_c = 4.0 \text{ kgm}^2$)

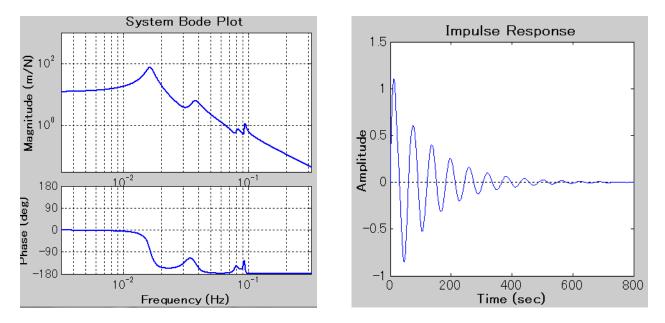


Figure 7: Transfer function and impulse response for optimized parameters ($I_c = 2.0 \text{ kgm}^2$)

8. Wire Design

I propose to make the wires thick from the viewpoint of the torsional mode damping, but a thick wire has a problem in horizontal attenuation. As the wires used in the GAS filter chain are relatively thick, the elasticity of the wire is not negligible. The elastic spring constant of the wire may worsen the horizontal attenuation performance of SAS, so we have to make the wires as thin as possible. However, this elastic spring constant only depends on the wire thickness in the first few cm near the filter [4], while the torsional spring constant depends on the thickness integrated over the wire length. So we can make the wire thin at the ends and thick in the middle like Fig. 8. The proposed wire parameters are shown in Tab. 7 (in the case of $I_c = 2.0 \text{ kgm}^2$).

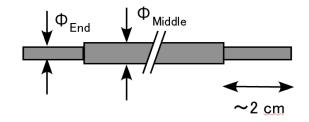


Figure 8: Proposed wire design

#	Diameter (End) [mm]	Diameter (Middle) [mm]	Length [m]
1	3.74	3.74	2.1
2	3.44	3.74	2.1
3	3.11	3.74	2.1
4	2.74	2.94	2.1
5	2.45	2.45	1.0

Table 7: Wire parameters ($I_c = 2.0 \text{ kgm}^2$)

There are other problems that may be caused by thickness of wires (center of percussion (COP) effects, bounce in the vertical direction, etc.). We have to investigate these effects carefully by simulating the horizontal and vertical attenuation of SAS with thickness of wires taken into account (my future works).

References

- [1] R. DeSalvo, *JGW Document*, **T1000249-v3** (2010)
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- [3] A. Takamori, PhD Thesis (2002)

[4] G. Hammond, LIGO Internal Document, T010171-00-D (2001)