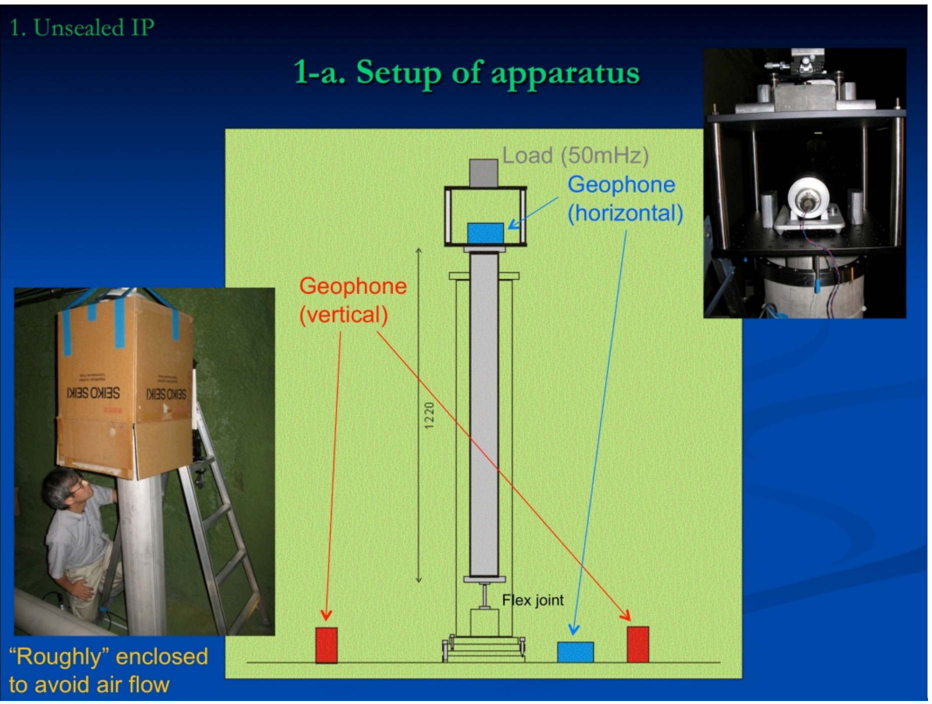
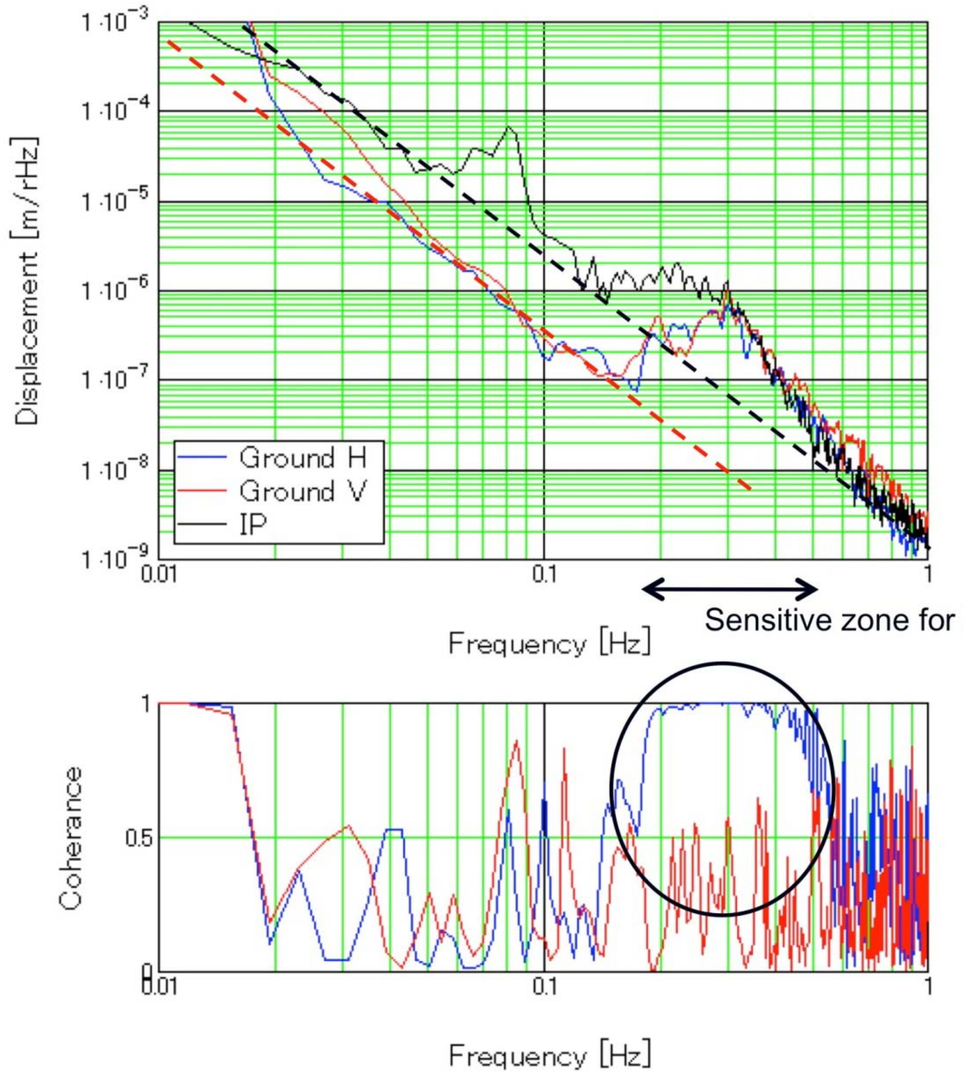
Behavior of an inverted pendulum in the Kamioka mine

R.Takahashi, A.Takamori, and E.Mojorona., *LCGT 1st f2f Meeting*, 2010/06/16.

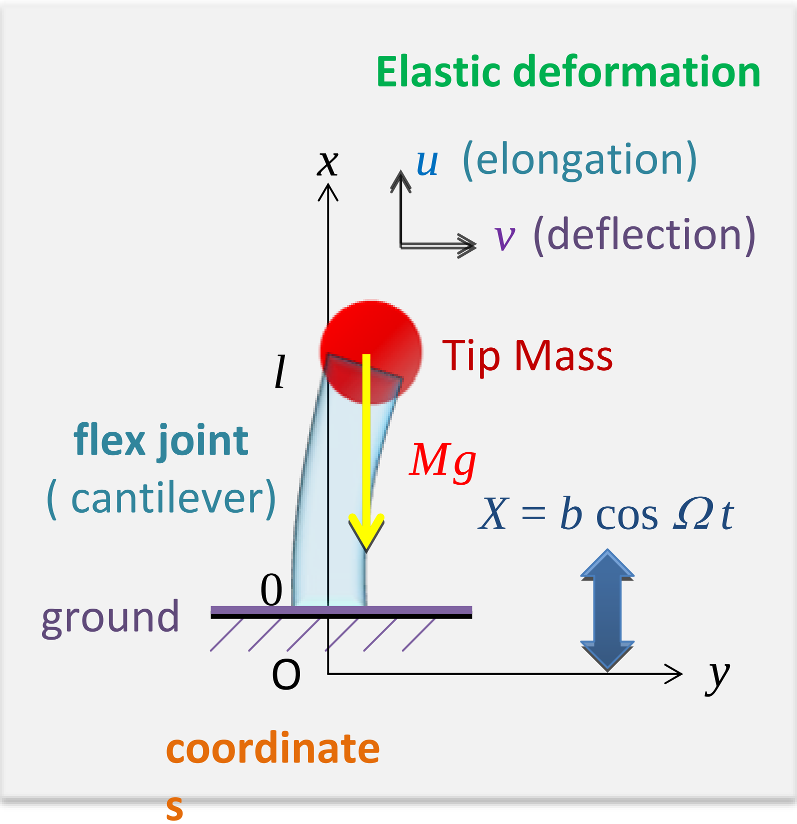




Observational data

Ishizaki's Model

Parametric Excitation



Flex joint was modelled in a continuous system as an elastic body.

The flex joint is excited vertically by the sinusoidal ground motion *X*.

$$X = b \cos \Omega t$$

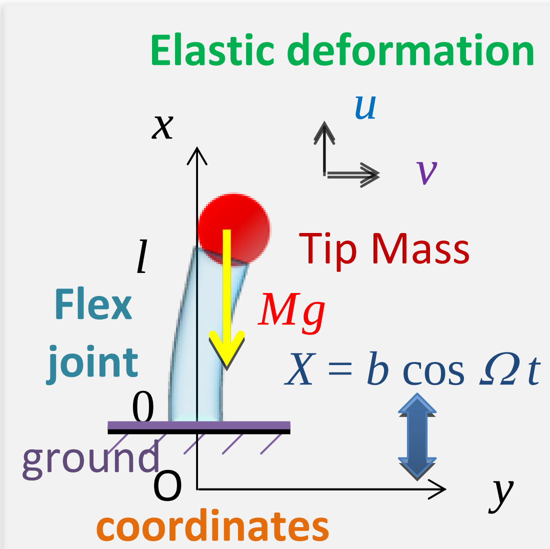
The deflection v is a function of coordinate x and time t.

$$v = v(x, t)$$

The equation of motion for the deflection:

$$\rho A \frac{\partial^2 v}{\partial t^2} + M\left(g + \frac{d^2 X}{dt^2}\right)\frac{\partial^2 v}{\partial t^2} + EI\frac{\partial^4 v}{\partial x^4} = 0.$$

ρ are density , A are cross section, and EI are flexural rigidity of the flex joint.
M is a mass of the tip mass. g is gravitational acceleration.



Parametric Excitation

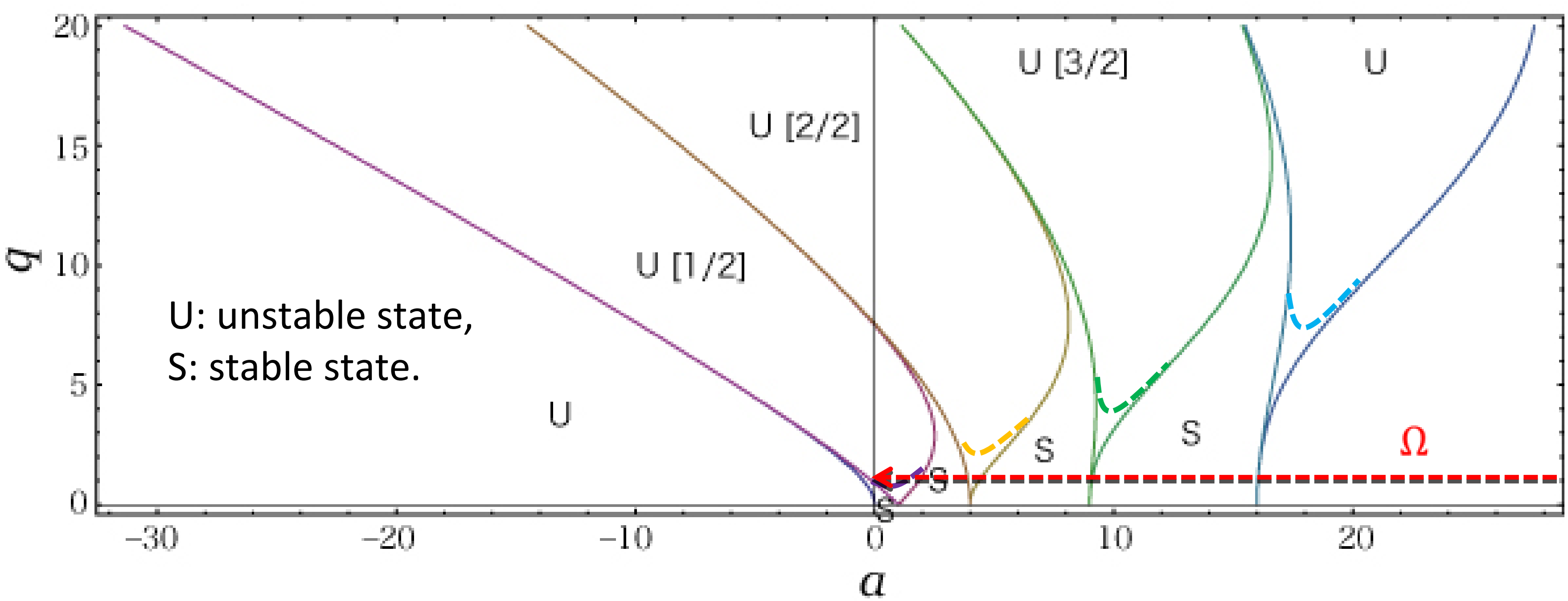
separation of variables: $v(x,t) = \xi(x)\sigma(t)$

$\xi(x)$: the first natural vibration mode (quarter cosine wave)

$$\frac{d^2\sigma}{d\boldsymbol{\tau}^2} + (\boldsymbol{a} - 2\boldsymbol{q}\cos 2\boldsymbol{\tau})\sigma = 0.$$

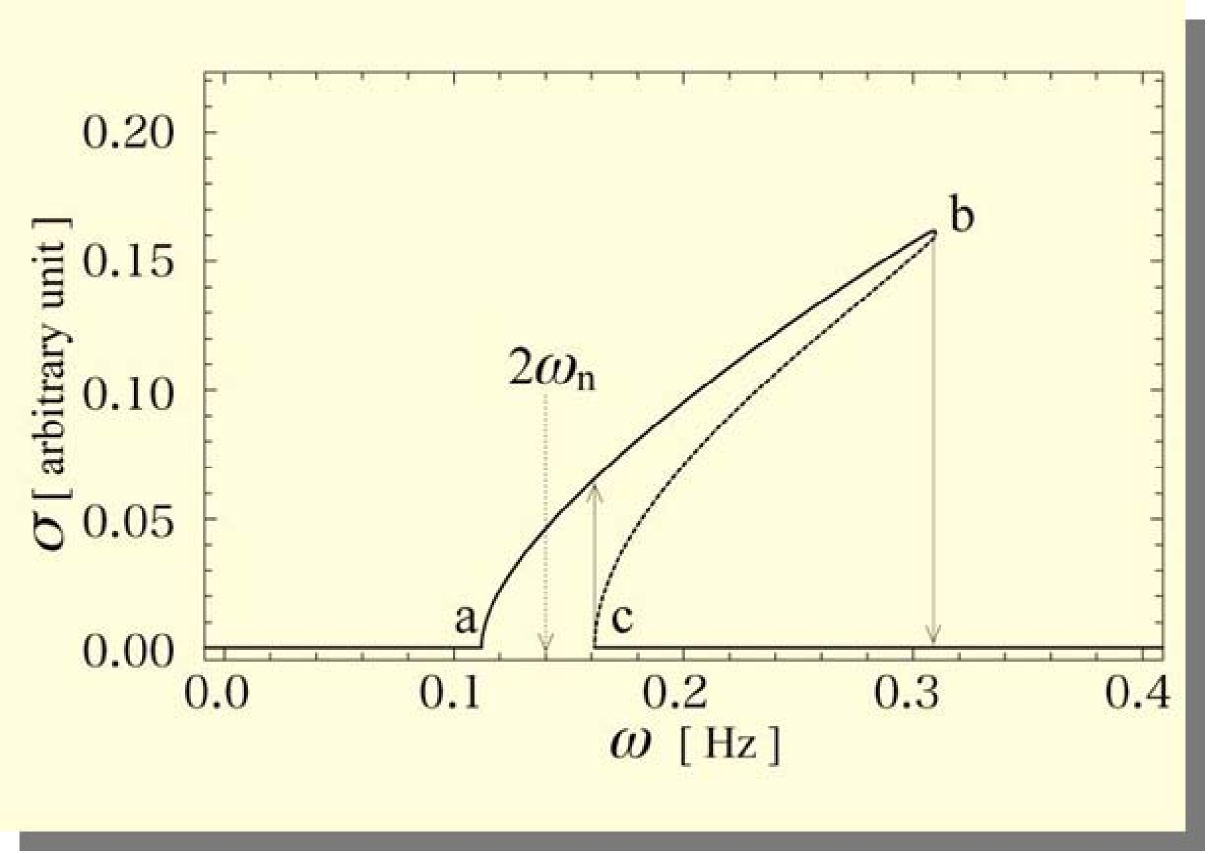
Mathieu's Equation

$$\boldsymbol{a} = \left(2\frac{\Omega_0}{\Omega}\right)^2 \left(1 - \frac{Mg}{P_{\mathrm{cr}}}\right), \qquad \boldsymbol{q} = 2\frac{M{\Omega_0}^2}{P_{\mathrm{cr}}}\, b, \qquad \boldsymbol{\tau} = \frac{\Omega\, t}{2}.$$



Parametric Excitation

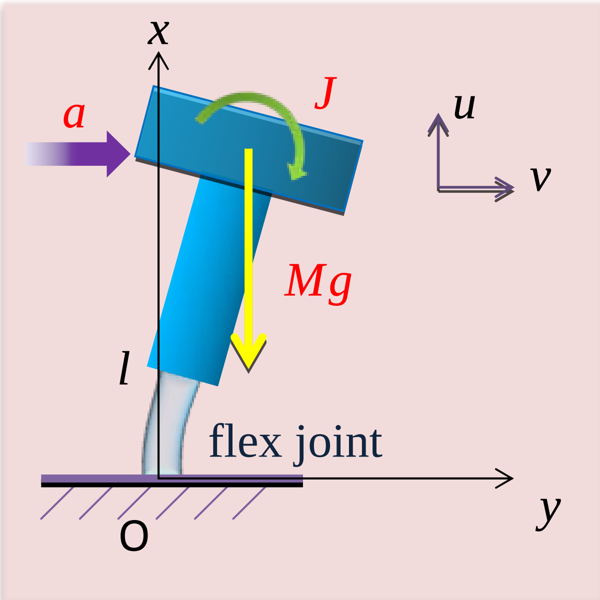
$$\frac{d^2\sigma}{dt^2} + 2\gamma\omega_n\frac{d\sigma}{dt} + \omega_n{}^2(1 - q'\cos\omega t)\sigma + \varepsilon\omega_n{}^2\sigma^3 = 0.$$

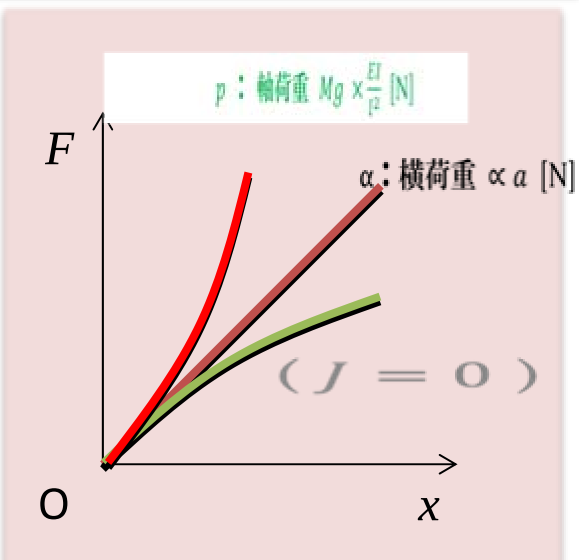


Frequency response of the non-linear Mathieu's equation.

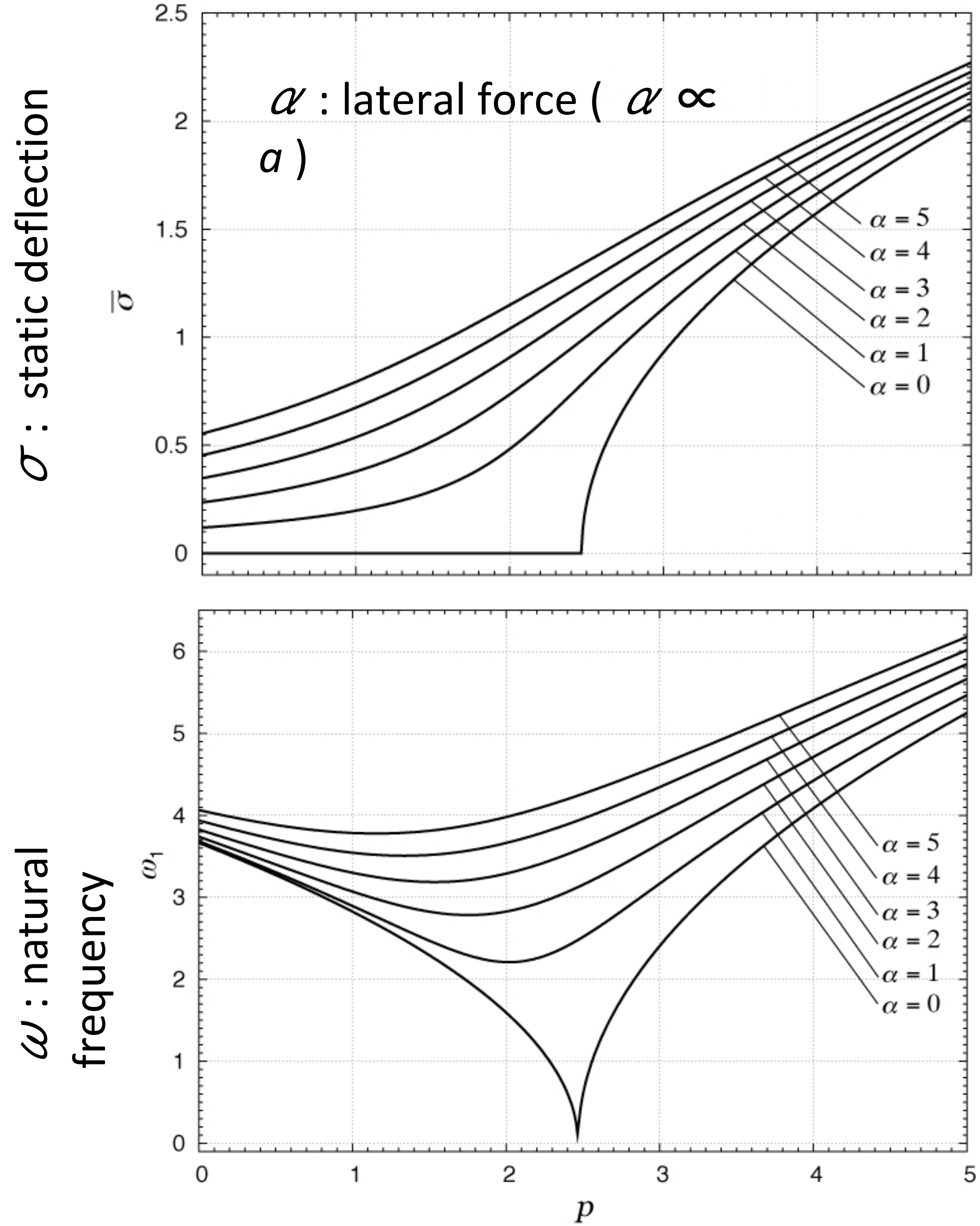
There is a resonance at twice frequency with the natural frequency.

buckling





$$\frac{F}{m} = \frac{k}{m}x \pm \frac{k}{m}\frac{k'}{k}x^3 = \omega_n{}^2 x \pm \omega_n{}^2 \varepsilon x^3.$$



p : axial force ($p \propto Mg$)