Shot noise with RF readout

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This note is a brief summary to estimate a shot-noise level with RF readout, which was originally written in Japanese for the LCGT project, and is translated to English with some changes, aimed at the shot-noise estimation of GEO.

With RF readout, in addition to the vacuum around the carrier light, the vacuum field around the double frequency of RF sidebands imposes noise at the demodulation process. As this noise level changes with the demodulation phase, it is called non-stationary shot noise. If we assume there is a pair of RF sidebands ($\pm f_{\rm m}$) around the carrier light and the demodulation function is a pure sin wave, it is only vacuum fields around $\pm 2f_{\rm m}$ that contributes as non-stationary shot noise. In reality, the demodulation will be done by a switching circuit, and the demodulation function will be rather a square wave; not so steep square wave as higher harmonics will be filtered out. In the case of GEO, another difference is that RF sidebands are unbalanced. It allows us to choose the quadrature [1] but the contribution of non-stationary shot noise will increase.

Reference [2] shows how to calculate non-stationary shot noise with a generic demodulation function. Let us pick some examples so that it will be easier to see it. At the dark port, RF sidebands consist of amplitude modulation $A(t)$ and phase modulation $P(t)$, then the power spectrum density of non-stationary shot noise is given as

$$S^{\rm add} = \frac{1}{T}\int_0^T D^2(A^2+P^2)dt - 1. \tag{1}$$

Here $D(t)$ is the demodulation function, which is normalized by

$$\left(\sum_{\rm k} D_{\rm k}^* A_{\rm k}\right)^2 + \left(\sum_{\rm k} D_{\rm k}^* P_{\rm k}\right)^2 = 1. \tag{2}$$

Subtitle k means k-th order harmonics. We can assume that harmonics fields will not appear at the dark port since they do not resonate in the power-recycling or in the signal-recycling cavity. Thus, $A(t) = 0$. Equation (1) tells us that the amplitude modulation only contributes to increase the noise level while the phase modulation contributes to obtain the signal. These facts have been reported since many years ago. [3][4][5]

First, let us assume there is only $\sin\omega_{\rm m}t$ at the dark port and the demodulation function can be written also by $\sin\omega_{\rm m}t$.

$$P(t) = \sqrt{2}\sin\omega_{\rm m}t \quad , \ D(t) = \sqrt{2}\sin\omega_{\rm m}t \ , \tag{3}$$

or

$$P_{\pm 1} = \frac{\pm i}{\sqrt{2}} \quad , \ D_{\pm 1} = \frac{\pm i}{\sqrt{2}}. \tag{4}$$

Equation (2) is satisfied. Then,

$$\begin{aligned} S^{\rm add} &= \frac{1}{T}\int_0^T 4\sin^4\omega_{\rm m}t dt - 1 \\ &= \frac{1}{2}, \end{aligned} \tag{5}$$

which shows the shot-noise level increases by $\sqrt{1.5}$.

Second, let us assume there is $\sin\omega_{\rm m}t$ at the dark port and the demodulation function is a square wave, which is described as

$$D(t) = \frac{\pi}{4}\left(\sin\omega_{\rm m}t + \frac{1}{3}\sin 3\omega_{\rm m}t + \frac{1}{5}\sin 5\omega_{\rm m}t + \cdots\right) \ . \tag{6}$$

Normalized modulation function is

$$P(t) = \frac{8}{\pi}\sin\omega_{\rm m}t. \tag{7}$$

Putting this into Eq. (1), we obtain

$$S^{\rm add} = \frac{\pi^2}{8} - 1 \simeq 0.2337. \tag{8}$$

This derivation is done by using Zeta function. This number should be used for the LCGT. It has been said that LCGT will see NS-NS binary at 200 Mpc (SN=10) with DC readout and at 160 Mpc with RF readout, but this number is derived with the sine-wave demodulation. It is 180 Mpc with RF readout if we calculate properly with the square-wave demodulation.

Now, let us reduce non-stationary shot noise more by adding harmonics to the modulation fields. We could make 3rd order harmonics resonate in the recycling cavities. Assuming the amplitude of the 3rd harmonics is x times that of the fundamental modulation, we have the normalized modulation function as

$$P(t) = \frac{8/\pi}{1+x/3}\left(\sin\omega_{\rm m}t + x\sin 3\omega_{\rm m}t\right), \tag{9}$$

then

$$S^{\rm add} = \frac{3\pi^2}{8}\frac{1+x^2}{(3+x)^2} - 1 \geq 0.1103 \ (x=1/3). \tag{10}$$

Sensitivity to NS-NS binaries becomes SN=10 at 190Mpc.

In the case of GEO, RF sidebands are unbalanced. Let us assume the upper sideband has an amplitude of X and the lower sideband has Y. Normalized with Eqs. (2) and (6), modulation functions are

$$A(t) = \frac{8}{\pi}\frac{X-Y}{X+Y}\cos\omega_{\rm m}t, \quad P(t) = \frac{8}{\pi}\sin\omega_{\rm m}t, \tag{11}$$

or

$$A_{\pm 1} = \frac{4}{\pi}\frac{X-Y}{X+Y}, \quad P_{\pm 1} = \frac{4}{\pi}i, \tag{12}$$

then

$$S^{\text{add}} = \frac{1}{2}\frac{X^2+Y^2}{(X+Y)^2}\Sigma_{k=0}^{\infty}\left(\frac{1}{2k+1}\right)^2 + \frac{XY}{(X+Y)^2}\left[-1+\Sigma_{k=0}^{\infty}\frac{2}{(2k+1)(2k+3)}\right] - 1$$
$$= \frac{\pi^2}{4}\frac{X^2+Y^2}{(X+Y)^2} - 1. \qquad (13)$$

One can see Eq. (13) becomes Eq. (8) with the balanced sidebands; $X = Y$. If the sidebands are totally unbalanced; $X = 1$ and $Y = 0$, it becomes

$$S^{\text{add}} = \frac{\pi^2}{4} - 1 \simeq 1.4674. \qquad (14)$$

Reference. [2] shows more things, which are how to optimize the demodulation function with each quadrature, there is a standard quantum limit for the optimized heterodyne detection, and so on. In fact, what we need now will be not such things but a shot-noise level with a fixed demodulation function. Equation (14) gives the answer. Non-stationary shot noise increases the noise level by 2.5 compared with the pure shot-noise level. It should better be adjusted with a bandwidth of the demodulator currently used and with a actual imbalance factor: $(X-Y)/(X+Y)$. At last, Fig. 1 will help intuitive understanding of non-stationary shot noise and its reduction by the harmonics demodulation.

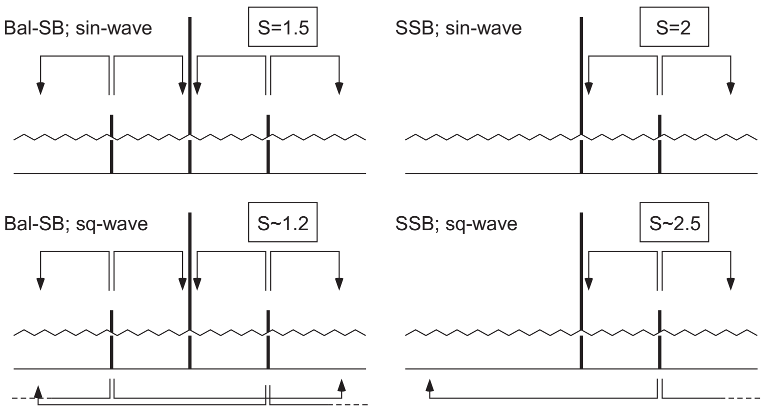

Figure 1: With the balanced phase modulation, square-wave demodulation reduces non-stationary shot noise due to a redundant measurement of vacuum by the upper and lower sidebands, but with the single sideband, square-wave demodulation increases the noise level.

References

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